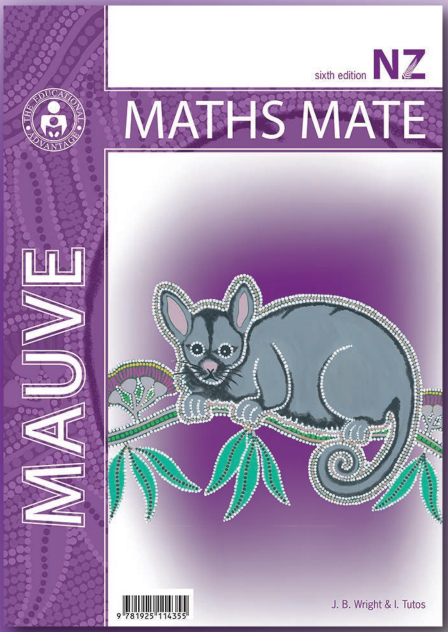
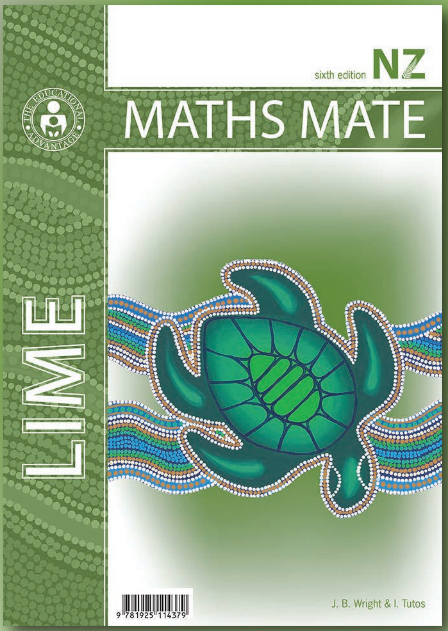
SKILL BUILDER

sixth edition NZ

MATHS MATE





J. B. Wright & I. Tutos

MATHS MATE

Skill Builder
Mauve/Lime

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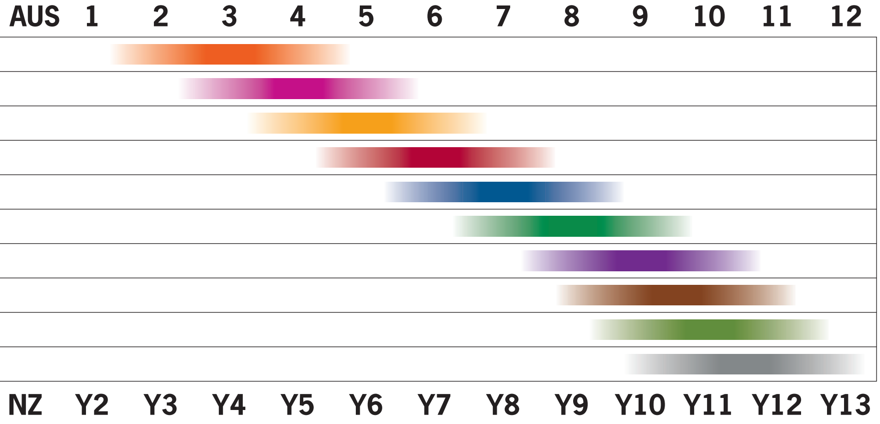
Maths Mate materials available for use

STUDENT PADS

GRADE / YEAR LEVEL INDICATOR

Maths Mate Orange	Student Pad - 2nd Ed.
Maths Mate Rose	Student Pad - 2nd Ed.
Maths Mate Yellow	Student Pad - 5th Ed.
Maths Mate Red	Student Pad - 5th Ed.
Maths Mate Blue	Student Pad - 6th Ed.
Maths Mate Green	Student Pad - 6th Ed.
Maths Mate Mauve	Student Pad - 6th Ed.
Maths Mate Coffee	Student Pad - 3rd Ed.
Maths Mate Lime	Student Pad - 6th Ed.
Maths Mate Silver	Student Pad - 3rd Ed.



TEACHER RESOURCES

Maths Mate Teacher Resources (for all year levels, PDF format)

SKILL BUILDERS

Maths Mate Skill Builder Orange/Rose	PDF format
Maths Mate Skill Builder Yellow/Red	PDF format
Maths Mate Skill Builder Blue/Green	PDF format
Maths Mate Skill Builder Mauve/Lime	PDF format

TEACHER'S GUIDE

FORWARD

Why use Skill Builders?

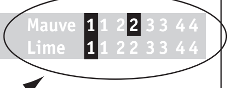
Too often, through the teaching, learning and assessment process, teachers identify weaknesses and gaps in student learning but the constraints of the classroom severely limit remediation opportunities.

The Maths Mate Skill Builder series was prepared in response to requests from teachers and parents who want an easy but effective way to help students who identify skill deficiencies using the Maths Mate Program, and are motivated to do something about them.

The Maths Mate record keeping sheets found at the start of each term in each Student Pad (and on the Teacher Resources ~ Record Keeping Sheets, pages 1 to 4) enable students to find out what they know and what they still need to learn and practise.

The Skill Builders extensively target through instruction and practice, all skills within the related Maths Mate Program except the problem solving questions. The Problem Solving Hints & Solutions (see Teacher Resources ~ Problem Solving Hints & Solutions) can be used by teachers to develop students' problem solving skills. The Skill Builders also contain a Glossary of important facts and reference material that will provide instant help when students present with difficulties.

Background to the design of Maths Mate and Skill Builders



Any question on the Maths Mate sheets is part of a set of 4 similar questions in the term. For example, consider sheets 1, 2, 3 and 4 in Maths Mate Mauve term 1. Question 10 on each sheet is similar in design, content and degree of difficulty. This grouping of question style is also true of the next set of four sheets and so on. Thus the Maths Mate tests made available in the Teacher Resources (see Teacher Resources ~ Test Masters, pages 1 to 32 and Test Answers, pages 1 to 32) also reflect this grouping of question style and substance. Generally too, the Skill Builders can be linked to each set of 4 similar questions. These links are identified in the grid at the title of each skill. The grid shown here for example, would relate a skill to questions in the first 4 sheets of MM Mauve term 1, the last 4 sheets of MM Mauve term 2 and the first 4 sheets of MM Lime term 1. Once understood, these links will be helpful to students in their selection of Skill Builders and to you in your allocation of Skill Builders to students.

On each Maths Mate worksheet, questions 1 through to 32 get progressively harder. (Refer - How to use the Skill Builders, page iv)

Suggestions for the preparation and organisation of Skill Builders

Teachers can either direct students to their digital copies or print copies of particular pages for students. Rather than photocopying Skill Builders one at a time, you may find it helpful to set up a file in a central area that contains perhaps five copies of each Skill Builder. In this way you will save time and be prepared in advance. Students should be reminded that the Glossary is a valuable resource that can be added to. The Glossary too can be photocopied for students as a resource.

How you can help

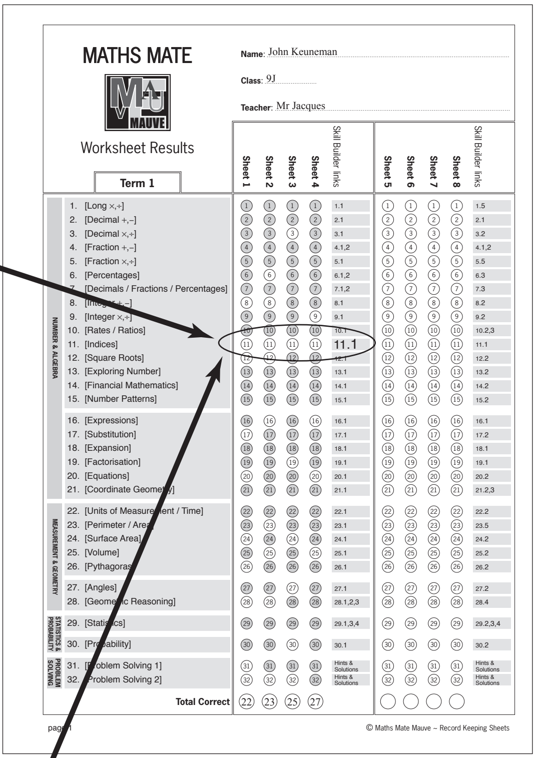
We are confident that your students will be rewarded for the effort you have made in making these worksheets available to them. As with any program, however, there is always room for improvement and we place great value in feedback from people like yourself. Please, if you have any suggestions at all, contact us.

HOW TO USE MATHS MATE SKILL BUILDERS

1. Determine which Maths Mate questions pose a difficulty

If a student gets one or more incorrect answers, represented by one or more successive unshaded boxes on their worksheet results sheet, then that question requires a Skill Builder.

For example, question 11 in Sheets 1, 2, 3 and 4 is not shaded, so Skill 11.1 from Skill Builder 11 needs to be handed to the student.



MATHS MATE

MAUVE

Worksheet Results

Term 1

Name: John Keuneman

Class: 9J

Teacher: Mr Jacques

		Sheet 1	Sheet 2	Sheet 3	Sheet 4	Skill Builder links	Sheet 5	Sheet 6	Sheet 7	Sheet 8	Skill Builder links
NUMBER & ALGEBRA	1. [Long ×,÷]	①	①	①	①	1.1	①	①	①	①	1.5
	2. [Decimal +,−]	②	②	②	②	2.1	②	②	②	②	2.1
	3. [Decimal ×,÷]	③	③	③	③	3.1	③	③	③	③	3.2
	4. [Fraction +,−]	④	④	④	④	4.1,2	④	④	④	④	4.1,2
	5. [Fraction ×,÷]	⑤	⑤	⑤	⑤	5.1	⑤	⑤	⑤	⑤	5.5
	6. [Percentages]	⑥	⑥	⑥	⑥	6.1,2	⑥	⑥	⑥	⑥	6.3
	7. [Decimals / Fractions / Percentages]	⑦	⑦	⑦	⑦	7.1,2	⑦	⑦	⑦	⑦	7.3
	8. [Integer +,−]	⑧	⑧	⑧	⑧	8.1	⑧	⑧	⑧	⑧	8.2
	9. [Integer ×,÷]	⑨	⑨	⑨	⑨	9.1	⑨	⑨	⑨	⑨	9.2
	10. [Rates / Ratios]	⑩	⑩	⑩	⑩	10.1	⑩	⑩	⑩	⑩	10.2,3
	11. [Indices]	⑪	⑪	⑪	⑪	11.1	⑪	⑪	⑪	⑪	11.1
	12. [Square Roots]	⑫	⑫	⑫	⑫	12.1	⑫	⑫	⑫	⑫	12.2
	13. [Exploring Number]	⑬	⑬	⑬	⑬	13.1	⑬	⑬	⑬	⑬	13.2
	14. [Financial Mathematics]	⑭	⑭	⑭	⑭	14.1	⑭	⑭	⑭	⑭	14.2
	15. [Number Patterns]	⑮	⑮	⑮	⑮	15.1	⑮	⑮	⑮	⑮	15.2
	16. [Expressions]	⑯	⑯	⑯	⑯	16.1	⑯	⑯	⑯	⑯	16.1
	17. [Substitution]	⑰	⑰	⑰	⑰	17.1	⑰	⑰	⑰	⑰	17.2
	18. [Expansion]	⑱	⑱	⑱	⑱	18.1	⑱	⑱	⑱	⑱	18.1
	19. [Factorisation]	⑲	⑲	⑲	⑲	19.1	⑲	⑲	⑲	⑲	19.1
	20. [Equations]	⑳	⑳	⑳	⑳	20.1	⑳	⑳	⑳	⑳	20.2
	21. [Coordinate Geometry]	㉑	㉑	㉑	㉑	21.1	㉑	㉑	㉑	㉑	21.2,3
MEASUREMENT & GEOMETRY	22. [Units of Measurement / Time]	㉒	㉒	㉒	㉒	22.1	㉒	㉒	㉒	㉒	22.2
	23. [Perimeter / Area]	㉓	㉓	㉓	㉓	23.1	㉓	㉓	㉓	㉓	23.5
	24. [Surface Area]	㉔	㉔	㉔	㉔	24.1	㉔	㉔	㉔	㉔	24.2
	25. [Volume]	㉕	㉕	㉕	㉕	25.1	㉕	㉕	㉕	㉕	25.2
	26. [Pythagoras]	㉖	㉖	㉖	㉖	26.1	㉖	㉖	㉖	㉖	26.2
	27. [Angles]	㉗	㉗	㉗	㉗	27.1	㉗	㉗	㉗	㉗	27.2
	28. [Geometric Reasoning]	㉘	㉘	㉘	㉘	28.1,2,3	㉘	㉘	㉘	㉘	28.4
STATISTICS & PROBABILITY	29. [Statistics]	㉙	㉙	㉙	㉙	29.1,3,4	㉙	㉙	㉙	㉙	29.2,3,4
	30. [Probability]	㉚	㉚	㉚	㉚	30.1	㉚	㉚	㉚	㉚	30.2
PROBLEM SOLVING	31. [Problem Solving 1]	㉛	㉛	㉛	㉛	Hints & Solutions	㉛	㉛	㉛	㉛	Hints & Solutions
	32. [Problem Solving 2]	㉜	㉜	㉜	㉜	Hints & Solutions	㉜	㉜	㉜	㉜	Hints & Solutions
	Total Correct	22	23	25	27		○	○	○	○	

page 1

© Maths Mate Mauve ~ Record Keeping Sheets

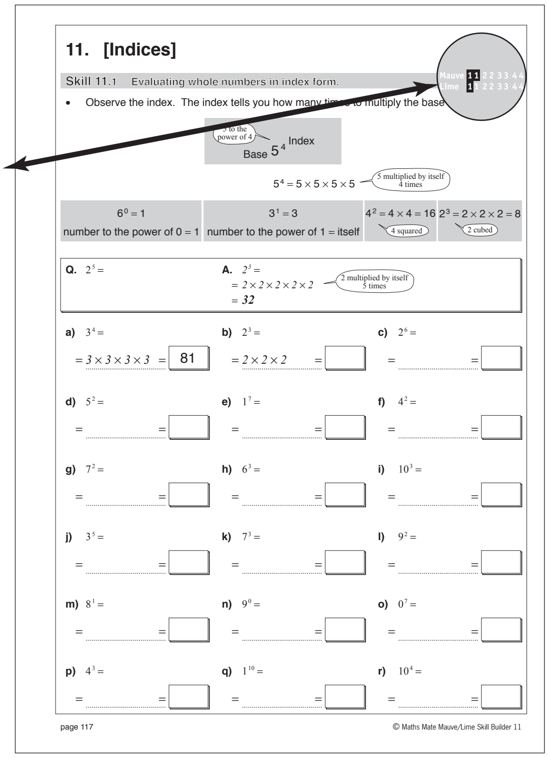
2. Find the relevant Skill Builder on the Maths Mate worksheet results sheet

Check across the question that is posing difficulties on the worksheet results sheet to find the list of skills within the Skill Builder that are most relevant to that question.

Obtain a copy of one or all of the skills listed for that question (pages 1 to 384). You can also double check with the grid at the right of each skill title, that the chosen skill is appropriate.

Remember, students should work through the skills in order. The skills where possible are arranged in increasing degree of difficulty.

Be aware that some skills may require the knowledge of previous skills, so when a student has several areas of weakness, they should work on the lowest numbered skill builders first. For example, a student struggling with Q4 and Q7 may need to build skills required for Q4 before they can improve Q7.



11. [Indices]

Skill 11.1 Evaluating whole numbers in index form.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Observe the index. The index tells you how many times to multiply the base

5 to the power of 4 — Base 5^4 Index

$5^4 = 5 \times 5 \times 5 \times 5$ — 5 multiplied by itself 4 times

$6^0 = 1$ number to the power of 0 = 1	$3^1 = 3$ number to the power of 1 = itself	$4^2 = 4 \times 4 = 16$ 4 squared	$2^3 = 2 \times 2 \times 2 = 8$ 2 cubed

Q. $2^5 =$

A. $2^5 =$
$= 2 \times 2 \times 2 \times 2 \times 2$ — 2 multiplied by itself 5 times
$= \mathbf{32}$

a) $3^4 =$ $= 3 \times 3 \times 3 \times 3 = \boxed{81}$

b) $2^3 =$ $= 2 \times 2 \times 2 = \boxed{}$

c) $2^6 =$ $= \ldots = \boxed{}$

d) $5^2 =$ $= \ldots = \boxed{}$

e) $1^7 =$ $= \ldots = \boxed{}$

f) $4^2 =$ $= \ldots = \boxed{}$

g) $7^2 =$ $= \ldots = \boxed{}$

h) $6^3 =$ $= \ldots = \boxed{}$

i) $10^3 =$ $= \ldots = \boxed{}$

j) $3^5 =$ $= \ldots = \boxed{}$

k) $7^3 =$ $= \ldots = \boxed{}$

l) $9^2 =$ $= \ldots = \boxed{}$

m) $8^1 =$ $= \ldots = \boxed{}$

n) $9^0 =$ $= \ldots = \boxed{}$

o) $0^7 =$ $= \ldots = \boxed{}$

p) $4^3 =$ $= \ldots = \boxed{}$

q) $1^{10} =$ $= \ldots = \boxed{}$

r) $10^4 =$ $= \ldots = \boxed{}$

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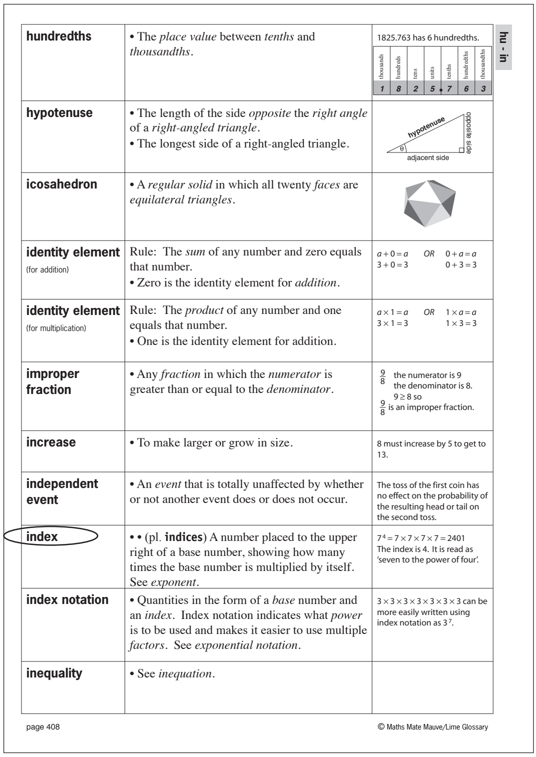
© Maths Mate Mauve/Lime Skill Builder 11

3. Look up any unknown terms in the Skill Builder glossary

The glossary (pages 385 to 450) is more than just a list of definitions. It contains a wealth of relevant information that may help the students to better understand the question at hand. Weaker students may find that referring to a copy of the glossary, and even building on it, is a helpful strategy for improving their overall mathematical competency.

For example, a student might need to look up the word "index" before attempting to complete Skill 11.1



hu - in

hundredths	• The *place value* between *tenths* and *thousandths*.	1825.763 has 6 hundredths. thousands 1, hundreds 8, tens 2, units 5 • tenths 7, hundredths 6, thousandths 3
hypotenuse	• The length of the side *opposite* the *right angle* of a *right-angled triangle*. • The longest side of a right-angled triangle.	hypotenuse opposite side adjacent side θ
icosahedron	• A *regular solid* in which all twenty *faces* are *equilateral triangles*.	
identity element (for addition)	Rule: The *sum* of any number and zero equals that number. • Zero is the identity element for *addition*.	$a + 0 = a$ *OR* $0 + a = a$ $3 + 0 = 3$ $0 + 3 = 3$
identity element (for multiplication)	Rule: The *product* of any number and one equals that number. • One is the identity element for addition.	$a \times 1 = a$ *OR* $1 \times a = a$ $3 \times 1 = 3$ $1 \times 3 = 3$
improper fraction	• Any *fraction* in which the *numerator* is greater than or equal to the *denominator*.	$\frac{9}{8}$ the numerator is 9 the denominator is 8. $9 \geq 8$ so $\frac{9}{8}$ is an improper fraction.
increase	• To make larger or grow in size.	8 must increase by 5 to get to 13.
independent event	• An *event* that is totally unaffected by whether or not another event does or does not occur.	The toss of the first coin has no effect on the probability of the resulting head or tail on the second toss.
index	• • (pl. **indices**) A number placed to the upper right of a base number, showing how many times the base number is multiplied by itself. See *exponent*.	$7^4 = 7 \times 7 \times 7 \times 7 = 2401$ The index is 4. It is read as 'seven to the power of four'.
index notation	• Quantities in the form of a *base* number and an *index*. Index notation indicates what *power* is to be used and makes it easier to use multiple *factors*. See *exponential notation*.	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ can be more easily written using index notation as 3^7.
inequality	• See *inequation*.	

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© Maths Mate Mauve/Lime Glossary

4. Complete the relevant Skill Builder

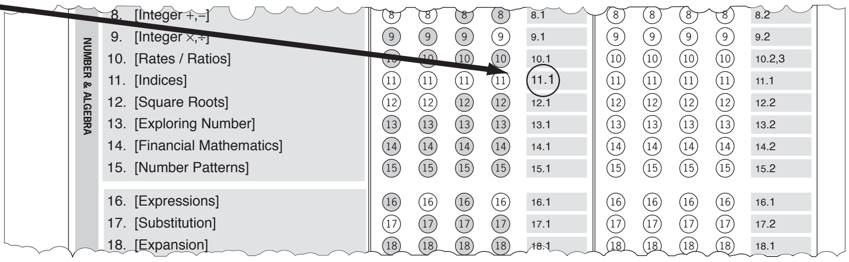
Work through the examples given for that skill, and complete the exercises.

There are many techniques or methods that can be used to teach the same basic skills, even something as simple as adding 7 and 9. It is good for a student to be given a range of alternatives appropriate for each skill but space restrictions make this impossible. These sheets often suggest an approach that may be different to a student's past experience. If a student feels more comfortable with his current technique, that is fine. In most cases it is the end result that counts.

It is possible to take a very weak student back to a Skill Builder from a lower level if this is necessary. It is also possible to use a higher level book for students to have further practice if required.

5. Correct the relevant Skill Builders from the Skill Builder answer sheets (from page 461)

6. Circle the completed skill numbers on the Maths Mate worksheet results sheet



NUMBER & ALGEBRA

8. [Integer +,−]	⑧	⑧	⑧	⑧	8.1	⑧	⑧	⑧	⑧	8.2
9. [Integer ×,÷]	⑨	⑨	⑨	⑨	9.1	⑨	⑨	⑨	⑨	9.2
10. [Rates / Ratios]	⑩	⑩	⑩	⑩	10.1	⑩	⑩	⑩	⑩	10.2,3
11. [Indices]	⑪	⑪	⑪	⑪	11.1	⑪	⑪	⑪	⑪	11.1
12. [Square Roots]	⑫	⑫	⑫	⑫	12.1	⑫	⑫	⑫	⑫	12.2
13. [Exploring Number]	⑬	⑬	⑬	⑬	13.1	⑬	⑬	⑬	⑬	13.2
14. [Financial Mathematics]	⑭	⑭	⑭	⑭	14.1	⑭	⑭	⑭	⑭	14.2
15. [Number Patterns]	⑮	⑮	⑮	⑮	15.1	⑮	⑮	⑮	⑮	15.2
16. [Expressions]	⑯	⑯	⑯	⑯	16.1	⑯	⑯	⑯	⑯	16.1
17. [Substitution]	⑰	⑰	⑰	⑰	17.1	⑰	⑰	⑰	⑰	17.2
18. [Expansion]	⑱	⑱	⑱	⑱	18.1	⑱	⑱	⑱	⑱	18.1

7. Go back and repeat previous Maths Mate questions

After completing a Skill Builder, students should be encouraged to go back and attempt again those particular questions on the recently completed Maths Mate worksheets.

Dear Parents

As part of their Mathematics program this year, all students have been given a weekly Maths Mate worksheet.

The program is now under way. The diagnostic nature of the worksheets helps students monitor their own progress. After they correct their worksheet and complete the record keeping sheet, over time, your child will be able to identify areas of strength and weakness in their mathematical learning.

If your child is having difficulty with a question for consecutive weeks or believes that their understanding is not at the level they would like, then Skill Builder sheets will be made available to develop each of the skills in the Maths Mate program. Each Skill Builder focuses on and explores one question from the Maths Mate worksheets.

As each question in the Maths Mate is generally more difficult than the last, finishing with the problem solving questions, then it would be advised that, if students are concerned with more than one question, they tackle lower numbered questions first.

The Skill Builders may also help to motivate students to make another attempt at mastering skills that they have found too difficult in the past, given that it will become clear to them that they will be confronted by the same type of question on a regular basis.

While we will be monitoring your child's progress and supporting their skill development in the school environment, it would be appreciated if you would complete the tear off slip at the bottom of this page so that we can be sure that you are aware of our expectations regarding both the Maths Mate worksheets and the availability of Skill Builder worksheets. We ask also that you continue to sign the completed worksheets each week so that we can ensure each student is working independently and regularly but with your support.

We thank you in anticipation of your involvement and remind you that you are encouraged to call and discuss your child's progress at any time.

Yours sincerely

Class Teacher

Principal

✂- -

Maths Mate Program - Skill Builder Return Slip

Student's Name: . . . . Class: . . . .

As a parent / guardian I have signed this form to indicate that I am aware of the support Maths Mate Skill Builders can give my child in their mathematical development.

Parent's Signature: . . . . Date: . . . .

CONTENTS

MM Question | SB Skill No. | [Maths Mate - Mathematical strand] Skill Builder - Skill description

1. [Long ×,÷]

Skill 1.1 Multiplying a large number by a multiple of 10.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Consider the zeros as making groups of 10s or 100s and place them at the end.
- Then multiply by the remaining digit as though it was a unit.

Q. $589 \times 700 =$

A. ***412 300***

$$\begin{array}{rrrrrrr} & & {}^{6}5 & {}^{6}8 & 9 & & \\ \times & & & & 7 & 0 & 0 \\ \hline & 4 & 1 & 2 & 3 & 0 & 0 \\ \hline \end{array}$$

Consider 700 as 7 groups of 100.

Multiply 589 by 7.

To show we want groups of 100, place two zeros after the 4123.

a) $67 \times 40 =$ 2680

$$\begin{array}{rrrr} & {}^{2}6 & 7 & \\ \times & & 4 & 0 \\ \hline 2 & 6 & 8 & 0 \\ \hline \end{array}$$

b) $58 \times 90 =$

$$\begin{array}{rrrr} & {}^{7}5 & 8 & \\ \times & & 9 & 0 \\ \hline & & 2 & 0 \\ \hline \end{array}$$

c) $74 \times 60 =$

$$\begin{array}{rrrr} & 7 & 4 & \\ \times & & 6 & 0 \\ \hline & & & 0 \\ \hline \end{array}$$

d) $89 \times 70 =$

$$\begin{array}{rrrr} & 8 & 9 & \\ \times & & 7 & 0 \\ \hline \\ \hline \end{array}$$

e) $483 \times 50 =$

$$\begin{array}{rrrrr} & 4 & 8 & 3 & \\ \times & & & 5 & 0 \\ \hline \\ \hline \end{array}$$

f) $790 \times 80 =$

$$\begin{array}{rrrrr} & 7 & 9 & 0 & \\ \times & & & 8 & 0 \\ \hline \\ \hline \end{array}$$

g) $890 \times 200 =$

$$\begin{array}{rrrrrr} & {}^{1}8 & 9 & 0 & & \\ \times & & & 2 & 0 & 0 \\ \hline 1 & 7 & 8 & 0 & 0 & 0 \\ \hline \end{array}$$

h) $647 \times 400 =$

$$\begin{array}{rrrrrr} & 6 & 4 & 7 & & \\ \times & & & 4 & 0 & 0 \\ \hline \\ \hline \end{array}$$

i) $2596 \times 200 =$

$$\begin{array}{rrrrrrr} & 2 & 5 & 9 & 6 & & \\ \times & & & & 2 & 0 & 0 \\ \hline \\ \hline \end{array}$$

j) $1516 \times 300 =$

k) $310 \times 2000 =$

l) $475 \times 2000 =$

Skill 1.2 Multiplying a large number by a two-digit number (1).

- Multiply by the units first.
- Then multiply by the tens.

Reminder: Put a zero in the units place before you start multiplying by the tens.

Q. 564 × 18 =

A. ***10152***

$$\begin{array}{rrrrrr} & & & \overset{5}{5} & \overset{3}{6} & 4 \\ & \times & & & 1 & 8 \\ \hline + & & \overset{1}{4} & 5 & 1 & 2 \\ & & 5 & 6 & 4 & 0 \\ \hline & 1 & 0 & 1 & 5 & 2 \\ \hline \end{array}$$

Multiply 564 by 8.

Then multiply 564 by 10.
Remember: Put a 0 in the units place.

Add the results.

This question can be thought of as:

$$\begin{array}{rrrr} & \overset{5}{5} & \overset{3}{6} & 4 \\ \times & & & 8 \\ \hline 4 & 5 & 1 & 2 \\ \hline \end{array} \quad \overset{\textit{plus}}{+} \quad \begin{array}{rrrr} & 5 & 6 & 4 \\ \times & & 1 & 0 \\ \hline 5 & 6 & 4 & 0 \\ \hline \end{array} = 10\,152$$

a) 19 × 15 = 285

$$\begin{array}{rrrr} & & \overset{4}{1} & 9 \\ & \times & 1 & 5 \\ \hline + & & 9 & 5 \\ & {}^{1}1 & 9 & 0 \\ \hline & 2 & 8 & 5 \\ \hline \end{array}$$

b) 27 × 13 =

$$\begin{array}{rrrr} & & \overset{2}{2} & 7 \\ & \times & 1 & 3 \\ \hline + & & 8 & 1 \\ & & & \\ \hline & & & \\ \hline \end{array}$$

c) 34 × 18 =

$$\begin{array}{rrr} & 3 & 4 \\ \times & 1 & 8 \\ \hline & & \\ & & \\ \hline & & \\ \hline \end{array}$$

d) 56 × 14 =

$$\begin{array}{rrr} & 5 & 6 \\ \times & 1 & 4 \\ \hline & & \\ & & \\ \hline & & \\ \hline \end{array}$$

e) 274 × 17 =

$$\begin{array}{rrrr} & 2 & 7 & 4 \\ \times & & 1 & 7 \\ \hline & & & \\ & & & \\ \hline & & & \\ \hline \end{array}$$

f) 456 × 19 =

$$\begin{array}{rrrr} & 4 & 5 & 6 \\ \times & & 1 & 9 \\ \hline & & & \\ & & & \\ \hline & & & \\ \hline \end{array}$$

g) 249 × 36 =

$$\begin{array}{rrrr} & 2 & 4 & 9 \\ \times & & 3 & 6 \\ \hline & & & \\ & & & \\ \hline & & & \\ \hline \end{array}$$

h) 237 × 28 =

$$\begin{array}{rrrr} & 2 & 3 & 7 \\ \times & & 2 & 8 \\ \hline & & & \\ & & & \\ \hline & & & \\ \hline \end{array}$$

i) 413 × 56 =

$$\begin{array}{rrrr} & 4 & 1 & 3 \\ \times & & 5 & 6 \\ \hline & & & \\ & & & \\ \hline & & & \\ \hline \end{array}$$

Skill 1.2 Multiplying a large number by a two-digit number (2).

j) $289 \times 47 =$ $\boxed{13583}$

$$\begin{array}{cccccc} & & & {}^{6}2 & {}^{6}8 & 9 \\ & & \times & {}^{3} & {}^{3}4 & 7 \\ \hline & & 2 & 0 & 2 & 3 \\ + & 1 & 1 & 5 & 6 & 0 \\ \hline & 1 & 3 & 5 & 8 & 3 \\ \hline \end{array}$$

k) $873 \times 35 =$

$$\begin{array}{cccc} & 8 & 7 & 3 \\ \times & & 3 & 5 \\ \hline \\ \hline \\ \hline \end{array}$$

l) $456 \times 64 =$

$$\begin{array}{cccc} & 4 & 5 & 6 \\ \times & & 6 & 4 \\ \hline \\ \hline \\ \hline \end{array}$$

m) $516 \times 33 =$

n) $934 \times 78 =$

o) $689 \times 56 =$

p) $2009 \times 96 =$

$$\begin{array}{ccccc} & 2 & 0 & 0 & 9 \\ \times & & & 9 & 6 \\ \hline \\ \hline \\ \hline \end{array}$$

q) $1087 \times 37 =$

$$\begin{array}{ccccc} & 1 & 0 & 8 & 7 \\ \times & & & 3 & 7 \\ \hline \\ \hline \\ \hline \end{array}$$

r) $3265 \times 73 =$

$$\begin{array}{ccccc} & 3 & 2 & 6 & 5 \\ \times & & & 7 & 3 \\ \hline \\ \hline \\ \hline \end{array}$$

s) $1989 \times 43 =$

t) $2701 \times 84 =$

u) $5678 \times 92 =$

Skill 1.3 Multiplying a large number by a large multiple of 10.

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 **3** 3 4 4

- Consider the zeros as making groups of 10's or 100's and place them at the end.
- Multiply by the units first, then by the tens.

Q. 382 × 230 =

A. ***87860***

$$\begin{array}{cccccc} & & \overset{2}{3} & 8 & 2 & \\ \times & & {}^{1} & 2 & 3 & 0 \\ \hline + & 1 & 1 & 4 & 6 & \\ & 7 & 6 & 4 & 0 & \\ \hline & 8 & 7 & 8 & 6 & 0 \\ \hline \end{array}$$

Consider 230 as 23 groups of 10.

Work with the 23 first.
Multiply 382 by 3.
Then multiply 382 by 20.
Add these results.

To show we want groups of 10, place a 0 after the 8786.

a) 358 × 130 = $\boxed{46540}$

$$\begin{array}{cccccc} & & \overset{1}{3} & \overset{2}{5} & 8 & \\ \times & & & 1 & 3 & 0 \\ \hline + & 1 & \overset{1}{0} & 7 & 4 & \\ & 3 & 5 & 8 & 0 & \\ \hline & 4 & 6 & 5 & 4 & 0 \\ \hline \end{array}$$

b) 469 × 210 = $\boxed{}$

$$\begin{array}{cccccc} & & 4 & 6 & 9 & \\ \times & & & 2 & 1 & 0 \\ \hline + & & & & & \\ & & & & 0 & \\ \hline & & & & & 0 \\ \hline \end{array}$$

c) 325 × 180 = $\boxed{}$

$$\begin{array}{cccccc} & & 3 & 2 & 5 & \\ \times & & & 1 & 8 & 0 \\ \hline & & & & & \\ \hline & & & & & \\ \hline \end{array}$$

d) 637 × 140 = $\boxed{}$

$$\begin{array}{cccccc} & & 6 & 3 & 7 & \\ \times & & & 1 & 4 & 0 \\ \hline & & & & & \\ \hline & & & & & \\ \hline \end{array}$$

e) 428 × 230 = $\boxed{}$

$$\begin{array}{cccccc} & & 4 & 2 & 8 & \\ \times & & & 2 & 3 & 0 \\ \hline & & & & & \\ \hline & & & & & \\ \hline \end{array}$$

f) 1865 × 390 = $\boxed{}$

$$\begin{array}{ccccccc} & & 1 & 8 & 6 & 5 & \\ \times & & & & 3 & 9 & 0 \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array}$$

g) 2904 × 420 = $\boxed{}$

$$\begin{array}{ccccccc} & & 2 & 9 & 0 & 4 & \\ \times & & & & 4 & 2 & 0 \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array}$$

h) 263 × 1500 = $\boxed{}$

$$\begin{array}{ccccccc} & & 2 & 6 & 3 & & \\ \times & & 1 & 5 & 0 & 0 & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array}$$

i) 457 × 1800 = $\boxed{}$

$$\begin{array}{ccccccc} & & 4 & 5 & 7 & & \\ \times & & 1 & 8 & 0 & 0 & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array}$$

Skill 1.4 Dividing a large number by a single digit.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Break down the division into smaller divisions.
- Work from left to right.

Q. 2835 ÷ 7 =

A. *405*

$$\begin{array}{r} 4\;0\;5 \\ 7\overline{)\cancel{2}\;{}^{2}8\;\cancel{3}\;{}^{3}5} \end{array}$$

Starting at the left, divide 7 into 2. 7 does not divide into 2 at least once so 'carry over' the 2 groups of 1000 and make 28 groups of 100.

7 divides into 28 four times and 0 remainder. Write a 4 above the 8.

Then divide 7 into 3. 7 does not divide into 3 at least once so 'carry over' the 3 groups of 10 and make 35 groups of 1. Write a 0 above the 3.

7 divides into 35 five times and 0 remainder. Write a 5 above the 5.

a) 756 ÷ 9 = 84

$$\begin{array}{r} 8\;4 \\ 9\overline{)\cancel{7}\;{}^{7}5\;{}^{3}6} \end{array}$$

b) 136 ÷ 8 =

$$\begin{array}{r} 1 \\ 8\overline{)\cancel{1}\;{}^{1}3\;6} \end{array}$$

c) 390 ÷ 6 =

$$6\overline{)\cancel{3}\;9\;0}$$

d) 496 ÷ 4 =

$$4\overline{)4\;9\;6}$$

e) 792 ÷ 3 =

$$3\overline{)7\;9\;2}$$

f) 854 ÷ 7 =

$$7\overline{)8\;5\;4}$$

g) 3324 ÷ 4 = 831

$$\begin{array}{r} 8\;3\;1 \\ 4\overline{)\cancel{3}\;{}^{3}3\;{}^{1}2\;4} \end{array}$$

h) 1491 ÷ 3 =

$$3\overline{)1\;4\;9\;1}$$

i) 4135 ÷ 5 =

$$5\overline{)4\;1\;3\;5}$$

j) 2384 ÷ 4 =

$$4\overline{)2\;3\;8\;4}$$

k) 5670 ÷ 6 =

$$6\overline{)5\;6\;7\;0}$$

l) 4383 ÷ 9 =

$$9\overline{)4\;3\;8\;3}$$

m) 6013 ÷ 7 =

$$7\overline{)6\;0\;1\;3}$$

n) 8560 ÷ 5 =

$$5\overline{)8\;5\;6\;0}$$

o) 9048 ÷ 8 =

$$8\overline{)9\;0\;4\;8}$$

Skill 1.5 Dividing a large number by a power of 10.

EITHER
When the whole number ends in the same number of zeros or more zeros than the power of 10:

- Take off as many zeros in the whole number as there are zeros in the power of 10.
  Example: $54\,00\cancel{0} \div 1\cancel{0} = 5400$
  $54\,0\cancel{0}\cancel{0} \div 1\cancel{0}\cancel{0} = 540$
  $54\,\cancel{0}\cancel{0}\cancel{0} \div 1\cancel{0}\cancel{0}\cancel{0} = 54$

OR
When the whole number ends in less zeros than the power of 10:

- Move the decimal place to the left as many places as there are zeros in the power of 10.
  Example: $3070 \div 100 = 30.\overset{\frown}{7}\overset{\frown}{\cancel{0}} = 30.7$
  Hints: Any zero at the end of the number and to the right of the decimal point can be removed.
  A decimal point would be at the end of a whole number but is not written by convention, e.g. $3070 = 3070.0$

Q. $48\,670 \div 1000 =$

A. $48\,670 \div 1000$
$= 48670.0 \div 1000$
$= 48.\overset{\frown}{6}\overset{\frown}{7}\overset{\frown}{\cancel{0}}$
$= \mathbf{48.67}$

There are 3 zeros in 1000 so move the decimal point 3 places to the left.

The zero on the right can be removed.

a) $12\,000 \div 100 =$
$= 12\,0\cancel{0}\cancel{0} \div 1\cancel{0}\cancel{0} =$ $\boxed{120}$

b) $15\,000 \div 10 =$
= .... = ☐

c) $13\,500 \div 10 =$
= .... = ☐

d) $98\,200 \div 100 =$
= .... = ☐

e) $3200 \div 100 =$
= .... = ☐

f) $80\,000 \div 100 =$
= .... = ☐

g) $543 \div 10 =$
$= 54.\overset{\frown}{3}$ = $\boxed{54.3}$

h) $278 \div 10 =$
= .... = ☐

i) $5466 \div 10 =$
= .... = ☐

j) $6450 \div 100 =$
= .... = ☐

k) $43\,070 \div 100 =$
= .... = ☐

l) $5507 \div 100 =$
= .... = ☐

m) $19\,034 \div 100 =$
= .... = ☐

n) $23\,790 \div 1000 =$
= .... = ☐

o) $42\,210 \div 1000 =$
= .... = ☐

Skill 1.6 Dividing a large number by a multiple of 10.

- If both the dividend and the divisor end in 0 or 00 then divide both numbers by 10 or 100 to remove both zero endings.
- Then divide by the remaining single digit.

Q. $34\,780 \div 20 =$

A. $34\,780 \div 20$

$= 34\,78\cancel{0} \div 2\cancel{0}$

$= \mathbf{1739}$

$$\begin{array}{l} \quad 1\;7\;3\;9 \\ 2\overline{)3\,{}^{1}4\;7\,{}^{1}8} \end{array}$$

Divide both numbers by 10 to remove the zeros.

Then complete the division.

a) $2460 \div 30 =$

$= 246\cancel{0} \div 3\cancel{0} = \boxed{82}$

$$\begin{array}{l} \quad 8\;2 \\ 3\overline{)\cancel{2}\,{}^{2}4\;6} \end{array}$$

b) $1760 \div 20 =$

$= 176\cancel{0} \div 2\cancel{0} = \boxed{}$

$$2\overline{)\cancel{1}\,{}^{1}7\;6}$$

c) $6950 \div 50 =$

$= \dots\dots = \boxed{}$

$$5\overline{)6\;9\;5}$$

d) $5480 \div 40 =$

$= \dots\dots = \boxed{}$

$$4\overline{)}$$

e) $9660 \div 70 =$

$= \dots\dots = \boxed{}$

$$7\overline{)}$$

f) $8220 \div 30 =$

$= \dots\dots = \boxed{}$

g) $39\,120 \div 40 =$

$= 39\,12\cancel{0} \div 4\cancel{0} = \boxed{978}$

$$\begin{array}{l} \quad 9\;7\;8 \\ 4\overline{)\cancel{3}\,{}^{3}9\,{}^{3}1\,{}^{3}2} \end{array}$$

h) $75\,980 \div 20 =$

$= \dots\dots = \boxed{}$

$$2\overline{)7\;5\;9\;8}$$

i) $37\,550 \div 50 =$

$= \dots\dots = \boxed{}$

$$5\overline{)}$$

j) $21\,420 \div 60 =$

$= \dots\dots = \boxed{}$

k) $50\,080 \div 80 =$

$= \dots\dots = \boxed{}$

l) $52\,380 \div 90 =$

$= \dots\dots = \boxed{}$

m) $137\,700 \div 300 =$

$= \dots\dots = \boxed{}$

n) $450\,400 \div 800 =$

$= \dots\dots = \boxed{}$

o) $142\,200 \div 600 =$

$= \dots\dots = \boxed{}$

Skill 1.7 Dividing a whole number by a two-digit number (1).

- Work from left to right.
- Break down the division into smaller divisions by dividing into only as many digits as you need to get an answer greater than 1.
- It may be difficult, so guess the number of divisions and multiply your guess to check.
- Subtract your answer from the original number to get the remainder, which must be less than the number you are dividing by.
- Continue in this way by bringing down the next digit to make the next number to divide into.
- Repeat until the result of the subtraction is zero.

Q. $1026 \div 19 =$

A. *54*

$$\begin{array}{rrrrr}
 & & & 5 & 4 \\ \cline{2-5}
19\big) & 1 & 0 & 2 & 6 \\
- & & 9 & 5 & \downarrow \\ \cline{2-5}
 & & & 7 & 6 \\
- & & & 7 & 6
\end{array}$$

Start at the left.
1 and 10 are too small to divide 19 into and get a result greater than 1.
Divide 102 ÷ 19 = ?
19 is nearly 20 so 5 is a good guess.
Check by multiplying 5 × 19 = 95
Subtract 102 – 95 = 7
Write 5 above the 2.

Bring down the 6
Divide 76 ÷ 19 = ? (Guess 4)
Check by multiplying 4 × 19 = 76
Subtract 76 – 76 = 0 (No remainder)
Write 4 above the 6.

$$\begin{array}{rrrrr}
 & & & 5 & 4 \\ \cline{2-5}
19\big) & \cancel{1} & \cancel{0} & {}^{10}2 & {}^{7}6
\end{array}$$

OR Work as a short division.

a) $476 \div 17 =$ $\boxed{28}$

$$\begin{array}{rrrr}
 & & 2 & 8 \\ \cline{2-4}
17\big) & 4 & 7 & 6 \\
- & 3 & 4 & \downarrow \\ \cline{2-4}
 & 1 & 3 & 6 \\
- & 1 & 3 & 6
\end{array}$$

b) $546 \div 13 =$ $\boxed{}$

$$\begin{array}{rrrr}
 & & 4 & \\ \cline{2-4}
13\big) & \cancel{5} & 4 & 6 \\
- & 5 & 2 & \downarrow \\ \cline{2-4}
 & & 2 & \\
- & & &
\end{array}$$

c) $645 \div 15 =$ $\boxed{}$

$$\begin{array}{rrrr}
 & & & \\ \cline{2-4}
15\big) & 6 & 4 & 5 \\
- & & & \\ \cline{2-4}
 & & & \\
- & & &
\end{array}$$

d) $792 \div 12 =$ $\boxed{}$

$$\begin{array}{rrrr}
 & & & \\ \cline{2-4}
12\big) & 7 & 9 & 2 \\
- & & & \\ \cline{2-4}
 & & & \\
- & & &
\end{array}$$

e) $728 \div 14 =$ $\boxed{}$

$$\begin{array}{rrrr}
 & & & \\ \cline{2-4}
14\big) & 7 & 2 & 8 \\
- & & & \\ \cline{2-4}
 & & & \\
- & & &
\end{array}$$

f) $578 \div 17 =$ $\boxed{}$

$$\begin{array}{rrrr}
 & & & \\ \cline{2-4}
17\big) & 5 & 7 & 8 \\
- & & & \\ \cline{2-4}
 & & & \\
- & & &
\end{array}$$

Skill 1.7 Dividing a whole number by a two-digit number (2).

g) 609 ÷ 21 = ☐

$$\begin{array}{r} 29\\ 21\overline{)\,6\;0\;9}\\ -\;4\;2\\ \hline 1\;8\;9\\ -\;1\;8\;9 \end{array}$$

h) 825 ÷ 25 = ☐

$$\begin{array}{r} 3\\ 25\overline{)\,8\;2\;5}\\ -\;\underline{\qquad\quad}\\ - \end{array}$$

i) 504 ÷ 14 = ☐

$$\begin{array}{r} 14\overline{)\,5\;0\;4}\\ -\;\underline{\qquad\quad}\\ - \end{array}$$

j) 432 ÷ 18 = ☐

$$\begin{array}{r} 18\overline{)\,4\;3\;2}\\ -\;\underline{\qquad\quad}\\ - \end{array}$$

k) 848 ÷ 16 = ☐

$$\begin{array}{r} 16\overline{)\,8\;4\;8}\\ -\;\underline{\qquad\quad}\\ - \end{array}$$

l) 814 ÷ 22 = ☐

$$\begin{array}{r} 22\overline{)\,8\;1\;4}\\ -\;\underline{\qquad\quad}\\ - \end{array}$$

m) 8055 ÷ 15 = ☐

$$\begin{array}{r} 15\overline{)\,8\;0\;5\;5}\\ -\;\underline{\qquad\qquad}\\ -\;\underline{\qquad\qquad}\\ - \end{array}$$

n) 1022 ÷ 14 = ☐

$$\begin{array}{r} 14\overline{)\,1\;0\;2\;2}\\ -\;\underline{\qquad\qquad}\\ - \end{array}$$

o) 3870 ÷ 18 = ☐

$$\begin{array}{r} 18\overline{)\,3\;8\;7\;0}\\ -\;\underline{\qquad\qquad}\\ -\;\underline{\qquad\qquad}\\ - \end{array}$$

p) 2686 ÷ 17 = ☐

$$\begin{array}{r} 17\overline{)\,2\;6\;8\;6}\\ -\;\underline{\qquad\qquad}\\ -\;\underline{\qquad\qquad}\\ - \end{array}$$

q) 2337 ÷ 19 = ☐

$$\begin{array}{r} 19\overline{)\,2\;3\;3\;7}\\ -\;\underline{\qquad\qquad}\\ -\;\underline{\qquad\qquad}\\ - \end{array}$$

r) 2608 ÷ 16 = ☐

$$\begin{array}{r} 16\overline{)\,2\;6\;0\;8}\\ -\;\underline{\qquad\qquad}\\ -\;\underline{\qquad\qquad}\\ - \end{array}$$

Skill 1.8 Dividing whole numbers - remainder.

- Place a decimal point and more zeros at the end of the whole number.
- Divide into the whole number and continue until you get an exact division with no remainder.
- Line up the decimal point in your answer.

Q. $3948 \div 8 =$

A. ***493.5***

$$\begin{array}{r} 4\;9\;3\;.\;5 \\ 8\overline{)\cancel{3}\;{}^{3}9\;{}^{7}4\;{}^{2}8\;.\;{}^{4}0} \end{array}$$

Divide 8 into 3948.0

Continue until you get an exact number with no remainder.

a) $639 \div 5 =$ 127.8

$$\begin{array}{r} 1\;2\;7\;.\;8 \\ 5\overline{)6\;{}^{1}3\;{}^{3}9\;.\;{}^{4}0} \end{array}$$

b) $315 \div 6 =$

$$\begin{array}{r} 5\;2\;.\; \\ 6\overline{)\cancel{3}\;{}^{3}1\;{}^{1}5\;.\;{}^{3}0} \end{array}$$

c) $366 \div 5 =$

$$\begin{array}{r} . \; \\ 5\overline{)\cancel{3}\;6\;6\;.\;0} \end{array}$$

d) $1379 \div 2 =$

$$\begin{array}{r} . \; \\ 2\overline{)1\;3\;7\;9\;.\;0} \end{array}$$

e) $4572 \div 8 =$

$$\begin{array}{r} . \; \\ 8\overline{)4\;5\;7\;2\;.\;0} \end{array}$$

f) $835 \div 4 =$

$$\begin{array}{r} . \; \\ 4\overline{)8\;3\;5\;.\;0\;0} \end{array}$$

g) $233 \div 4 =$

$$\begin{array}{r} 5\;8\;.\;2\;5 \\ 4\overline{)\cancel{2}\;{}^{2}3\;{}^{3}3\;.\;{}^{1}0\;{}^{2}0} \end{array}$$

h) $590 \div 8 =$

$$\begin{array}{r} . \; \\ 8\overline{)5\;9\;0\;.\;0\;0} \end{array}$$

i) $2058 \div 12 =$

$$\begin{array}{r} . \; \\ 12\overline{)2\;0\;5\;8\;.\;0} \end{array}$$

j) $1706 \div 20 =$

$$\begin{array}{r} . \; \\ 20\overline{)1\;7\;0\;6\;.\;0} \end{array}$$

–

–

–

k) $5187 \div 15 =$

$$\begin{array}{r} . \; \\ 15\overline{)5\;1\;8\;7\;.\;0} \end{array}$$

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–

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l) $988 \div 16 =$

$$\begin{array}{r} . \; \\ 16\overline{)9\;8\;8\;.\;0\;0} \end{array}$$

–

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Skill 1.9 Dividing whole numbers - recurring remainder.

- Place a decimal point and more zeros at the end of the whole number.
- Divide into the whole number and continue until:
  - The last digit keeps repeating - place a dot above the recurring digit.
  - Two or more digits repeat in a pattern - place a dot above the pattern of recurring digits.
- Line up the decimal point in your answer.

Q. 698 ÷ 6 =

A. *$116.\dot{3}$*

$$\begin{array}{r} 1\;1\;6\,.\,3\;3\;3 \\ 6\,\overline{)\,6\;9\;{}^{3}8\,.\,{}^{2}0\;{}^{2}0\;{}^{2}0} \end{array}$$

Divide 6 into 698.000 using as many zeros as you like.

Continue until you are sure of the pattern of the remainder.

a) 650 ÷ 9 = $72.\dot{2}$

$$\begin{array}{r} 7\;2\,.\,2\;2 \\ 9\,\overline{)\,\cancel{6}\;{}^{6}5\;{}^{2}0\,.\,{}^{2}0\;{}^{2}0} \end{array}$$

b) 445 ÷ 3 =

$$\begin{array}{r} 1\;\;\;\;\;.\;\;\;\; \\ 3\,\overline{)\,4\;{}^{1}4\;5\,.\,0\;0} \end{array}$$

c) 370 ÷ 6 =

$$\begin{array}{r} .\;\;\;\; \\ 6\,\overline{)\,3\;7\;0\,.\,0\;0} \end{array}$$

d) 434 ÷ 3 =

$$\begin{array}{r} . \\ 3\,\overline{)\,4\;3\;4\,.\;\;\;} \end{array}$$

e) 938 ÷ 6 =

$$\begin{array}{r} . \\ 6\,\overline{)\,9\;3\;8\,.\;\;\;} \end{array}$$

f) 962 ÷ 9 =

$$9\,\overline{)\,9\;6\;2\;\;\;\;}$$

g) 5870 ÷ 6 =

$$6\,\overline{)\,5\;8\;7\;0\;\;\;\;}$$

h) 1304 ÷ 9 =

$$9\,\overline{)\,1\;3\;0\;4\;\;\;\;}$$

i) 985 ÷ 11 = $89.\dot{5}\dot{4}$

$$\begin{array}{r} 8\;9\,.\,5\;4\;5\;4 \\ 11\,\overline{)\,\cancel{9}\;{}^{9}8\;{}^{10}5\,.\,{}^{6}0\;{}^{5}0\;{}^{6}0\;{}^{5}0} \end{array}$$

j) 1547 ÷ 21 =

$$21\,\overline{)\,1\;5\;4\;7\;\;\;\;}$$

–

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–

k) 754 ÷ 22 =

$$22\,\overline{)\,7\;5\;4\;\;\;\;}$$

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–

l) 4586 ÷ 15 =

$$15\,\overline{)\,4\;5\;8\;6\;\;\;\;}$$

–

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2. [Decimal +,–]

Skill 2.1 Adding decimal numbers (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Work vertically, lining up the decimal points.
- Add from right to left in the columns.

Q. 3.42 + 0.38 =

A. ***3.80***

$$\begin{array}{r} 3.\overset{1}{4}2 \\ +\ 0.38 \\ \hline \mathbf{3.80} \\ \hline \end{array}$$

2 + 8 = 10 carry 1, write 0
4 + 3 + carry 1 = 8 write 8
3 + 0 = 3 write 3

a) 0.14 + 0.08 = 0.22

$$\begin{array}{r} 0.\overset{1}{1}4 \\ +\ 0.08 \\ \hline 0.22 \\ \hline \end{array}$$

b) 7.5 + 3.8 =

$$\begin{array}{r} \overset{1}{7}.5 \\ +\ 3.8 \\ \hline .3 \\ \hline \end{array}$$

c) 0.65 + 0.27 =

$$\begin{array}{r} 0.65 \\ +\ 0.27 \\ \hline . \quad \\ \hline \end{array}$$

d) 4.34 + 3.81 =

$$\begin{array}{r} 4.34 \\ +\ 3.81 \\ \hline . \quad \\ \hline \end{array}$$

e) 13.007 + 16.507 =

$$\begin{array}{r} 13.007 \\ +\ 16.507 \\ \hline \\ \hline \end{array}$$

f) 1.492 + 5.774 =

$$\begin{array}{r} 1.492 \\ +\ \quad . \quad \\ \hline \\ \hline \end{array}$$

g) 3.45 + 2.05 =

$$\begin{array}{r} 3.45 \\ +\ \quad . \quad \\ \hline \\ \hline \end{array}$$

h) 16.097 + 11.332 =

$$\begin{array}{r} . \quad \\ +\ \quad . \quad \\ \hline \\ \hline \end{array}$$

i) 7.035 + 2.97 =

$$\begin{array}{r} 7.035 \\ +\ \quad . \quad \\ \hline \\ \hline \end{array}$$

j) 8.3 + 2.65 =

$$\begin{array}{r} 8.3 \\ +\ \quad . \quad \\ \hline \\ \hline \end{array}$$

Skill 2.1 Adding decimal numbers (2).

k) 33.46 + 8 = 41.46

$$\begin{array}{rrcrr} {}^{1}3 & 3 & . & 4 & 6 \\ + & 8 & . & 0 & 0 \\ \hline 4 & 1 & . & 4 & 6 \\ \hline \end{array}$$

l) 14.245 + 9 =

$$\begin{array}{rrcrrr} 1 & 4 & . & 2 & 4 & 5 \\ + & & . & & & \\ \hline & & . & & & \\ \hline \end{array}$$

m) 5.09 + 6.005 =

$$\begin{array}{rcrr} & . & & \\ + & . & & \\ \hline & . & & \\ \hline \end{array}$$

n) 4.263 + 7 =

$$\begin{array}{rcrr} & . & & \\ + & . & & \\ \hline & . & & \\ \hline \end{array}$$

o) 5.43 + 0.593 =

$$\begin{array}{rcrr} & . & & \\ + & . & & \\ \hline & . & & \\ \hline \end{array}$$

p) 0.82 + 19.6 =

$$\begin{array}{rrcr} & & . & \\ + & & . & \\ \hline & & . & \\ \hline \end{array}$$

q) 0.618 + 1.382 =

$$\begin{array}{rc} & . \\ + & . \\ \hline & . \\ \hline \end{array}$$

r) 3.8 + 12.207 =

$$\begin{array}{rc} & . \\ + & . \\ \hline & . \\ \hline \end{array}$$

s) 2.045 + 0.309 =

$$\begin{array}{r} + \\ \hline \\ \hline \end{array}$$

t) 0.008 + 10.04 =

$$\begin{array}{r} + \\ \hline \\ \hline \end{array}$$

u) 4.747 + 47.47 =

$$\begin{array}{r} + \\ \hline \\ \hline \end{array}$$

v) 0.407 + 0.605 =

$$\begin{array}{r} + \\ \hline \\ \hline \end{array}$$

Skill 2.2 Subtracting decimal numbers (1).

- Write the numbers vertically, lining up the decimal points.
- Subtract from right to left in the columns.
- If there is no digit in a space after the last digit of a decimal number, then you can add a zero if you need one.

Q. 5.09 – 0.983 =

A. *4.107*

$$\begin{array}{r} 5\,.\,{}^{1}0\,9\,{}^{1}0 \\ -\,{}_{1}0\,.\,9\,{}_{1}8\,3 \\ \hline 4\,.\,1\,0\,7 \\ \hline \end{array}$$

Add a 0 after the 9.
0 – 3 = ? Borrow 1 hundredth or 10 thousandths.
10 – 3 = 7 write 7
Pay back 1 hundredth.
8 + 1 = 9
9 – 9 = 0 write 0
0 – 9 = ? Borrow 1 unit or 10 tenths.
10 – 9 = 1 write 1
Pay back 1 unit.
0 + 1 = 1
5 – 1 = 4 write 4

a) 5.62 – 0.51 = 5.11

$$\begin{array}{r} 5\,.\,6\,2 \\ -\;0\,.\,5\,1 \\ \hline 5\,.\,1\,1 \\ \hline \end{array}$$

b) 9.6 – 7.8 =

$$\begin{array}{r} 9\,.\,{}^{1}6 \\ -\;{}_{1}7\,.\,8 \\ \hline .\,8 \\ \hline \end{array}$$

c) 9.07 – 4.56 =

$$\begin{array}{r} 9\,.\,0\,7 \\ -\;4\,.\,5\,6 \\ \hline . \\ \hline \end{array}$$

d) 1.068 – 0.097 =

$$\begin{array}{r} 1\,.\,0\,6\,8 \\ -\;0\,.\,0\,9\,7 \\ \hline . \\ \hline \end{array}$$

e) 25.45 – 15.72 =

$$\begin{array}{r} 2\,5\,.\,4\,5 \\ -\;1\,5\,.\,7\,2 \\ \hline . \\ \hline \end{array}$$

f) 10.37 – 4.6 =

$$\begin{array}{r} 1\,0\,.\,3\,7 \\ -\;4\,.\,6 \\ \hline . \\ \hline \end{array}$$

g) 14.072 – 8 =

$$\begin{array}{r} 1\,4\,.\,0\,7\,2 \\ -\;\,. \\ \hline . \\ \hline \end{array}$$

h) 4.043 – 0.807 =

$$\begin{array}{r} . \\ -\;. \\ \hline . \\ \hline \end{array}$$

Skill 2.2 Subtracting decimal numbers (2).

i) 1.36 – 0.058 = 1.302

$$\begin{array}{r} 1.36\overset{1}{0} \\ -\;0.0{}_{1}5\,8 \\ \hline 1.302 \\ \hline \end{array}$$

j) 5.4 – 0.75 =

$$\begin{array}{r} 5.4\,0 \\ -\quad . \quad\; \\ \hline . \quad\; \\ \hline \end{array}$$

k) 12.5 – 0.047 =

$$\begin{array}{r} 12.5\,0\,0 \\ -\quad . \quad\quad \\ \hline . \quad\quad \\ \hline \end{array}$$

l) 0.086 – 0.009 =

–

m) 6.02 – 0.055 =

–

n) 24.9 – 6.33 =

–

o) 1.9 – 1.743 =

–

p) 15.08 – 0.491 =

–

q) 2.006 – 1.4 =

–

r) 23.33 – 16.66 =

–

s) 14.14 – 1.41 =

–

t) 5.3 – 0.503 =

–

Skill 2.3 Subtracting a decimal number from a whole number (1).

- Write a decimal point at the end of the whole number.
- Add as many zeros after the decimal point as there are decimal places in the decimal number.
- Work vertically, lining up the decimal points.
- Subtract from right to left in the columns.

Q. 10 − 0.38 =

A. *9.62*

$$\begin{array}{r} 1\ {}^{1}0\,.\,{}^{1}0\ {}^{1}0 \\ -\ {}_{1}\ {}_{1}0\,.\,{}_{1}3\ 8 \\ \hline 9\,.\,6\ 2 \\ \hline \end{array}$$

0 − 8 = ? Borrow 10 hundredths.
10 − 8 = 2 write 2
Pay back 10 hundredths or 1 tenth.
3 + 1 = 4
0 − 4 = ? Borrow 10 tenths.
10 − 4 = 6 write 6
Pay back 10 tenths or 1 unit.
0 + 1 = 1
0 − 1 = ? Borrow 10 units.
10 − 1 = 9 write 9
Pay back 10 units or 1 ten.
1 − 1 = 0

a) 1 − 0.07 = 0.93

$$\begin{array}{r} 1\,.\,{}^{1}0\ {}^{1}0 \\ -\ {}_{1}0\,.\,{}_{1}0\ 7 \\ \hline 0\,.\,9\ 3 \\ \hline \end{array}$$

b) 1 − 0.09 =

$$\begin{array}{r} 1\,.\,{}^{1}0\ {}^{1}0 \\ -\ 0\,.\,{}_{1}0\ 9 \\ \hline .\ \ \ 1 \\ \hline \end{array}$$

c) 10 − 2.7 =

$$\begin{array}{r} 1\ 0\,.\,0 \\ -\ \ 2\,.\,7 \\ \hline . \ \ \\ \hline \end{array}$$

d) 1 − 0.034 =

$$\begin{array}{r} 1\,.\,0\ 0\ 0 \\ -\ 0\,.\,0\ 3\ 4 \\ \hline . \ \ \ \ \ \ \\ \hline \end{array}$$

e) 10 − 0.84 =

$$\begin{array}{r} 1\ 0\,.\,0\ 0 \\ -\ \ 0\,.\,8\ 4 \\ \hline . \ \ \ \ \\ \hline \end{array}$$

f) 10 − 5.637 =

$$\begin{array}{r} 1\ 0\,.\,0\ 0\ 0 \\ -\ \ \ \ \ .\ \ \ \ \ \ \\ \hline . \ \ \ \ \ \ \\ \hline \end{array}$$

g) 100 − 8.3 =

$$\begin{array}{r} 1\ 0\ 0\,.\,0 \\ -\ \ \ \ \ \ .\ \ \\ \hline . \ \ \\ \hline \end{array}$$

h) 100 − 0.23 =

$$\begin{array}{r} 1\ 0\ 0\,.\,0\ 0 \\ -\ \ \ \ \ \ .\ \ \ \ \\ \hline . \ \ \ \ \\ \hline \end{array}$$

Skill 2.3 Subtracting a decimal number from a whole number (2).

i) 8 – 0.62 = $\boxed{7.38}$

$$\begin{array}{r} 8\,.\,{}^{1}0\;{}^{1}0 \\ -\;{}_{1}0\,.\,{}_{1}6\;2 \\ \hline 7\,.\,3\;8 \\ \hline \end{array}$$

j) 5 – 0.27 = $\boxed{}$

$$\begin{array}{r} 5\,.\,0\;0 \\ -\quad.\quad\;\; \\ \hline .\quad\;\; \\ \hline \end{array}$$

k) 9 – 0.604 = $\boxed{}$

$$\begin{array}{r} 9\,.\,0\;0\;0 \\ -\quad.\quad\quad\;\; \\ \hline .\quad\quad\;\; \\ \hline \end{array}$$

l) 6 – 2.71 = $\boxed{}$

$$\begin{array}{r} .\quad\;\; \\ -\quad.\quad\;\; \\ \hline .\quad\;\; \\ \hline \end{array}$$

m) 2 – 0.077 = $\boxed{}$

$$\begin{array}{r} .\quad\quad\;\; \\ -\quad.\quad\quad\;\; \\ \hline .\quad\quad\;\; \\ \hline \end{array}$$

n) 13 – 3.78 = $\boxed{}$

$$\begin{array}{r} .\quad\;\; \\ -\quad.\quad\;\; \\ \hline .\quad\;\; \\ \hline \end{array}$$

o) 14 – 9.567 = $\boxed{}$

$$\begin{array}{r} .\quad\quad\quad\;\; \\ -\quad.\quad\quad\quad\;\; \\ \hline .\quad\quad\quad\;\; \\ \hline \end{array}$$

p) 18 – 0.515 = $\boxed{}$

$$\begin{array}{r} .\quad\quad\quad\;\; \\ -\quad.\quad\quad\quad\;\; \\ \hline .\quad\quad\quad\;\; \\ \hline \end{array}$$

q) 20 – 2.02 = $\boxed{}$

$$\begin{array}{l} -\quad\quad\quad\quad \\ \hline \\ \hline \end{array}$$

r) 43 – 0.307 = $\boxed{}$

$$\begin{array}{l} -\quad\quad\quad\quad \\ \hline \\ \hline \end{array}$$

s) 31 – 6.003 = $\boxed{}$

$$\begin{array}{l} -\quad\quad\quad\quad \\ \hline \\ \hline \end{array}$$

t) 13 – 0.333 = $\boxed{}$

$$\begin{array}{l} -\quad\quad\quad\quad \\ \hline \\ \hline \end{array}$$

Skill 2.4 Adding and subtracting decimal numbers (1).

- Use BODMAS rules and add or subtract from left to right across the equation.
- Complete the operations one at a time.
- Work each operation vertically, lining up the decimal points.
- When in column format, add or subtract from right to left in the columns.

Q. 7.3 + 6.41 − 0.08 = **A.** ***13.63***

Complete the operations from left to right

$$\begin{array}{r} {}^{1}\;7.3 \\ +\;6.41 \\ \hline \mathbf{13.71} \\ \\ 13.7\,{}^{1}1 \\ -\;\;0.{}_{1}08 \\ \hline \mathbf{13.63} \end{array}$$

First add 7.3 + 6.41

Then subtract 0.08 from the result.

a) 5.238 − 1.45 + 0.66 = 4.448

$$\begin{array}{r} 5.{}^{1}2\,{}^{1}38 \\ -\;{}_{1}1.{}_{1}450 \\ \hline 3.788 \\ \\ {}^{1}3.{}^{1}788 \\ +\;0.660 \\ \hline 4.448 \end{array}$$

b) 4.7 + 1.05 − 0.5 =

$$\begin{array}{r} 4.70 \\ +\;1.05 \\ \hline 5.75 \\ \\ 5.75 \\ -\;0.50 \\ \hline . \end{array}$$

c) 13.3 − 4.6 + 0.77 =

$$\begin{array}{r} 1\,{}^{1}3.{}^{1}3 \\ -\;{}_{1}\;{}_{1}4.6 \\ \hline . \\ \\ . \\ +\quad . \\ \hline . \end{array}$$

d) 1 + 2.309 − 0.92 =

$$\begin{array}{r} 1.000 \\ +\;2.309 \\ \hline . \\ \\ . \\ -\quad . \\ \hline . \end{array}$$

e) 20.3 − 5.4 + 1.78 =

$$\begin{array}{r} 2\,{}^{1}0.{}^{1}3 \\ -\;{}_{1}\;{}_{1}5.4 \\ \hline . \\ \\ . \\ +\quad . \\ \hline . \end{array}$$

f) 3 + 6.049 − 1.796 =

$$\begin{array}{r} 3.000 \\ +\;6.049 \\ \hline . \\ \\ . \\ -\quad . \\ \hline . \end{array}$$

Skill 2.4 Adding and subtracting decimal numbers (2).

g) 0.99 + 1.067 – 0.48 =

$^{1}0\,.\,^{1}9\;9$
$+\;1\,.\,0\;6\;7$
.
.
$-\;0\,.\,4\;8$
.

h) 21 – 14.46 + 3.9 =

$2\;1\,.\,0\;0$
$-\;1\;4\,.\,4\;6$
.
.
$+$.
.

i) 16.7 + 0.336 – 1.59 =

$1\;6\,.\,7$
$+\;0\,.\,3\;3\;6$
.
.
$-$.
.

j) 10.504 – 3.07 – 1.06 =

$1\;0\,.\,5\;0\;4$
$-$.
.
.
$-$.
.

k) 0.223 + 11.094 – 8.4 =

$+$

$-$

l) 21.75 – 0.336 – 1.9 =

$-$

$-$

m) 18.093 – 13.007 + 0.256 =

$-$

$+$

n) 1.08 – 0.241 + 12.121 =

$-$

$+$

3. [Decimal ×,÷]

Skill 3.1 Multiplying a decimal number by a whole number.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Ignore any decimal points and complete the multiplication from right to left.
- Count the number of decimal places in the question.
- Move the decimal point the same number of places from the right in the answer.

Q. 5.49 × 6 =

A. ***32.94***

$$\begin{array}{r} {}^{2}5\,.\,{}^{5}\textcircled{4}\textcircled{9} \\ \times \qquad 6 \\ \hline 3\,2\,.\,9\,4 \\ \hline \end{array}$$

6 × 9 = 54	carry 5, write 4
6 × 4 + carry 5 = 29	carry 2, write 9
6 × 5 + carry 2 = 32	write 32

2 decimal places in question so move decimal point 2 places from right

a) 0.7 × 4 = $\boxed{2.8}$

$$\begin{array}{r} {}^{2}0\,.\,\textcircled{7} \\ \times \quad 4 \\ \hline 2\,.\,8 \\ \hline \end{array}$$

b) 0.9 × 8 =

$$\begin{array}{r} {}^{7}0\,.\,\textcircled{9} \\ \times \quad 8 \\ \hline .\,2 \\ \hline \end{array}$$

c) 2.6 × 7 =

$$\begin{array}{r} {}^{4}2\,.\,6 \\ \times \quad 7 \\ \hline \\ \hline \end{array}$$

d) 4.8 × 5 =

$$\begin{array}{r} 4\,.\,8 \\ \times \quad 5 \\ \hline \\ \hline \end{array}$$

e) 3.26 × 7 =

$$\begin{array}{r} 3\,.\,2\;\;6 \\ \times \qquad 7 \\ \hline \\ \hline \end{array}$$

f) 2.08 × 3 =

$$\begin{array}{r} 2\,.\,0\;\;8 \\ \times \qquad 3 \\ \hline \\ \hline \end{array}$$

g) 12.23 × 6 =

$$\begin{array}{r} 1\;\;2\,.\,2\;\;3 \\ \times \qquad\quad 6 \\ \hline \\ \hline \end{array}$$

h) 1.507 × 9 =

$$\begin{array}{r} 1\,.\,5\;\;0\;\;7 \\ \times \qquad\quad 9 \\ \hline \\ \hline \end{array}$$

i) 21.37 × 7 =

$$\begin{array}{r} 2\;\;1\,.\,3\;\;7 \\ \times \qquad\quad 7 \\ \hline \\ \hline \end{array}$$

j) 14.3 × 8 =

$$\begin{array}{r} 1\;\;4\,.\,3 \\ \times \qquad 8 \\ \hline \\ \hline \end{array}$$

k) 3.056 × 9 =

$$\begin{array}{r} 3\,.\,0\;\;5\;\;6 \\ \times \qquad\quad 9 \\ \hline \\ \hline \end{array}$$

l) 48.27 × 3 =

$$\begin{array}{r} 4\;\;8\,.\,2\;\;7 \\ \times \qquad\quad 3 \\ \hline \\ \hline \end{array}$$

Skill 3.2 Multiplying a decimal number by powers and multiples of 10.

Multiplying by a power of 10

- Count the number of zeros in the power of 10.
- Move the decimal point to the right as many places as there are zeros in the power of 10.
- Add zeros as place holders, if necessary.

  Example: $4.5 \times 100 = 4.\overset{\frown\frown}{50} \times 100 = 450$

Multiplying by a multiple of 10

- Disregard the 0's in the multiple of 10 and multiply by the remaining digit.
  (see skill 3.1, page 21)
- Move the decimal point to the right as many places as there are zeros in the multiple of 10.

Q. $0.005 \times 80 =$

A.

$$\begin{array}{r} 0\,.\,\textcircled{0}\overset{4}{\textcircled{0}}\,5 \\ \times \qquad 8 \\ \hline 0\,.\,0\,4\,0 \\ \hline \end{array}$$

$8 \times 5 = 40$	carry 4, write 0
$8 \times 0 + \text{carry } 4 = 4$	write 4
$8 \times 0 = 0$	write 0
$8 \times 0 = 0$	write 0

$0.005 \times 8\textcircled{0}$

$= 0.\overset{\frown}{04}0 \times 10$

$= \mathbf{0.4}$

1 zero in multiple so move decimal point 1 place right

a) $6.37 \times 1\textcircled{00} =$ 2 zeros, 2 places right

$= 6\overset{\frown\frown}{37}.$ $= \boxed{637}$

b) $3.98 \times 1\textcircled{0} =$

$= 39\overset{\frown}{.}8$ $= \boxed{\quad}$

c) $0.03 \times 10 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

d) $4.29 \times 100 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

e) $100 \times 3.007 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

f) $21.88 \times 100 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

g) $100 \times 0.005 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

h) $0.8 \times 100 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

i) $100 \times 0.12 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

j) $0.039 \times 10 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

k) $0.73 \times 10 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

l) $1\textcircled{000} \times 0.57 =$

$= 0.\overset{\frown\frown\frown}{57}$ $= \boxed{570}$

Add zeros as place holders

m) $50 \times 8.6 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

n) $0.0058 \times 40 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

o) $0.64 \times 200 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

p) $30 \times 0.0309 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

q) $0.004 \times 200 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

r) $60 \times 0.704 =$

$= \ldots\ldots\ldots = \boxed{\quad}$

Skill 3.3 Multiplying a decimal number by a negative power of 10 (e.g. 0.1)

Mauve 1 1 **2** 2 3 3 4 4
Lime 1 **1** 2 2 3 3 4 4

- Move the decimal point to the right in the power of 10, as many places as you need to make 1.

  Example: In $0.\overset{\frown}{0}\overset{\frown}{1}$ the decimal point must move two places to the right to make 1.
- Then move the decimal point the same number of places to the left in the other number.
- Add zeros as place holders, if necessary.

  Example: $2.4 \times 0.01 = 0\overset{\frown}{0}\overset{\frown}{2}.4 \times 0.\overset{\frown}{0}\overset{\frown}{1} = 0.024 \times 1 = 0.024$
- If the result is less than 1, write a zero in the units place.

  Example: By convention 0.37 rather than .37

Q. $0.01 \times 3.9 =$

A. 0.01×3.9

$= 0.\overset{\frown}{0}\overset{\frown}{1} \times \overset{\frown}{0}\overset{\frown}{3}.9$

$= 1 \times 0.039$

$= \mathbf{0.039}$

2 places right makes 1 so move decimal point 2 places left

< 1 so write zero in units place

a) $\overset{\frown}{7}.84 \times 0.\overset{\frown}{1} =$

$= 0.784 \times 1 = \boxed{0.784}$

b) $\overset{\frown}{4}.2 \times 0.\overset{\frown}{1} =$

= .... = ☐

c) $0.1 \times 68.5 =$

= .... = ☐

d) $0.01 \times 593.2 =$

= .... = ☐

e) $484.5 \times 0.01 =$

= .... = ☐

f) $0.01 \times 223.7 =$

= .... = ☐

g) $0.001 \times 31.3 =$

$= 0.\overset{\frown}{0}\overset{\frown}{0}\overset{\frown}{1} \times 0\overset{\frown}{0}\overset{\frown}{3}\overset{\frown}{1}.3$

$= 1 \times 0.0313 = \boxed{0.0313}$

Use zeros as place holders

h) $0.001 \times 9090.9 =$

= ....

= .... = ☐

i) $0.001 \times 1234.5 =$

= ....

= .... = ☐

j) $0.01 \times 12.8 =$

= ....

= .... = ☐

k) $32.5 \times 0.01 =$

= ....

= .... = ☐

l) $0.01 \times 13.9 =$

= ....

= .... = ☐

m) $530.8 \times 0.001 =$

= ....

= .... = ☐

n) $0.01 \times 1.02 =$

= ....

= .... = ☐

o) $5.4 \times 0.001 =$

= ....

= .... = ☐

Skill 3.4 Multiplying a decimal number by a decimal number.

- Neglect any decimal points and complete the multiplication from right to left.
- Count the number of decimal places in the question.
- Move the decimal point the same number of places from the right in the answer.
- Use zeros as place holders, if necessary.
  Example: $0.02 \times 0.3 = 0.006$
- If the result is less than 1, write a zero in the units place.
  Example: By convention 0.37 not .37
- Remove any zeros at the end of the decimal number, after the decimal point, if necessary.
- Remove any zeros at the start of the decimal number, up to zero units, if necessary.

Q. $15.4 \times 0.03 =$

A. ***0.462***

$$\begin{array}{r} {}^{1}1\ \ {}^{1}5\ .\enclose{circle}{4} \\ \times\ \ 0\ .\enclose{circle}{0}\ \enclose{circle}{3} \\ \hline 0\ .\ 4\ \ 6\ \ 2 \end{array}$$

$3 \times 4 = 12$ carry 1, write 2
$3 \times 5 + \text{carry } 1 = 16$ carry 1, write 6
$3 \times 1 + \text{carry } 1 = 4$ write 4

< 1 so write zero in units place

3 decimal places in question so move decimal point 3 places from right

a) $0.6 \times 0.7 = \boxed{0.42}$

$$\begin{array}{r} {}^{4}0\ .\enclose{circle}{6} \\ \times\ 0\ .\enclose{circle}{7} \\ \hline 0\ .\ 4\ \ 2 \end{array}$$

2 decimal places
2 places from right
< 1 so write zero in units place

b) $0.8 \times 0.4 = \boxed{}$

$$\begin{array}{r} {}^{3}0\ .\enclose{circle}{8} \\ \times\ 0\ .\enclose{circle}{4} \\ \hline 2 \end{array}$$

c) $0.9 \times 0.5 = \boxed{}$

$$\begin{array}{r} {}^{4}0\ .\ 9 \\ \times\ 0\ .\ 5 \\ \hline \end{array}$$

d) $3.6 \times 0.6 = \boxed{}$

$$\begin{array}{r} 3\ .\ 6 \\ \times\ 0\ .\ 6 \\ \hline \end{array}$$

e) $0.7 \times 4.58 = \boxed{}$

$$\begin{array}{r} 4\ .\ 5\ \ 8 \\ \times\ \ \ 0\ .\ 7 \\ \hline \end{array}$$

f) $0.17 \times 0.08 = \boxed{}$

$$\begin{array}{r} 0\ .\ 1\ \ 7 \\ \times\ 0\ .\ 0\ \ 8 \\ \hline \end{array}$$

g) $3.9 \times 0.09 = \boxed{}$

$$\begin{array}{r} 3\ .\ 9 \\ \times\ 0\ .\ 0\ \ 9 \\ \hline \end{array}$$

h) $0.03 \times 2.98 = \boxed{}$

$$\begin{array}{r} 2\ .\ 9\ \ 8 \\ \times\ 0\ .\ 0\ \ 3 \\ \hline \end{array}$$

i) $32.5 \times 0.09 = \boxed{}$

$$\begin{array}{r} 3\ \ 2\ .\ 5 \\ \times\ \ 0\ .\ 0\ \ 9 \\ \hline \end{array}$$

j) $2.75 \times 6.7 = \boxed{}$

$$\begin{array}{r} 2\ .\ 7\ \ 5 \\ \times\ \ \ 6\ .\ 7 \\ \hline \\ \hline \\ \hline \end{array}$$

k) $9.15 \times 2.3 = \boxed{}$

$$\begin{array}{r} 9\ .\ 1\ \ 5 \\ \times\ \ \ 2\ .\ 3 \\ \hline \\ \hline \\ \hline \end{array}$$

l) $12.8 \times 0.43 = \boxed{}$

$$\begin{array}{r} 1\ \ 2\ .\ 8 \\ \times\ \ 0\ .\ 4\ \ 3 \\ \hline \\ \hline \\ \hline \end{array}$$

Skill 3.5 Dividing a decimal number by a whole number.

Mauve 11 22 33 44
Lime 11 22 33 44

- Break down the division into smaller divisions.
- Divide from left to right.
- Line up the decimal point in your answer with the decimal point in the question.

Q. 208.2 ÷ 6 =

A. ***34.7***

$$\begin{array}{r} 3\;4\;.\;7 \\ 6\,\overline{)\,\cancel{2}\;{}^{2}0\;{}^{2}8\;.\;{}^{4}2} \end{array}$$

÷ from left

Line up decimal places

2 ÷ 6 = ?	carry 2	
20 ÷ 6 = 3	carry 2	write 3
28 ÷ 6 = 4	carry 4	write 4
42 ÷ 6 = 7		write 7

a) 163.2 ÷ 8 = 20.4

÷ from left

$$\begin{array}{r} 2\;0\;.\;4 \\ 8\,\overline{)\,\cancel{1}\;{}^{1}6\;\cancel{3}\;.\;{}^{3}2} \end{array}$$

Line up decimal places

b) 76.8 ÷ 4 =

$$4\,\overline{)\,7\;{}^{3}6\;.\;8}$$

c) 46.5 ÷ 5 =

$$5\,\overline{)\,\cancel{4}\;{}^{4}6\;.\;5}$$

d) 140.4 ÷ 6 =

$$6\,\overline{)\,1\;4\;0\;.\;4}$$

e) 145.8 ÷ 3 =

$$3\,\overline{)\,1\;4\;5\;.\;8}$$

f) 130.9 ÷ 7 =

$$7\,\overline{)\,1\;3\;0\;.\;9}$$

g) 31.05 ÷ 5 =

$$5\,\overline{)\,3\;1\;.\;0\;5}$$

h) 79.48 ÷ 2 =

$$2\,\overline{)\,7\;9\;.\;4\;8}$$

i) 96.56 ÷ 8 =

$$8\,\overline{)\,9\;6\;.\;5\;6}$$

j) 104.24 ÷ 4 =

$$4\,\overline{)\,1\;0\;4\;.\;2\;4}$$

k) 153.54 ÷ 9 =

$$9\,\overline{)\,1\;5\;3\;.\;5\;4}$$

l) 794.78 ÷ 7 =

$$7\,\overline{)\,7\;9\;4\;.\;7\;8}$$

m) 201.12 ÷ 8 =

n) 391.26 ÷ 6 =

o) 150.625 ÷ 5 =

Skill 3.6 Dividing a decimal number by a power of 10.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Move the decimal place to the left as many places as there are zeros in the power of 10.
  Example: $30\overset{\frown\frown}{72} \div 100 = 30.72$
- Add zeros as place holders, if necessary.
  Example: $4.5 \div 100 = 0\overset{\frown\frown}{04}.5 \div 100 = 0.045$
- If the result is less than 1, write a zero in the units place.
  Example: By convention 0.37 not .37

Q. $0.97 \div 10 =$

A. $0.97 \div 10$ 1 zero, 1 place left
$= \overset{\frown}{0}.97 \div 10$
$= \mathbf{0.097}$
< 1 so write zero in units place

a) $6.7 \div 100 =$ 2 zeros, 2 places left
$= 0\overset{\frown\frown}{06}.7 \div 100 = \boxed{0.067}$
Add zeros as place holders

b) $230.6 \div 10 =$
= .... = $\boxed{\quad}$

c) $15.3 \div 10 =$
= .... = $\boxed{\quad}$

d) $3.35 \div 10 =$
= .... = $\boxed{\quad}$

e) $800.9 \div 100 =$
= .... = $\boxed{\quad}$

f) $32.4 \div 100 =$
= .... = $\boxed{\quad}$

g) $0.36 \div 10 =$
$= 0\overset{\frown}{0}.36 \div 10 = \boxed{\quad}$
Add zeros as place holders

h) $0.08 \div 10 =$
= .... = $\boxed{\quad}$

i) $65.3 \div 100 =$
= .... = $\boxed{\quad}$

j) $49.2 \div 100 =$
= .... = $\boxed{\quad}$

k) $6.8 \div 100 =$
= .... = $\boxed{\quad}$

l) $0.74 \div 100 =$
= .... = $\boxed{\quad}$

m) $2972.5 \div 1000 =$
= .... = $\boxed{\quad}$

n) $33.1 \div 1000 =$
= .... = $\boxed{\quad}$

o) $0.5 \div 1000 =$
= .... = $\boxed{\quad}$

p) $0.015 \div 10 =$
= .... = $\boxed{\quad}$

q) $0.6 \div 100 =$
= .... = $\boxed{\quad}$

r) $1.02 \div 100 =$
= .... = $\boxed{\quad}$

s) $3.25 \div 1000 =$
= .... = $\boxed{\quad}$

t) $42.6 \div 1000 =$
= .... = $\boxed{\quad}$

u) $0.23 \div 100 =$
= .... = $\boxed{\quad}$

Skill 3.7 Dividing a decimal number by a negative power of 10 (e.g. 0.1)

- Move the decimal point to the right in the power of 10, as many places as you need to make 1.

  Example: In $0.\overset{\frown}{0}\overset{\frown}{1}$ the decimal point must move two places to the right to make 1.
- Move the decimal point the same number of places to the right in the dividend.

  Example: $4.\overset{\frown}{5}\overset{\frown}{2} \div 0.\overset{\frown}{0}\overset{\frown}{1} = 452 \div 1 = 452$
- Add zeros as place holders, if necessary.

  Example: $4.5 \div 0.01 = 4.\overset{\frown}{5}\overset{\frown}{0} \div 0.\overset{\frown}{0}\overset{\frown}{1} = 450 \div 1 = 450$

Q. $0.85 \div 0.01 =$

A. $0.85 \div 0.01$

$= 0.\overset{\frown}{8}\overset{\frown}{5} \div 0.\overset{\frown}{0}\overset{\frown}{1}$ 2 places right makes 1

$= 85 \div 1$ so 2 places right

$= \mathbf{85}$

a) $5.\overset{\frown}{6} \div 0.\overset{\frown}{1} =$ 1 place right makes 1

$= 56 \div 1 = \boxed{56}$

b) $3.03 \div 0.1 =$

= .... =

c) $2.4 \div 0.1 =$

= .... =

d) $0.058 \div 0.1 =$

= .... =

e) $42.7 \div 0.1 =$

= .... =

f) $0.38 \div 0.1 =$

= .... =

g) $0.76 \div 0.01 =$

= .... =

h) $0.09 \div 0.01 =$

= .... =

i) $65.3 \div 0.01 =$

= .... =

j) $0.005 \div 0.01 =$

= .... =

k) $0.89 \div 0.01 =$

= .... =

l) $7.153 \div 0.001 =$

= .... =

m) $0.048 \div 0.1 =$

= .... =

n) $12.4 \div 0.1 =$

= .... =

o) $0.75 \div 0.01 =$

= .... =

p) $1.2 \div 0.01 =$ Add zeros as place holders

$= 1.\overset{\frown}{2}\overset{\frown}{0} \div 0.\overset{\frown}{0}\overset{\frown}{1}$ 2 places right

$= 120 \div 1 = \boxed{120}$

q) $23.2 \div 0.01 =$

= ....

= .... =

r) $3.58 \div 0.001 =$

= ....

= .... =

s) $0.4 \div 0.01 =$

= ....

= .... =

t) $0.03 \div 0.001 =$

= ....

= .... =

u) $8.04 \div 0.001 =$

= ....

= .... =

Skill 3.8 Dividing a decimal number by a decimal number.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 **3** 4 **4**

- Move the decimal point to the right in the divisor, as many places as you need to make it a whole number.
- Then move the decimal point the same number of places to the right in the dividend.
  Example: $4.\overset{\frown\frown}{53} \div 0.\overset{\frown\frown}{02} = 453.0 \div 2 = 226.5$
- Add zeros as place holders, if necessary.
  Example: $3.6 \div 0.06 = 3.\overset{\frown\frown}{60} \div 0.\overset{\frown\frown}{06} = 360 \div 6 = 60$ (See also example above.)
- Break down the division into smaller divisions.
- Divide from left to right.
- Line up the decimal point in your answer with the decimal point in the question.

Q. $3.68 \div 0.8 =$

A. $3.\overset{\frown}{6}8 \div 0.\overset{\frown}{8}$

$= 36.8 \div 8$

$= \mathbf{4.6}$

$$\begin{array}{r} 4.6 \\ 8\overline{)\cancel{3}\,{}^{3}6.{}^{4}8} \end{array}$$

÷ from left

Line up decimal places

1 place right makes a whole number

a) $9.\overset{\frown}{8}4 \div 0.\overset{\frown}{8} =$ so 1 place right

$= 98.4 \div 8$ = $\boxed{12.3}$

$$\begin{array}{r} 12.3 \\ 8\overline{)9\,{}^{1}8.{}^{2}4} \end{array}$$

b) $0.6 \div 0.2 =$

= .... = ☐

c) $2.8 \div 0.4 =$

= .... = ☐

d) $0.03 \div 0.003 =$

= .... = ☐

e) $4.68 \div 0.4 =$

= .... = ☐

f) $8.61 \div 0.7 =$

= .... = ☐

g) $35.6 \div 3.56 =$

= .... = ☐

h) $5.68 \div 0.8 =$

= .... = ☐

i) $13.35 \div 0.5 =$

= .... = ☐

j) $2.4 \div 0.06 =$

= .... = ☐

k) $0.4 \div 0.004 =$

= .... = ☐

l) $0.2 \div 0.25 =$

= .... = ☐

m) $3.675 \div 0.15 =$

= .... = ☐

n) $37.8 \div 1.2 =$

= .... = ☐

o) $1.75 \div 1.4 =$

= .... = ☐

Skill 3.9 Dividing a whole number by a decimal number.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Move the decimal point to the right in the divisor, as many places as you need to make a whole number.
- Then move the decimal point the same number of places to the right in the dividend.
  Example: $45 \div 0.02 = 45.\overset{\frown}{00} \div 0.\overset{\frown}{02} = 4500 \div 2 = 2250$
- Add zeros as place holders, if necessary.
  Example: $36 \div 0.6 = 36.\overset{\frown}{0} \div 0.\overset{\frown}{6} = 360 \div 6 = 60$ (See also example above)
- Break down the division into smaller divisions.
- Divide from left to right.
- Line up the decimal point in your answer with the decimal point in the question.

Q. $60 \div 0.2 =$

A. $60 \div 0.2$

$= 60.\overset{\frown}{0} \div 0.\overset{\frown}{2}$

$= 600 \div 2$

$= \mathbf{300}$

1 place right makes a whole number

Add zeros as place holders

÷ from left

$$\begin{array}{r} 3\,0\,0 \\ 2\overline{)6\,0\,0} \end{array}$$

2 places right make a whole number

a) $9 \div 0.03 =$

$= 9.\overset{\frown}{00} \div 0.\overset{\frown}{03} = \boxed{300}$

Add zeros as place holders

$$\begin{array}{r} 3\,0\,0 \\ 3\overline{)9\,0\,0} \end{array}$$

b) $7 \div 0.02 =$

= .... = ☐

$2\overline{)}$

c) $80 \div 0.4 =$

= .... = ☐

d) $27 \div 0.9 =$

= .... = ☐

e) $18 \div 0.04 =$

= .... = ☐

f) $32 \div 0.8 =$

= .... = ☐

g) $45 \div 0.05 =$

= .... = ☐

h) $50 \div 0.25 =$

= .... = ☐

i) $60 \div 0.12 =$

= .... = ☐

j) $30 \div 0.15 =$

= .... = ☐

k) $96 \div 0.8 =$

= .... = ☐

l) $14 \div 0.5 =$

= .... = ☐

4. [Fraction +,–]

Skill 4.1 Adding fractions with the same denominator (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Add the numerators (top numbers of the fractions).
- Do not change the denominators.
- Simplify the resulting fraction and/or change it to a mixed number if necessary.

To simplify a fraction

Hint: If the numbers are both even then you can start with dividing by 2.

$\frac{4}{16}$ numerator (even) denominator (even)

- Divide both the numerator and the denominator by the same number.

$\frac{4^{\div 2}}{16_{\div 2}} = \frac{2^{\div 2}}{8_{\div 2}} = \frac{1}{4}$

To change an improper fraction to a mixed number

$\frac{7}{3}$ numerator IMPROPER FRACTION denominator

- Divide the numerator by the denominator.

$\frac{7}{3} = 7 \div 3 = 2$ remainder 1

- Write the result as the whole number and the remainder over the denominator.

$\frac{7}{3} = 7 \div 3 = 2\frac{1}{3}$

To change a mixed number to an improper fraction

MIXED NUMBER

$3\frac{2}{5}$ whole number proper fraction

- Multiply the whole number by the denominator and then add the result to the numerator.

$3\frac{2}{5}$ + × $3 \times 5 + 2 = 17$

- Rewrite the total over the denominator.

$3\frac{2}{5} = \frac{17}{5}$

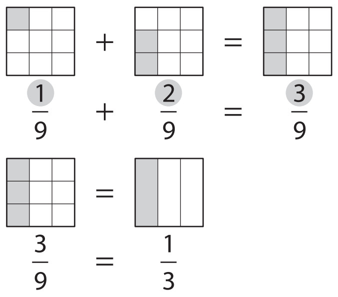
Q. $\frac{1}{9} + \frac{2}{9} =$

A. $\frac{1}{9} + \frac{2}{9} =$ Add the top numbers only

$= \frac{1+2}{9} =$

$= \frac{3^{\div 3}}{9_{\div 3}}$ Simplify

$= \frac{1}{3}$



a) $\frac{3}{8} + \frac{2}{8} =$ Add the top numbers only

$= \frac{3+2}{8}$ $= \boxed{\frac{5}{8}}$

b) $\frac{1}{6} + \frac{4}{6} =$

= .... = ☐

c) $\frac{4}{9} + \frac{4}{9} =$

= .... = ☐

d) $\frac{3}{11} + \frac{4}{11} =$

= .... = ☐

e) $\frac{2}{9} + \frac{5}{9} =$

= .... = ☐

f) $\frac{7}{13} + \frac{5}{13} =$

= .... = ☐

Skill 4.1 Adding fractions with the same denominator (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

g) $\frac{5}{7}+\frac{6}{7}=$

$=\frac{11}{7}$ Change to mixed number

$=11 \div 7 \qquad = \boxed{1\frac{4}{7}}$

h) $\frac{4}{5}+\frac{4}{5}=$

$=\frac{8}{5}$

$=8 \div 5 \qquad = \boxed{}$

i) $\frac{5}{9}+\frac{8}{9}=$

$=$

$= \qquad = \boxed{}$

j) $\frac{6}{11}+\frac{7}{11}=$

$=$

$= \qquad = \boxed{}$

k) $\frac{11}{17}+\frac{10}{17}=$

$=$

$= \qquad = \boxed{}$

l) $\frac{13}{15}+\frac{6}{15}=$

$=$

$= \qquad = \boxed{}$

m) $\frac{1}{6}+\frac{1}{6}=$ Simplify

$=\frac{2\,{}^{\div 2}}{6\,{}_{\div 2}} \qquad = \boxed{\frac{1}{3}}$

n) $\frac{2}{10}+\frac{3}{10}=$

$=\frac{5}{10} \qquad = \boxed{}$

o) $\frac{1}{8}+\frac{5}{8}=$

$= \qquad = \boxed{}$

p) $\frac{5}{12}+\frac{4}{12}=$

$= \qquad = \boxed{}$

q) $\frac{4}{15}+\frac{6}{15}=$

$= \qquad = \boxed{}$

r) $\frac{3}{10}+\frac{1}{10}=$

$= \qquad = \boxed{}$

s) $\frac{3}{4}+\frac{3}{4}=$ Simplify

$=\frac{6}{4} \qquad = \frac{2\,{}^{\div 2}}{4\,{}_{\div 2}} \qquad = \boxed{1\frac{1}{2}}$

t) $\frac{7}{8}+\frac{1}{8}=$

$=\frac{8\,{}^{\div 8}}{8\,{}_{\div 8}} \qquad = \frac{1}{1} \qquad = \boxed{}$

u) $\frac{5}{6}+\frac{5}{6}=$

$= \qquad = \qquad = \boxed{}$

v) $\frac{9}{7}+\frac{5}{7}=$

$= \qquad = \boxed{}$

w) $\frac{4}{9}+\frac{8}{9}=$

$= \qquad = \boxed{}$

x) $\frac{11}{18}+\frac{13}{18}=$

$= \qquad = \boxed{}$

y) $\frac{11}{10}+\frac{9}{10}=$

$= \qquad = \boxed{}$

z) $\frac{7}{12}+\frac{7}{12}=$

$= \qquad = \boxed{}$

zz) $\frac{11}{15}+\frac{7}{15}=$

$= \qquad = \boxed{}$

Skill 4.2 Subtracting fractions with the same denominator.

- Subtract the numerators (top numbers of the fractions).
- Do not change the denominators.
- Simplify the resulting fraction and/or change it to a mixed number if necessary.
  (see skill 4.1, page 31)

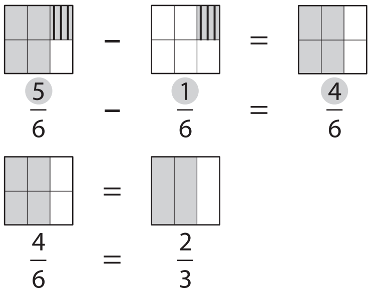
Q. $\frac{5}{6} - \frac{1}{6} =$

A. $\frac{5}{6} - \frac{1}{6} =$ Subtract the top numbers only

$= \frac{5-1}{6} =$

$= \frac{4^{\div 2}}{6_{\div 2}}$ Simplify

$= \frac{2}{3}$



$\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$

$\frac{4}{6} = \frac{2}{3}$

a) $\frac{4}{5} - \frac{2}{5} =$ Subtract the top numbers only

$= \frac{4-2}{5}$ ……………… $= \boxed{\frac{2}{5}}$

b) $\frac{7}{8} - \frac{4}{8} =$

$=$ ……………… $= \boxed{}$

c) $\frac{8}{9} - \frac{4}{9} =$

$=$ ……………… $= \boxed{}$

d) $\frac{17}{9} - \frac{4}{9} =$

$= \frac{13}{9}$ Change to mixed number ………………

$= 13 \div 9$ ……………… $= \boxed{1\frac{4}{9}}$

e) $\frac{19}{10} - \frac{2}{10} =$

$=$ ………………

$=$ ……………… $= \boxed{}$

f) $\frac{18}{7} - \frac{2}{7} =$

$=$ ………………

$=$ ……………… $= \boxed{}$

g) $\frac{8}{9} - \frac{2}{9} =$ Simplify

$= \frac{6^{\div 3}}{9_{\div 3}}$ ……………… $= \boxed{}$

h) $\frac{7}{12} - \frac{5}{12} =$

$=$ ……………… $= \boxed{}$

i) $\frac{5}{8} - \frac{1}{8} =$

$=$ ……………… $= \boxed{}$

j) $\frac{11}{18} - \frac{5}{18} =$

$=$ ……………… $= \boxed{}$

k) $\frac{13}{15} - \frac{10}{15} =$

$=$ ……………… $= \boxed{}$

l) $\frac{11}{16} - \frac{7}{16} =$

$=$ ……………… $= \boxed{}$

m) $\frac{17}{5} - \frac{2}{5} =$

$= \frac{15^{\div 5}}{5_{\div 5}} = \frac{3}{1}$ ……………… $= \boxed{}$

n) $\frac{15}{8} - \frac{5}{8} =$

$=$ ……… $=$ ……… $= \boxed{}$

o) $\frac{19}{12} - \frac{5}{12} =$

$=$ ……… $=$ ……… $= \boxed{}$

Skill 4.3 Adding mixed numbers with the same denominator.

Mauve 1 1 2 **2** 3 3 4 **4**
Lime 1 **1** 2 2 3 **3** 4 4

- Add the whole numbers first.
- Add the numerators (top numbers of the fractions).
- Do not change the denominators.
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

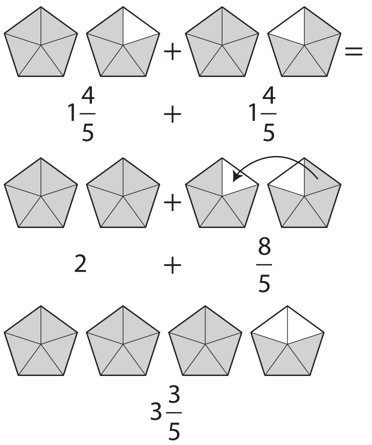
Q. $1\frac{4}{5}+1\frac{4}{5}=$

A. $1+1+\frac{4}{5}+\frac{4}{5}=$ Add the top numbers only

$=2+\frac{8}{5}$ Change to mixed number

$=2+1\frac{3}{5}$ Add the whole numbers

$=\mathbf{3\frac{3}{5}}$



a) $\overset{+}{\overbrace{2\frac{4}{7}+3}}\frac{2}{7}=$ Add the whole numbers

$=5+\frac{6}{7}$ = $5\frac{6}{7}$

b) $2\frac{4}{9}+1\frac{4}{9}=$

= ………… =

c) $1\frac{3}{11}+4\frac{7}{11}=$

= ………… =

d) $\overset{+}{\overbrace{1\frac{1}{6}+1}}\frac{5}{6}=$

$=2+\frac{6}{6}$

$=2+1$ =

e) $2\frac{7}{10}+3\frac{1}{10}=$

= …………

= ………… =

f) $2\frac{5}{12}+1\frac{4}{12}=$

= …………

= ………… =

g) $\overset{+}{\overbrace{2\frac{5}{7}+1}}\frac{3}{7}=$

$=3+\frac{8}{7}$ Change to mixed number

$=3+1\frac{1}{7}$ =

h) $2\frac{4}{9}+2\frac{7}{9}=$

= …………

= ………… =

i) $3\frac{9}{10}+5\frac{8}{10}=$

= …………

= ………… =

j) $\overset{+}{\overbrace{1\frac{3}{8}+2}}\frac{7}{8}=$

$=3+\frac{10}{8}$

$=3+1\frac{2^{\div 2}}{8_{\div 2}}$ Simplify

$=3+1\frac{1}{4}$ =

k) $2\frac{5}{6}+3\frac{5}{6}=$

= …………

= …………

= ………… =

l) $3\frac{5}{12}+2\frac{10}{12}=$

= …………

= …………

= ………… =

Skill 4.4 Subtracting mixed numbers with the same denominator (1).

Mauve 1 1 **2** 2 3 3 4 **4**
Lime **1** 1 2 2 **3** 3 4 4

- Change mixed numbers to improper fractions before subtracting. (see skill 4.1, page 31)
- Subtract the numerators.
- Do not change the denominators.
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

Q. $3\frac{3}{8} - 1\frac{5}{8} =$

A. $3\frac{3}{8} - 1\frac{5}{8} =$ Change to improper fractions

$3\frac{3}{8} = \frac{3\times 8+3}{8} = \frac{27}{8}$

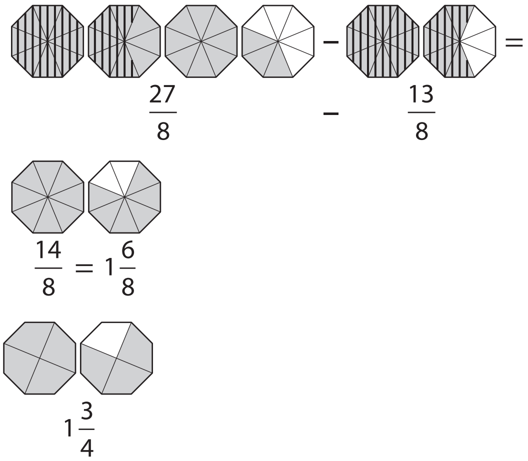
$1\frac{5}{8} = \frac{1\times 8+5}{8} = \frac{13}{8}$

$= \frac{27}{8} - \frac{13}{8}$ Subtract the top numbers only

$= \frac{14}{8}$ Change to mixed number

$= 1\frac{6^{\div 2}}{8_{\div 2}}$ Simplify

$= 1\frac{3}{4}$



a) $3\frac{1}{5} - 1\frac{4}{5} =$

$= \frac{16}{5} - \frac{9}{5}$ Subtract the top numbers only

$= \frac{7}{5}$ $= \boxed{1\frac{2}{5}}$

b) $5\frac{5}{7} - 2\frac{6}{7} =$

=

= =

c) $4\frac{3}{11} - 2\frac{9}{11} =$

=

= =

d) $4\frac{7}{8} - 1\frac{5}{8} =$

$= \frac{39}{8} - \frac{13}{8}$

$= \frac{26}{8}$ Change to mixed number

$= 3\frac{2^{\div 2}}{8_{\div 2}}$ =

e) $4\frac{9}{12} - 2\frac{5}{12} =$

=

=

= =

f) $5\frac{7}{9} - 3\frac{1}{9} =$

=

=

= =

g) $4\frac{2}{9} - 2\frac{5}{9} =$

=

=

= =

h) $3\frac{3}{8} - 1\frac{7}{8} =$

=

=

= =

i) $4\frac{3}{10} - 2\frac{7}{10} =$

=

=

= =

Skill 4.4 Subtracting mixed numbers with the same denominator (2).

Mauve 1 1 2 **2** 3 3 4 **4**
Lime 1 **1** 2 2 3 **3** 4 4

- Subtract the whole numbers first.
- Subtract the numerators.
- Do not change the denominators.
- Simplify the resulting fraction if necessary. (see skill 4.1, page 31)

Hint: For subtractions you may need to convert 1 to an equivalent fraction.

Example: 1 whole circle

$1 = \frac{3}{3} = \frac{5}{5}$ numerator = denominator

Q. $3\frac{3}{8}-1\frac{5}{8}=$

A. $\overset{\overset{-}{\frown}}{3\frac{3}{8}-1}\frac{5}{8}=$

$=2+\frac{3}{8}-\frac{5}{8}$

$=1+1+\frac{3}{8}-\frac{5}{8}$

$=1+\boxed{\frac{8}{8}}+\frac{3}{8}-\frac{5}{8}$

$=1+\frac{11}{8}-\frac{5}{8}$

$=1+\frac{6^{\div 2}}{8_{\div 2}}$

$=1+\frac{3}{4}$

$=\mathbf{1\frac{3}{4}}$

$3-1=2$ and $\frac{3}{8}-\frac{5}{8}=?$

$\frac{5}{8}$ can not be subtracted from $\frac{3}{8}$ and give a positive answer, so borrow a 1 from the 2.

$1=\frac{8}{8}$ (see hint)

$\frac{8}{8}+\frac{3}{8}=\frac{8+3}{8}=\frac{11}{8}$

$\frac{11}{8}-\frac{5}{8}=\frac{11-5}{8}=\frac{6}{8}$

Simplify.

j) $\overset{\overset{-}{\frown}}{5\frac{7}{8}-3}\frac{1}{8}=$

$=2+\frac{7}{8}-\frac{1}{8}$

$=2+\frac{6^{\div 2}}{8_{\div 2}}$

$=2+\frac{3}{4}$ $=\boxed{2\frac{3}{4}}$

k) $4\frac{11}{12}-1\frac{1}{12}=$

=

=

= $=\boxed{}$

l) $3\frac{11}{15}-1\frac{2}{15}=$

=

=

= $=\boxed{}$

m) $\overset{\overset{-}{\frown}}{5\frac{1}{4}-3}\frac{3}{4}=$

$=2+\frac{1}{4}-\frac{3}{4}$

$=1+1+\frac{1}{4}-\frac{3}{4}$

$=1+\boxed{\frac{4}{4}}+\frac{1}{4}-\frac{3}{4}$

$=1+\frac{2^{\div 2}}{4_{\div 2}}$

$=1+\frac{1}{2}$ $=\boxed{}$

n) $3\frac{1}{3}-1\frac{2}{3}=$

=

=

=

=

= $=\boxed{}$

o) $3\frac{1}{15}-1\frac{6}{15}=$

=

=

=

=

= $=\boxed{}$

Skill 4.5 Subtracting a mixed number from a whole number (1).

Mauve 11 22 33 44
Lime 11 22 33 44

- Change the mixed number to an improper fraction before subtracting. (see skill 4.1, page 31)
- Write the whole number as an improper fraction with the same denominator as the mixed number.
- Subtract the numerators.
- Do not change the denominators.
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

Q. $3-1\frac{2}{9}=$

A. $3-1\frac{2}{9}=$ Change to improper fractions

$=\frac{3}{1}-\frac{11}{9}=$

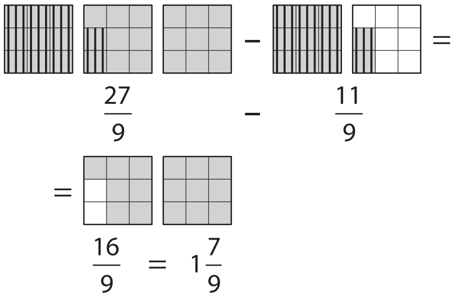
$=\frac{27}{9}-\frac{11}{9}$ Subtract the top numbers only

$=\frac{16}{9}$ Change to mixed number

$=\mathbf{1\frac{7}{9}}$

3 can be written as: $\frac{3}{1}$

$\frac{3}{1}=\frac{27}{9}$ and $1\frac{2}{9}=\frac{1\times 9+2}{9}=\frac{11}{9}$



a) $2-\frac{2}{7}=$

$=\frac{2}{1}-\frac{2}{7}$ ....

$=\frac{14}{7}-\frac{2}{7}$ Subtract the top numbers only ....

$=\frac{12}{7}$ .... $=\boxed{1\frac{5}{7}}$

b) $3-\frac{4}{9}=$

= ....

= ....

= .... =

c) $2-\frac{3}{7}=$

= ....

= ....

= .... =

d) $2-\frac{6}{13}=$

= ....

= ....

= .... =

e) $6-\frac{5}{8}=$

= ....

= ....

= .... =

f) $4-\frac{5}{12}=$

= ....

= ....

= .... =

g) $4-2\frac{1}{6}=$

= ....

= ....

= .... =

h) $5-1\frac{3}{7}=$

= ....

= ....

= .... =

i) $7-3\frac{5}{8}=$

= ....

= ....

= .... =

Skill 4.5 Subtracting a mixed number from a whole number (2).

- Subtract the whole numbers first.
- Borrow 1 from the whole number and write it as a fraction with the same denominator.
- Subtract the numerators.
- Do not change the denominators.

Q. $3-1\frac{2}{9}=$

A. $2-\frac{2}{9}=$

$=1+1-\frac{2}{9}$

$=1+\frac{9}{9}-\frac{2}{9}$

$=1+\frac{7}{9}$

$=\mathbf{1\frac{7}{9}}$

3 – 1 = 2 and 2 = 1 + 1

$1=\frac{9}{9}$

$\frac{9}{9}-\frac{2}{9}=\frac{9-2}{9}=\frac{7}{9}$

j) $5-\frac{2}{3}=$

$=4+1-\frac{2}{3}$

$=4+\frac{3}{3}-\frac{2}{3}$

$=4+\frac{1}{3}$ $=\boxed{4\frac{1}{3}}$

k) $2-\frac{3}{4}=$

$=$

$=$

$=$ $=\boxed{}$

l) $6-\frac{4}{7}=$

$=$

$=$

$=$ $=\boxed{}$

m) $4-\frac{1}{2}=$

$=$

$=$

$=$ $=\boxed{}$

n) $3-\frac{7}{8}=$

$=$

$=$

$=$ $=\boxed{}$

o) $5-\frac{7}{10}=$

$=$

$=$

$=$ $=\boxed{}$

p) $3-1\frac{3}{5}=$

$=2-\frac{3}{5}$

$=1+1-\frac{3}{5}$

$=1+\frac{5}{5}-\frac{3}{5}$

$=1+\frac{2}{5}$ $=\boxed{}$

q) $5-3\frac{3}{8}=$

$=$

$=$

$=$

$=$ $=\boxed{}$

r) $4-1\frac{5}{6}=$

$=$

$=$

$=$

$=$ $=\boxed{}$

Skill 4.6 Adding fractions with different denominators - one denominator divides evenly into the other denominator (1).

- Find the lowest common denominator of the fractions, which is the Lowest Common Multiple (LCM) of the denominators. In this case the LCM is the largest denominator.
- Change the fractions to equivalent fractions with the lowest common denominator.
- Add the fractions with the same denominators. (see skill 4.1, page 31)
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

Hint: If unsure which is the LCM of the denominators, use their product as the common denominator.

Examples:

$$\frac{5}{6}+\frac{1^{\times 3}}{2_{\times 3}}=\frac{5}{6}+\frac{3}{6}=\frac{8^{\div 2}}{6_{\div 2}}=\frac{4}{3}=1\frac{1}{3}$$ (LCM of 6 and 2 is 6, because 2 divides evenly into 6)

OR

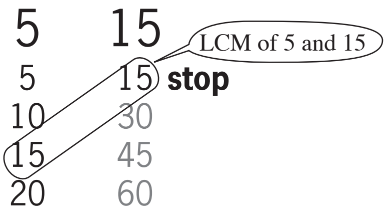
$$\frac{5}{6}+\frac{1}{2}=\frac{10}{12}+\frac{6}{12}=\frac{16^{\div 4}}{12_{\div 4}}=\frac{4}{3}=1\frac{1}{3}$$ (common denominator of 6 and 2 is $6 \times 2 = 12$)

To find the Lowest Common Multiple (LCM) of two numbers

- Write in ascending order some multiples of the smaller number first.
- Write in ascending order some multiples of the bigger number and stop when you find a multiple that appears in the first list ⇒ Lowest Common Multiple (LCM).

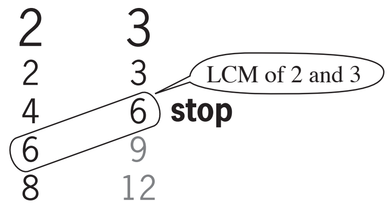
Hint: The lowest common multiple is the smallest number that the two numbers divide into.

LCM when one number divides evenly into the other number



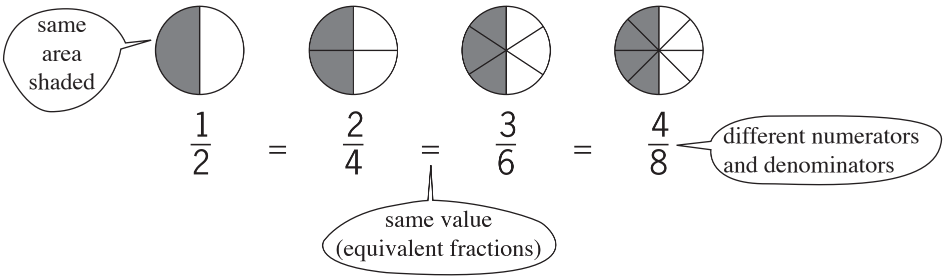
Hint: LCM is the largest number.

LCM when the numbers have NO common factors other than 1



Hint: LCM is the product of the numbers.

To find equivalent fractions



$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}$$

$$\frac{1^{\times 2}}{2_{\times 2}}=\frac{2}{4} \qquad \frac{1^{\times 3}}{2_{\times 3}}=\frac{3}{6} \qquad \frac{1^{\times 4}}{2_{\times 4}}=\frac{4}{8}$$

Equivalent fractions have the same value.
Equivalent fractions are formed by multiplying the numerator and denominator by the same number.

Skill 4.6 Adding fractions with different denominators - one denominator divides evenly into the other denominator (2).

Q. $\frac{1}{4}+\frac{3}{8}=$

A. $\frac{1}{4}+\frac{3}{8}=$ LCM of 4 and 8 is 8

$=\frac{1^{\times 2}}{4_{\times 2}}+\frac{3}{8}$ Multiply the numerator and denominator by 2

$\frac{1^{\times 2 = 2}}{4}$ because 8 ÷ 4 = 2

$=\frac{2+3}{8}$ Add the top numbers only

$=\frac{5}{8}$

OR

A. $\frac{1}{4}+\frac{3}{8}=$ Use 4 × 8 = 32 as the common denominator

$=\frac{1^{\times 8}}{4_{\times 8}}+\frac{3^{\times 4}}{8_{\times 4}}$

$\frac{1^{\times 8 = 8}}{4}$ because 32 ÷ 4 = 8

$=\frac{8+12}{32}$

$\frac{3^{\times 4 = 12}}{8}$ because 32 ÷ 8 = 4

$=\frac{20^{\div 4}}{32_{\div 4}}$ Simplify

$=\frac{5}{8}$

a) $\frac{7}{10}+\frac{3}{20}=$ LCM of 10 and 20 is 20

$=\frac{7^{\times 2}}{10_{\times 2}}+\frac{3}{20}$

$=\frac{14+3}{20}$ $=\boxed{\quad}$

b) $\frac{2}{7}+\frac{2}{21}=$

$=$

$=$ $=\boxed{\quad}$

c) $\frac{1}{6}+\frac{5}{12}=$

$=$

$=$ $=\boxed{\quad}$

d) $\frac{1}{5}+\frac{3}{10}=$ LCM of 5 and 10 is 10

$=\frac{1^{\times 2}}{5_{\times 2}}+\frac{3}{10}$

$=\frac{2+3}{10}$ Add the top numbers only

$=\frac{5^{\div 5}}{10_{\div 5}}$ $=\boxed{\quad}$

e) $\frac{2}{3}+\frac{5}{6}=$

$=$

$=$

$=$ $=\boxed{\quad}$

f) $\frac{5}{8}+\frac{1}{2}=$

$=$

$=$

$=$ $=\boxed{\quad}$

g) $\frac{3}{4}+\frac{5}{8}=$

$=$

$=$

$=$ $=\boxed{\quad}$

h) $\frac{4}{5}+\frac{7}{10}=$

$=$

$=$

$=$ $=\boxed{\quad}$

i) $\frac{1}{5}+\frac{1}{20}=$

$=$

$=$

$=$ $=\boxed{\quad}$

j) $\frac{3}{4}+\frac{11}{12}=$

$=$

$=$

$=$ $=\boxed{\quad}$

k) $\frac{3}{10}+\frac{3}{50}=$

$=$

$=$

$=$ $=\boxed{\quad}$

l) $\frac{2}{3}+\frac{1}{12}=$

$=$

$=$

$=$ $=\boxed{\quad}$

Skill 4.7 Adding fractions with different denominators - the HCF of the denominators is 1 (e.g. 2 and 3, 5 and 6).

- Find the lowest common denominator of the fractions, which is the Lowest Common Multiple (LCM) of the denominators. In this case the LCM is the product of the denominators. (see skill 4.6, page 39)
- Change the fractions to equivalent fractions with the lowest common denominator.
- Add the fractions with the same denominators. (see skill 4.1, page 31)
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

Q. $\frac{1}{2}+\frac{1}{3}=$

A. $\frac{1}{2}+\frac{1}{3}=$ LCM of 2 and 3 is 6

$\frac{1^{\times 3 = \boxed{3}}}{2}$ because 6 ÷ 2 = 3

$\frac{1^{\times 2 = \boxed{2}}}{3}$ because 6 ÷ 3 = 2

Multiply the numerator and denominator by 3 $= \frac{1^{\times 3}}{2_{\times 3}}+\frac{1^{\times 2}}{3_{\times 2}}$ Multiply the numerator and denominator by 2

$= \frac{\boxed{3}+\boxed{2}}{6}$ Add the top numbers only

$= \frac{5}{6}$

a) $\frac{2}{5}+\frac{1}{8}=$ LCM of 5 and 8 is 40

= ....

= .... = ☐

b) $\frac{1}{3}+\frac{3}{10}=$

= ....

= .... = ☐

c) $\frac{2}{3}+\frac{1}{11}=$

= ....

= .... = ☐

d) $\frac{1}{3}+\frac{3}{4}=$ LCM of 3 and 4 is 12

$= \frac{1^{\times 4}}{3_{\times 4}}+\frac{3^{\times 3}}{4_{\times 3}}$ ....

$= \frac{4+9}{12}$ Add the top numbers only ....

$= \frac{13}{12}$ .... = ☐

e) $\frac{2}{3}+\frac{2}{5}=$

= ....

= ....

= .... = ☐

f) $\frac{3}{4}+\frac{3}{5}=$

= ....

= ....

= .... = ☐

g) $\frac{4}{5}+\frac{1}{2}=$

= ....

= ....

= .... = ☐

h) $\frac{4}{7}+\frac{1}{2}=$

= ....

= ....

= .... = ☐

i) $\frac{1}{3}+\frac{7}{8}=$

= ....

= ....

= .... = ☐

Skill 4.8 Adding fractions with different denominators - the denominators have common factors ≠ 1.

- Find the lowest common denominator of the fractions, which is the Lowest Common Multiple (LCM) of the denominators. (see skill 4.6, page 39)
- Change the fractions to equivalent fractions with the lowest common denominator.
- Add the fractions with the same denominators. (see skill 4.1, page 31)
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

  Hint: If unsure which is the LCM of the denominators, use their product as the common denominator.

Q. $\frac{1}{4}+\frac{1}{6}=$

A. $\frac{1}{4}+\frac{1}{6}=$ LCM of 4 and 6 is 12

$=\frac{1^{\times 3}}{4_{\times 3}}+\frac{1^{\times 2}}{6_{\times 2}}$ $\quad \frac{1^{\times 3 = 3}}{4}$ because $12 \div 4 = 3$

$=\frac{3+2}{12}$ $\quad \frac{1^{\times 2 = 2}}{6}$ because $12 \div 6 = 2$

$=\mathbf{\frac{5}{12}}$

OR

A. $\frac{1}{4}+\frac{1}{6}=$ Use $4 \times 6 = 24$ as the common denominator

$=\frac{1^{\times 6}}{4_{\times 6}}+\frac{1^{\times 4}}{6_{\times 4}}$ $\quad \frac{1^{\times 6 = 6}}{4}$ because $24 \div 4 = 6$

$=\frac{6+4}{24}$ $\quad \frac{1^{\times 4 = 4}}{6}$ because $24 \div 6 = 4$

$=\frac{10^{\div 2}}{24_{\div 2}}$ Simplify

$=\mathbf{\frac{5}{12}}$

a) $\frac{5}{6}+\frac{1}{8}=$ LCM of 6 and 8 is 24

$=\frac{5^{\times 4}}{6_{\times 4}}+\frac{1^{\times 3}}{8_{\times 3}}$ Add the top numbers only

$=\frac{20+3}{24} = \boxed{\frac{23}{24}}$

b) $\frac{1}{4}+\frac{3}{10}=$

= ……………………………

= ……………………… = $\boxed{}$

c) $\frac{1}{6}+\frac{1}{15}=$

= ……………………………

= ……………………… = $\boxed{}$

d) $\frac{1}{12}+\frac{2}{9}=$

= ……………………………

= ……………………… = $\boxed{}$

e) $\frac{1}{10}+\frac{2}{25}=$ LCM of 10 and 25 is 50

= ……………………………

= ……………………… = $\boxed{}$

f) $\frac{3}{10}+\frac{4}{15}=$

= ……………………………

= ……………………… = $\boxed{}$

g) $\frac{3}{10}+\frac{5}{6}=$

= ……………………………

= ……………………………

= ……………………… = $\boxed{}$

h) $\frac{3}{4}+\frac{5}{6}=$

= ……………………………

= ……………………………

= ……………………… = $\boxed{}$

i) $\frac{3}{8}+\frac{7}{10}=$

= ……………………………

= ……………………………

= ……………………… = $\boxed{}$

Skill 4.9 Subtracting fractions with different denominators - one denominator divides evenly into the other denominator.

- Find the lowest common denominator of the fractions, which is the Lowest Common Multiple (LCM) of the denominators. In this case the LCM is the largest denominator. (see skill 4.6, page 39)
- Change the fractions to equivalent fractions with the lowest common denominator.
- Subtract the fractions with the same denominators. (see skill 4.2, page 33)
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

Hint: If unsure which is the LCM of the denominators, use their product as the common denominator.

Q. $\frac{2}{3}-\frac{4}{9}=$

A. $\frac{2}{3}-\frac{4}{9}=$ LCM of 3 and 9 is 9

$=\frac{2^{\times 3}}{3_{\times 3}}-\frac{4}{9}$ Multiply the numerator and denominator by 3

$\frac{2^{\times 3 = 6}}{3}$ because $9 \div 3 = 3$

$=\frac{6-4}{9}$ Subtract the top numbers only

$=\frac{2}{9}$

OR **A.** $\frac{2}{3}-\frac{4}{9}=$ Use 3 × 9 = 27 as the common denominator

$=\frac{2^{\times 9}}{3_{\times 9}}-\frac{4^{\times 3}}{9_{\times 3}}$

$\frac{2^{\times 9 = 18}}{3}$ because $27 \div 3 = 9$

$\frac{4^{\times 3 = 12}}{9}$ because $27 \div 9 = 3$

$=\frac{18-12}{27}$

$=\frac{6^{\div 3}}{27_{\div 3}}$ Simplify

$=\frac{2}{9}$

a) $\frac{3}{4}-\frac{1}{8}=$ LCM of 4 and 8 is 8

$=\frac{3^{\times 2}}{4_{\times 2}}-\frac{1}{8}$ ....

$=\frac{6-1}{8}$ Subtract the top numbers only .... $=\boxed{\frac{5}{8}}$

b) $\frac{3}{4}-\frac{3}{16}=$ LCM of 4 and 16 is 16

$=$ ....

$=$ .... $=$ ☐

c) $\frac{3}{7}-\frac{1}{14}=$

$=$ ....

$=$ .... $=$ ☐

d) $\frac{1}{5}-\frac{1}{15}=$

$=$ ....

$=$ .... $=$ ☐

e) $\frac{1}{5}-\frac{2}{25}=$

$=$ ....

$=$ .... $=$ ☐

f) $\frac{7}{12}-\frac{1}{2}=$

$=$ ....

$=$ .... $=$ ☐

g) $\frac{3}{4}-\frac{3}{20}=$

$=$ ....

$=$ ....

$=$ .... $=$ ☐

h) $\frac{1}{6}-\frac{1}{24}=$

$=$ ....

$=$ ....

$=$ .... $=$ ☐

i) $\frac{4}{18}-\frac{1}{9}=$

$=$ ....

$=$ ....

$=$ .... $=$ ☐

Skill 4.10 Subtracting fractions with different denominators - the HCF of the denominators is 1 (e.g. 2 and 3, 5 and 6).

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 **3** 3 4 4

- Find the lowest common denominator of the fractions, which is the Lowest Common Multiple (LCM) of the denominators. In this case the LCM is the product of the denominators. (see skill 4.6, page 39)
- Change the fractions to equivalent fractions with the lowest common denominator.
- Subtract the fractions with the same denominators. (see skill 4.2, page 33)
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

Q. $\frac{1}{3} - \frac{1}{5} =$

A. $\frac{1}{3} - \frac{1}{5} =$ LCM of 3 and 5 is 15

$\frac{1}{3}^{\times 5} = 5$ because 15 ÷ 3 = 5

$\frac{1}{5}^{\times 3} = 3$ because 15 ÷ 5 = 3

Multiply the numerator and denominator by 5

$= \frac{1^{\times 5}}{3_{\times 5}} - \frac{1^{\times 3}}{5_{\times 3}}$

Multiply the numerator and denominator by 3

$= \frac{5-3}{15}$ Subtract the top numbers only

$= \mathbf{\frac{2}{15}}$

a) $\frac{3}{8} - \frac{1}{5} =$ LCM of 8 and 5 is 40

$= \frac{3^{\times 5}}{8_{\times 5}} - \frac{1^{\times 8}}{5_{\times 8}}$

$= \frac{15-8}{40} = \boxed{\frac{7}{40}}$

b) $\frac{3}{5} - \frac{1}{2} =$

$=$

$=$ $=$ ☐

c) $\frac{3}{5} - \frac{1}{6} =$

$=$

$=$ $=$ ☐

d) $\frac{2}{3} - \frac{5}{8} =$

$=$

$=$ $=$ ☐

e) $\frac{2}{3} - \frac{1}{4} =$

$=$

$=$ $=$ ☐

f) $\frac{7}{9} - \frac{1}{2} =$

$=$

$=$ $=$ ☐

g) $\frac{2}{5} - \frac{2}{7} =$

$=$

$=$ $=$ ☐

h) $\frac{6}{7} - \frac{1}{2} =$ LCM of 7 and 2 is 14

$=$

$=$ $=$ ☐

i) $\frac{2}{5} - \frac{1}{4} =$

$=$

$=$ $=$ ☐

j) $\frac{4}{7} - \frac{3}{8} =$

$=$

$=$ $=$ ☐

k) $\frac{7}{10} - \frac{2}{3} =$

$=$

$=$ $=$ ☐

l) $\frac{9}{11} - \frac{2}{3} =$

$=$

$=$ $=$ ☐

Skill 4.11 Subtracting fractions with different denominators - the denominators have common factors ≠ 1.

- Find the lowest common denominator of the fractions, which is the Lowest Common Multiple (LCM) of the denominators. (see skill 4.6, page 39)
- Change the fractions to equivalent fractions with the lowest common denominator.
- Subtract the fractions with the same denominators. (see skill 4.2, page 33)
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

Hint: If unsure which is the LCM of the denominators, use their product as the common denominator.

Q. $\frac{3}{8}-\frac{1}{6}=$

A. $\frac{3}{8}-\frac{1}{6}=$ LCM of 8 and 6 is 24

$=\frac{3^{\times 3}}{8_{\times 3}}-\frac{1^{\times 4}}{6_{\times 4}}$ $\qquad \frac{3^{\times 3 = 9}}{8}$ because $24 \div 8 = 3$

$=\frac{9-4}{24}$ $\qquad \frac{1^{\times 4 = 4}}{6}$ beacuse $24 \div 6 = 4$

$=\frac{5}{24}$

OR

A. $\frac{3}{8}-\frac{1}{6}=$ Use 8 × 6 = 48 as the common denominator

$=\frac{3^{\times 6}}{8_{\times 6}}-\frac{1^{\times 8}}{6_{\times 8}}$ $\qquad \frac{3^{\times 6 = 18}}{8}$ because $48 \div 8 = 6$

$=\frac{18-8}{48}$ $\qquad \frac{1^{\times 8 = 8}}{6}$ beacuse $48 \div 6 = 8$

$=\frac{10^{\div 2}}{48_{\div 2}}$ Simplify

$=\frac{5}{24}$

a) $\frac{9}{10}-\frac{3}{4}=$ LCM of 10 and 4 is 20

$=\frac{9^{\times 2}}{10_{\times 2}}-\frac{3^{\times 5}}{4_{\times 5}}$ Subtract the top numbers only

$=\frac{18-15}{20} = \boxed{\frac{3}{20}}$

b) $\frac{3}{8}-\frac{1}{10}=$

= ....

= .... = $\boxed{}$

c) $\frac{3}{10}-\frac{4}{15}=$

= ....

= .... = $\boxed{}$

d) $\frac{4}{9}-\frac{5}{12}=$

= ....

= .... = $\boxed{}$

e) $\frac{3}{8}-\frac{1}{12}=$ LCM of 8 and 12 is 24

= ....

= .... = $\boxed{}$

f) $\frac{5}{9}-\frac{4}{15}=$

= ....

= .... = $\boxed{}$

g) $\frac{4}{15}-\frac{1}{6}=$

= ....

= ....

= .... = $\boxed{}$

h) $\frac{3}{10}-\frac{5}{25}=$

= ....

= ....

= .... = $\boxed{}$

i) $\frac{3}{10}-\frac{1}{6}=$

= ....

= ....

= .... = $\boxed{}$

Skill 4.12 Adding and subtracting fractions with different denominators.

- Find the lowest common denominator of the fractions, which is the Lowest Common Multiple (LCM) of the denominators. (see skills 4.6 to 4.11, pages 39 to 45)
- Change the fractions to equivalent fractions with the lowest common denominator.
- Add and/or subtract the fractions with the same denominators. (see skills 4.1, page 31 and 4.2, page 33)
- Simplify the resulting fraction and/or change it to a mixed number if necessary. (see skill 4.1, page 31)

Q. $\frac{3}{5}-\frac{1}{4}+\frac{3}{20}=$

A. $\frac{3}{5}-\frac{1}{4}+\frac{3}{20}=$

$=\frac{3^{\times 4}}{5_{\times 4}}-\frac{1^{\times 5}}{4_{\times 5}}+\frac{3}{20}$

$=\frac{12-5+3}{20}$

$=\frac{10^{\div 10}}{20_{\div 10}}$

$=\frac{\mathbf{1}}{\mathbf{2}}$

20 is the lowest common denominator.
5 divides into 20 four times.
Multiply the numerator and denominator of $\frac{3}{5}$ by 4.
4 divides into 20 five times.
Multiply the numerator and denominator of $\frac{1}{4}$ by 5.
20 divides into 20 once.
Leave $\frac{3}{20}$ unchanged.

a) $\frac{1}{2}+\frac{3}{8}-\frac{1}{4}=$ LCM of 2, 8 and 4 is 8

$=\frac{1^{\times 4}}{2_{\times 4}}+\frac{3}{8}-\frac{1^{\times 2}}{4_{\times 2}}$ Add and subtract numerators only

$=\frac{4+3-2}{8}$ $=\boxed{\frac{5}{8}}$

b) $\frac{1}{8}+\frac{7}{16}-\frac{1}{2}=$

=

= =

c) $\frac{5}{6}-\frac{1}{2}+\frac{1}{12}=$

=

= =

d) $\frac{1}{2}+\frac{1}{3}-\frac{3}{5}=$ LCM of 2, 3 and 5 is 30

=

= =

e) $\frac{2}{3}+\frac{1}{2}-\frac{6}{7}=$

=

= =

f) $\frac{3}{5}-\frac{1}{2}+\frac{1}{9}=$

=

= =

g) $\frac{7}{11}+\frac{5}{22}-\frac{1}{2}=$

=

=

= =

h) $\frac{7}{10}-\frac{2}{5}+\frac{1}{2}=$

=

=

= =

i) $\frac{6}{7}-\frac{1}{2}-\frac{1}{14}=$

=

=

= =

Skill 4.13 Adding or subtracting mixed numbers with different denominators.

- Add or subtract the whole numbers first.
- Add or subtract the fractions by finding the common denominator.
  (see skills 4.6 to 4.11, pages 39 to 45)

Q. $1\frac{1}{5}+1\frac{2}{3}=$

A. $1+1=2$ Add whole numbers.

$\frac{1}{5}+\frac{2}{3}$

$=\frac{1^{\times 3}}{5_{\times 3}}+\frac{2^{\times 5}}{3_{\times 5}}$

$=\frac{3+10}{15}$

$=\frac{13}{15}$

$=2+\frac{13}{15}=\mathbf{2\frac{13}{15}}$

15 is the lowest common denominator.
5 divides into 15 three times.
Multiply the numerator and denominator of $\frac{1}{5}$ by 3.
3 divides into 15 five times.
Multiply the numerator and denominator of $\frac{2}{3}$ by 5.

a) $1\frac{1}{6}+2\frac{2}{9}=3+\ldots.$ Add whole numbers

$\frac{1}{6}+\frac{2}{9}$ LCM of 6 and 9 is 18

$=\frac{1^{\times 3}}{6_{\times 3}}+\frac{2^{\times 2}}{9_{\times 2}}=\frac{3+4}{18}=\frac{7}{18}$

$\Rightarrow 3+\frac{7}{18}=\boxed{3\frac{7}{18}}$

b) $2\frac{3}{4}+2\frac{3}{10}=$

=

=

$\Rightarrow \quad = \boxed{}$

c) $1\frac{3}{8}+1\frac{1}{10}=$

=

=

$\Rightarrow \quad = \boxed{}$

d) $1\frac{1}{2}+1\frac{1}{3}=$

=

=

$\Rightarrow \quad = \boxed{}$

e) $1\frac{2}{5}+1\frac{1}{2}=$

=

=

$\Rightarrow \quad = \boxed{}$

f) $1\frac{4}{7}+1\frac{1}{3}=$ LCM of 7 and 3 is 21

=

=

$\Rightarrow \quad = \boxed{}$

g) $2\frac{5}{7}-1\frac{1}{2}=$

=

=

$\Rightarrow \quad = \boxed{}$

h) $4\frac{3}{4}-1\frac{1}{3}=$

=

=

$\Rightarrow \quad = \boxed{}$

i) $3\frac{5}{6}-1\frac{2}{3}=$

=

=

$\Rightarrow \quad = \boxed{}$

5. [Fraction ×,÷]

Skill 5.1 Multiplying a fraction by a whole number (1).

Mauve 1 1 22 33 44
Lime 11 22 33 44

- Multiply the numerator of the fraction by the whole number.
- Do not change the denominator.
- Simplify the resulting fraction and/or change it to a mixed number if necessary.

EITHER
- Cross simplify where possible before multiplying.

OR
- Simplify at the end.

To find the Highest Common Factor (HCF) of two numbers

- Write all the factors of each number (the factors must divide exactly into the number).
- Find the largest number that appears on both lists.
  Hint: The Highest Common Factor is the largest number that divides evenly into both numbers.

HCF for Identical numbers

5	**5**
1	1
5	5

HCF of 5 and 5

Hint: 5 is the HCF of 5 and 5 because 5 is the largest number that divides into 5 and 5.

HCF when one number divides evenly into the other number

4	**12**
1	1
2	2
4	3
	4
	6
	12

HCF of 4 and 12

Hint: 4 is the HCF of 4 and 12 because 4 is the largest number that divides into 4 and 12.

To change an improper fraction to a mixed number

numerator

$\frac{7}{2}$ IMPROPER FRACTION

denominator

- Divide the numerator by the denominator.

  $\frac{7}{2} = 7 \div 2 = 3$ remainder 1
- Write the result as the whole number and the remainder over the denominator.

  3 remainder 1 = $3\frac{1}{2}$

To cross multiply a fraction and a whole number

- Simplify the denominator of the fraction and the whole number. This means to divide them by the same number, usually by their Highest Common Factor.
- Cross out the denominator of the fraction and the whole number.
- Write the result of the division next to each crossed number.
- Multiply the top numbers together.

$$\frac{3}{10}\times 5 = \frac{3}{10_{\div 5}}\times 5^{\div 5}$$

Divide 5 and 10 by 5

$$= \frac{3}{{}_{2}\cancel{10}}\times \cancel{5}^{1}$$

5 ÷ 5 = 1
10 ÷ 5 = 2

$$= \frac{3}{2} = 1\frac{1}{2}$$

Skill 5.1 Multiplying a fraction by a whole number (2).

Q. $\frac{3}{4}\times 2=$

A. $\frac{3}{\cancel{4}_{2}}\times \cancel{2}^{1}=$ Divide 4 and 2 by 2

$=\frac{3\times 1}{2}$

$=\frac{3}{2}$ Change to mixed number

$=1\frac{1}{2}$

OR

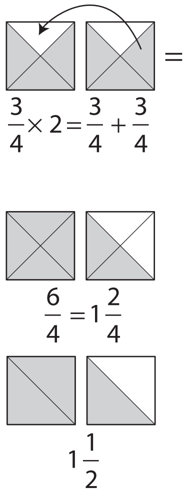
A. $\frac{3}{4}\times 2=$ Multiply 3 by 2

$=\frac{3\times 2}{4}$

$=\frac{6}{4}$

$=1\frac{2^{\div 2}}{4_{\div 2}}$ Simplify

$=1\frac{1}{2}$



=

$\frac{3}{4}\times 2=\frac{3}{4}+\frac{3}{4}$

=

$\frac{6}{4}=1\frac{2}{4}$

=

$1\frac{1}{2}$

a) $4\times\frac{3}{7}=$

$=\frac{4\times 3}{7}$

$=\frac{12}{7}$ = $\boxed{1\frac{5}{7}}$

b) $\frac{2}{9}\times 5=$

= ……………………

= …………………… = ☐

c) $6\times\frac{2}{5}=$

= ……………………

= …………………… = ☐

d) $\frac{7}{10}\times 3=$

= ……………………

= …………………… = ☐

e) $2\times\frac{6}{7}=$

= ……………………

= …………………… = ☐

f) $\frac{1}{3}\times 4=$

= ……………………

= …………………… = ☐

g) $6\times\frac{1}{12}=$

$=\cancel{6}^{1}\times\frac{1}{\cancel{12}_{2}}$ Divide 6 and 12 by 6

$=\frac{1\times 1}{2}$ = ☐

h) $3\times\frac{5}{9}=$

= ……………………

= …………………… = ☐

i) $2\times\frac{1}{8}=$

= ……………………

= …………………… = ☐

j) $5\times\frac{3}{10}=$

= ……………………

= …………………… = ☐

k) $10\times\frac{2}{15}=$

= ……………………

= …………………… = ☐

l) $8\times\frac{5}{6}=$

= ……………………

= …………………… = ☐

Skill 5.2 Multiplying two fractions (1).

- Multiply the numerators of the fractions.
- Multiply the denominators of the fractions.

To simplify:

EITHER
- Simplify where possible before multiplying.

OR
- Simplify at the end.

To cross multiply two fractions

- Simplify the numbers in the fractions diagonally (in a cross). This means to divide top and bottom numbers by the same number, usually by their Highest Common Factor. (see skill 5.1, page 49)
- Cross out the numbers in the fractions diagonally (in a cross).
- Write the result of the division next to each crossed number.
- Multiply the top results together.
- Multiply the bottom results together.

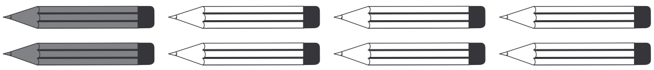
Divide 3 and 9 by 3

Divide 4 and 8 by 4

$$\frac{3}{4}\times\frac{8}{9}=\frac{\overset{\div 3}{3}}{\underset{\div 4}{4}}\times\frac{\overset{\div 4}{8}}{\underset{\div 3}{9}}=\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}}\times\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}}=\frac{1\times 2}{1\times 3}=\frac{2}{3}$$

“Of” means “times”

A quarter of a box of 8 pencils equals 2.



4 ÷ 4 = 1
8 ÷ 4 = 2

$$\frac{1}{4}\text{ of }8=\frac{1}{\underset{1}{\cancel{4}}}\times\overset{2}{\cancel{8}}=2$$

Divide 4 and 8 by 4

Q. $\frac{2}{3}\times\frac{3}{4}=$

A. $\frac{2}{3}\times\frac{3}{4}=$

$$=\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}}$$

Divide 3 and 3 by 3

Divide 2 and 4 by 2

$$=\frac{1\times 1}{1\times 2}$$

$$=\boldsymbol{\frac{1}{2}}$$

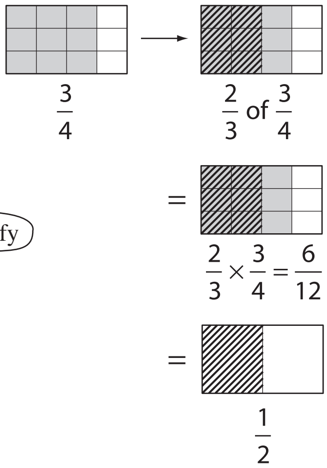
OR **A.** $\frac{2}{3}\times\frac{3}{4}=$

$$=\frac{2\times 3}{3\times 4}$$

$$=\frac{6^{\div 6}}{12_{\div 6}}$$

Simplify

$$=\boldsymbol{\frac{1}{2}}$$



a) $\frac{3}{4}\times\frac{1}{5}=$

$=\frac{3\times 1}{4\times 5}$ .... $=\boxed{\frac{3}{20}}$

b) $\frac{2}{3}\times\frac{1}{7}=$

$=$ .... $=\boxed{}$

c) $\frac{1}{6}\times\frac{5}{8}=$

$=$ .... $=\boxed{}$

Skill 5.2 Multiplying two fractions (2).

d) $\frac{5}{8}\times\frac{3}{10}=$

$=\frac{\overset{1}{\cancel{5}}}{8}\times\frac{3}{\underset{2}{\cancel{10}}}$

Divide
5 and 10 by 5

$=\frac{1\times 3}{8\times 2}=\boxed{}$

e) $\frac{3}{4}\times\frac{2}{5}=$

$=$

$= \qquad = \boxed{}$

f) $\frac{2}{3}\times\frac{6}{11}=$

$=$

$= \qquad = \boxed{}$

g) $\frac{2}{3}\times\frac{3}{8}=$

$=\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{8}}}$

Divide
3 and 3 by 3

Divide
2 and 8 by 2

$=\frac{1\times 1}{1\times 4}=\boxed{}$

h) $\frac{7}{10}\times\frac{2}{7}=$

$=$

$= \qquad = \boxed{}$

i) $\frac{4}{9}\times\frac{9}{20}=$

$=$

$= \qquad = \boxed{}$

j) $\frac{5}{16}\times\frac{4}{5}=$

$=$

$= \qquad = \boxed{}$

k) $\frac{2}{11}\times\frac{11}{12}=$

$=$

$= \qquad = \boxed{}$

l) $\frac{3}{20}\times\frac{5}{6}=$

$=$

$= \qquad = \boxed{}$

m) $\frac{6}{10}\times\frac{5}{24}=$

$=$

$= \qquad = \boxed{}$

n) $\frac{5}{7}\times\frac{14}{25}=$

$=$

$= \qquad = \boxed{}$

o) $\frac{11}{12}\times\frac{6}{22}=$

$=$

$= \qquad = \boxed{}$

p) $\frac{2}{15}\times\frac{5}{6}=$

$=$

$= \qquad = \boxed{}$

q) $\frac{7}{27}\times\frac{18}{35}=$

$=$

$= \qquad = \boxed{}$

r) $\frac{6}{40}\times\frac{8}{15}=$

$=$

$= \qquad = \boxed{}$

s) $\frac{4}{9}\times\frac{3}{10}=$

$=$

$= \qquad = \boxed{}$

t) $\frac{7}{8}\times\frac{6}{21}=$

$=$

$= \qquad = \boxed{}$

u) $\frac{8}{15}\times\frac{3}{10}=$

$=$

$= \qquad = \boxed{}$

Skill 5.3 Multiplying a mixed number by a fraction or by another mixed number.

- Change the mixed numbers to improper fractions before multiplying. (see skill 5.1, page 49)
- Multiply the numerators of the fractions.
- Multiply the denominators of the fractions.

  To simplify:

EITHER
- Cross simplify where possible before multiplying.
  (see skill 5.2, page 51)

OR
- Simplify at the end.

Q. $2\frac{3}{5}\times1\frac{1}{4}=$

A. $2\frac{3}{5}\times1\frac{1}{4}=$

Change to improper fractions

$=\frac{13}{\cancel{5}_{1}}\times\frac{\overset{1}{\cancel{5}}}{4}$

Divide 5 and 5 by 5

$=\frac{13\times1}{1\times4}$

Change to mixed number

$=\frac{13}{4}$

$=\mathbf{3\frac{1}{4}}$

OR

A. $2\frac{3}{5}\times1\frac{1}{4}=$

$=\frac{13}{5}\times\frac{5}{4}$

$=\frac{13\times5}{5\times4}$

$=\frac{65^{\div5}}{20_{\div5}}$

Simplify

$=\frac{13}{4}=\mathbf{3\frac{1}{4}}$

a) $1\frac{1}{2}\times\frac{5}{6}=$

$=\frac{\overset{1}{\cancel{3}}}{2}\times\frac{5}{\cancel{6}_{2}}$

Divide 3 and 6 by 3

$=\frac{1\times5}{2\times2}$

$=\frac{5}{4}$ $=\boxed{1\frac{1}{4}}$

b) $1\frac{1}{11}\times3\frac{2}{3}=$

=

=

= =

c) $3\frac{3}{4}\times1\frac{1}{5}=$

=

=

= =

d) $4\frac{1}{2}\times\frac{4}{15}=$

=

=

= =

e) $1\frac{1}{5}\times1\frac{7}{8}=$

=

=

= =

f) $1\frac{4}{5}\times\frac{5}{12}=$

=

=

= =

g) $3\frac{1}{6}\times\frac{3}{4}=$

=

=

= =

h) $2\frac{7}{10}\times1\frac{2}{3}=$

=

=

= =

i) $\frac{5}{6}\times1\frac{5}{7}=$

=

=

= =

Skill 5.4 Multiplying three fractions.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 **4** 4

- Multiply the numerators of the fractions.
- Multiply the denominators of the fractions.

To simplify:

EITHER
- Cross simplify where possible before multiplying.

(see skill 5.2, page 51)

OR
- Simplify at the end.

Q. $\frac{1}{8}\times\frac{6}{7}\times\frac{7}{9}=$

A. $\frac{1}{8}\times\frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{7}}}\times\frac{\overset{1}{\cancel{7}}}{\underset{3}{\cancel{9}}}=$

Divide 7 and 7 by 7

Divide 6 and 9 by 3

$=\frac{1}{\underset{4}{\cancel{8}}}\times\frac{\overset{1}{\cancel{2}}}{1}\times\frac{1}{3}$

Divide 8 and 2 by 2

$=\frac{1\times1\times1}{4\times1\times3}$

$=\boldsymbol{\frac{1}{12}}$

OR **A.** $\frac{1}{8}\times\frac{6}{7}\times\frac{7}{9}=$

$=\frac{1\times6\times7}{8\times7\times9}$

$=\frac{42}{504}\begin{smallmatrix}\div6\\ \div6\end{smallmatrix}$ Simplify

$=\frac{7}{84}\begin{smallmatrix}\div7\\ \div7\end{smallmatrix}$ Simplify

$=\boldsymbol{\frac{1}{12}}$

a) $\frac{1}{2}\times\frac{1}{4}\times\frac{4}{5}=$

Divide 4 and 4 by 4

$=\frac{1}{2}\times\frac{1}{\underset{1}{\cancel{4}}}\times\frac{\overset{1}{\cancel{4}}}{5}$

$=\frac{1\times1\times1}{2\times1\times5}=\boxed{\frac{1}{10}}$

b) $\frac{2}{5}\times\frac{2}{3}\times\frac{1}{2}=$

= ……………………

= ……………… = ☐

c) $\frac{2}{3}\times\frac{1}{8}\times\frac{3}{4}=$

= ……………………

= ……………… = ☐

d) $\frac{5}{6}\times\frac{3}{4}\times\frac{2}{5}=$

= ……………………

= ……………… = ☐

e) $\frac{3}{4}\times\frac{3}{10}\times\frac{5}{6}=$

= ……………………

= ……………… = ☐

f) $\frac{7}{10}\times\frac{2}{3}\times\frac{6}{7}=$

= ……………………

= ……………… = ☐

g) $\frac{7}{9}\times\frac{2}{14}\times\frac{3}{4}=$

= ……………………

= ……………… = ☐

h) $\frac{2}{3}\times\frac{9}{10}\times\frac{4}{9}=$

= ……………………

= ……………… = ☐

i) $\frac{3}{10}\times\frac{5}{12}\times\frac{6}{15}=$

= ……………………

= ……………… = ☐

j) $\frac{4}{5}\times\frac{10}{11}\times\frac{3}{8}=$

= ……………………

= ……………… = ☐

k) $\frac{5}{6}\times\frac{4}{15}\times\frac{9}{16}=$

= ……………………

= ……………… = ☐

l) $\frac{5}{11}\times\frac{11}{18}\times\frac{6}{25}=$

= ……………………

= ……………… = ☐

Skill 5.5 Dividing two fractions.

- Copy the first fraction and change “divide by” (÷) into “times” (×).
- Invert the second fraction.
- Multiply the numerators of the fractions.
- Multiply the denominators of the fractions.

To simplify:

EITHER

- Cross simplify where possible before multiplying.
(see skill 5.2, page 51)

OR

- Simplify at the end.

Q. $\frac{1}{6}\div\frac{3}{8}=$

A. $\frac{1}{6}\div\frac{3}{8}=$

Invert second fraction

$=\frac{1}{6}\times\frac{8}{3}$

Divide 6 and 8 by 2

$=\frac{1}{\cancel{6}_{3}}\times\frac{\cancel{8}^{4}}{3}$

$=\frac{1\times 4}{3\times 3}$

$=\mathbf{\frac{4}{9}}$

OR **A.** $\frac{1}{6}\div\frac{3}{8}=$

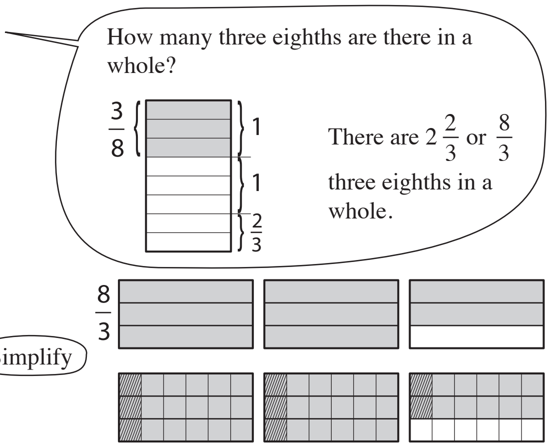
$=\frac{1}{6}\times\frac{8}{3}$

$=\frac{1\times 8}{6\times 3}$

$=\frac{8^{\div 2}}{18_{\div 2}}$

Simplify

$=\mathbf{\frac{4}{9}}$



$\frac{1}{6}$ of $\frac{8}{3}=\frac{1}{6}\times\frac{8}{3}=\frac{8}{18}$

a) $\frac{1}{8}\div\frac{3}{5}=$

$=\frac{1}{8}\times\frac{5}{3}$

$=\frac{1\times 5}{8\times 3}$ $=\boxed{\frac{5}{24}}$

b) $\frac{1}{3}\div\frac{3}{4}=$

=

= ☐

c) $\frac{2}{7}\div\frac{5}{8}=$

=

= ☐

d) $\frac{3}{10}\div\frac{3}{4}=$

$=\frac{3}{10}\times\frac{4}{3}$

Divide 3 and 3 by 3

$=\frac{{}^{1}\cancel{3}}{{}_{5}\cancel{10}}\times\frac{\cancel{4}^{2}}{\cancel{3}_{1}}$

Divide 10 and 4 by 2

$=\frac{1\times 2}{5\times 1}$ = ☐

e) $\frac{5}{8}\div\frac{1}{2}=$

=

=

= ☐

f) $\frac{2}{3}\div\frac{1}{6}=$

=

=

= ☐

g) $\frac{5}{8}\div\frac{5}{12}=$

=

=

= ☐

h) $\frac{9}{10}\div\frac{3}{8}=$

=

=

= ☐

i) $\frac{5}{9}\div\frac{15}{18}=$

=

=

= ☐

Skill 5.6 Dividing a whole number by a fraction.

- Copy the whole number and change "divide by" (÷) into "times" (×).
- Invert the fraction.
- Multiply the whole number by the numerator of the fraction. Don't change the denominator. To simplify:

EITHER

- Cross simplify where possible before multiplying. (see skill 5.2, page 51)

OR

- Simplify at the end.

Q. $2 \div \frac{1}{4} =$

A. $2 \div \frac{1}{4} =$ Invert fraction

$= 2 \times \frac{4}{1}$

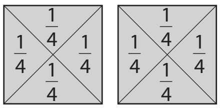
$= \frac{2 \times 4}{1}$

$= \frac{8}{1}$

$= \boldsymbol{8}$

How many quarters are there in two wholes?

There are 8 quarters in two wholes.



$2 \div \frac{1}{4} = 2 \times 4 = 8$

a) $5 \div \frac{5}{6} =$ Invert fraction

$= 5 \times \frac{6}{5}$ ....

$= \overset{1}{\cancel{5}} \times \frac{6}{\cancel{5}_1}$ Divide 5 and 5 by 5 ....

$= \frac{1 \times 6}{1}$ .... $= \boxed{6}$

b) $2 \div \frac{2}{7} =$

= ....

= ....

= .... = ☐

c) $7 \div \frac{7}{9} =$

= ....

= ....

= .... = ☐

d) $4 \div \frac{2}{5} =$

= ....

= ....

= .... = ☐

e) $9 \div \frac{3}{8} =$

= ....

= ....

= .... = ☐

f) $8 \div \frac{1}{2} =$

= ....

= ....

= .... = ☐

g) $12 \div \frac{8}{11} =$

= ....

= ....

= .... = ☐

h) $8 \div \frac{6}{7} =$

= ....

= ....

= .... = ☐

i) $10 \div \frac{4}{9} =$

= ....

= ....

= .... = ☐

Skill 5.7 Dividing a fraction by a whole number.

- Copy the fraction and write the whole number as an improper fraction with denominator 1.
- Change “divide by” (÷) into “times” (×).
- Invert the second fraction.
- Multiply the numerators of the fractions.
- Multiply the denominators of the fractions.

To simplify:

EITHER

- Cross simplify where possible before multiplying.
(see skill 5.2, page 51)

OR

- Simplify at the end.

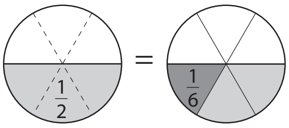
Q. $\frac{1}{2} \div 3 =$

A. $\frac{1}{2} \div 3 =$

$= \frac{1}{2} \div \boxed{\frac{3}{1}} =$ Invert second fraction

$= \frac{1}{2} \times \boxed{\frac{1}{3}} = \frac{1 \times 1}{2 \times 3} = \mathbf{\frac{1}{6}}$

What is one half divided into 3 equal parts?



$\frac{1}{2} \div 3 = \frac{1}{6}$

a) $\frac{4}{5} \div 8 =$

$= \frac{4}{5} \div \boxed{\frac{8}{1}}$

$= \frac{4}{5} \times \boxed{\frac{1}{8}}$ Divide 4 and 8 by 4

$= \frac{\overset{1}{\cancel{4}}}{5} \times \frac{1}{\underset{2}{\cancel{8}}} = \boxed{\frac{1}{10}}$

b) $\frac{3}{5} \div 9 =$

= ……………………

= ……………………

= …………………… = ☐

c) $\frac{6}{7} \div 3 =$

= ……………………

= ……………………

= …………………… = ☐

d) $\frac{6}{7} \div 12 =$

= ……………………

= ……………………

= …………………… = ☐

e) $\frac{3}{8} \div 15 =$

= ……………………

= ……………………

= …………………… = ☐

f) $\frac{8}{9} \div 16 =$

= ……………………

= ……………………

= …………………… = ☐

g) $\frac{4}{11} \div 20 =$

= ……………………

= ……………………

= …………………… = ☐

h) $\frac{5}{12} \div 25 =$

= ……………………

= ……………………

= …………………… = ☐

i) $\frac{6}{13} \div 15 =$

= ……………………

= ……………………

= …………………… = ☐

Skill 5.8 Dividing a mixed number by a fraction or by another mixed number.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 **4**

- Change the mixed numbers to improper fractions before dividing. (see skill 5.1, page 49)
- Copy the first fraction and change “divide by” (÷) into “times” (×).
- Invert the second fraction.
- Multiply the numerators of the fractions.
- Multiply the denominators of the fractions.

To simplify:

EITHER
- Cross simplify where possible before multiplying. (see skill 5.2, page 51)

OR
- Simplify at the end.

Q. $2\frac{2}{3} \div 1\frac{1}{3} =$

A. $\frac{8}{3} \div \frac{4}{3} =$ Invert second fraction

$= \frac{8}{3} \times \boxed{\frac{3}{4}}$

Divide 8 and 4 by 4

Divide 3 and 3 by 3

$= \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} = \frac{2}{1} = \mathbf{2}$

OR

A. $\frac{8}{3} \div \frac{4}{3} =$ Invert second fraction

$= \frac{8}{3} \times \boxed{\frac{3}{4}}$

$= \frac{8 \times 3}{3 \times 4}$

Simplify

$= \frac{24^{\div 12}}{12_{\div 12}} = \frac{2}{1} = \mathbf{2}$

a) $2\frac{1}{4} \div \frac{3}{4} =$ Change to improper fraction

$= \frac{9}{4} \div \frac{3}{4}$

$= \frac{9}{4} \times \boxed{\frac{4}{3}}$

$= \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}$ $= \boxed{3}$

b) $1\frac{3}{5} \div \frac{4}{5} =$

$=$ ....

$=$ ....

$=$ .... $= \boxed{}$

c) $5\frac{1}{3} \div 4 =$

$=$ ....

$=$ ....

$=$ .... $= \boxed{}$

d) $2\frac{3}{4} \div 6\frac{3}{5} =$

$=$ ....

$=$ ....

$=$ .... $= \boxed{}$

e) $2\frac{2}{9} \div 1\frac{3}{7} =$

$=$ ....

$=$ ....

$=$ .... $= \boxed{}$

f) $3\frac{1}{3} \div 3\frac{3}{4} =$

$=$ ....

$=$ ....

$=$ .... $= \boxed{}$

g) $2\frac{2}{5} \div \frac{6}{25} =$

$=$ ....

$=$ ....

$=$ .... $= \boxed{}$

h) $2\frac{1}{3} \div 1\frac{5}{9} =$

$=$ ....

$=$ ....

$=$ .... $= \boxed{}$

i) $1\frac{5}{6} \div 1\frac{5}{12} =$

$=$ ....

$=$ ....

$=$ .... $= \boxed{}$

6. [Percentages]

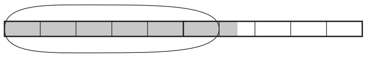
Skill 6.1 Estimating a percentage.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

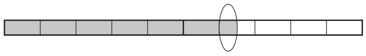
- Picture the amount shaded as out of 100.
- Count the known parts.
- Compare to common parts like one half equals 50%.

Q. What percentage is shown on the bar?

A. *65%*



6 out of 10 parts are shaded.
That much is 60%



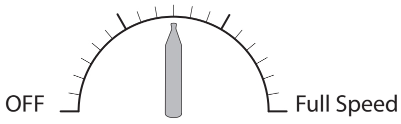
Plus half of another part.
So add another 5%.

a) Estimate the percentage of the line between the arrows.

2 out of 5 parts

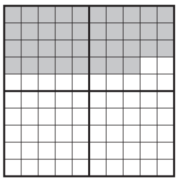
40%

b) What percentage of full speed has been reached?

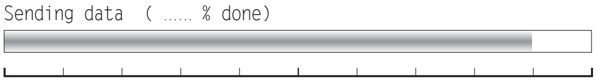


c) What percentage of the file has been transferred?

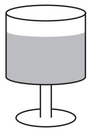
d) What percentage of the grid is shaded?



e) What percentage of data has been sent?



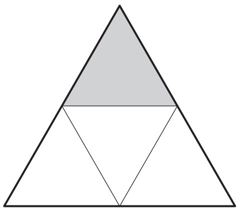
f) Which glass is 75% full?

A) 

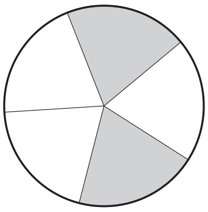
B) 

C) 

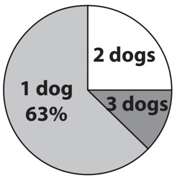
g) What percentage of the diagram is shaded?



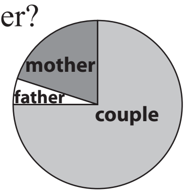
h) What percentage of the diagram is shaded?



i) What percentage of dog owners own 3 dogs?



j) In Australia 5% of adolescents live with their father. What percentage of adolescents live with their mother?



Skill 6.2 Finding the remaining percentage.

- Subtract the given percentages from one whole or 100% to find the remaining percentage.

Q. An aluminium solder is made up of 65% zinc, 20% aluminium and the rest copper. What percentage is copper?

A. *100% – 65% – 20%*
= ***15%***

a) A lingerie item was made up of 67% polyamide, 14% elastane and cotton. What percentage was cotton?

100% – 67% – 14% = 19%

b) German silver is made up of 55% copper, 25% zinc and the rest nickel. What percentage is nickel?

.... = ☐

c) The energy in walnuts comes from proteins, carbohydrates and fats. If 5% comes from proteins and 5% from carbohydrates, how much energy is supplied by fats?

.... = ☐

d) Australia's water use is divided between 16% urban/industrial, 77% irrigation and the rest, "other rural". What percentage is "other rural"?

.... = ☐

e) Lou's coat is made up of 3% spandex, 21% nylon and the rest rayon. How much rayon is in the coat?

.... = ☐

f) A cafe latte is made up of 30% coffee, 5% froth and the rest is milk. What percentage of a cafe latte is milk?

.... = ☐

g) The eastern states and territories of Australia make up 37% of Australia's area. If the central region makes up for another 30%, what percentage does Western Australia cover?

.... = ☐

h) Of the slopes at Perisher Blue 22% are classified Beginner, 60% Intermediate and the rest Advanced. What percentage of slopes are Advanced?

.... = ☐

i) 1% of the earth's atmosphere is a mixture of gases, 78% is nitrogen, and the rest is oxygen. How much of our atmosphere is oxygen?

.... = ☐

j) Normann Stadler, winner of the 2006 World Triathlon Championships, spent 11% of his time swimming, 53% riding and the rest running. What percentage of time did he run?

.... = ☐

Skill 6.3 Finding a percentage of a multiple of 100.

EITHER
- Change the percentage to a fraction out of 100.

  Example: $40\% = \frac{40}{100}$
- Rewrite the question as a multiplication (change "of" to "×").
- Change the whole number to a fraction over 1.

  Example: $7 = \frac{7}{1}$
- Cross simplify the fractions before multiplying.
  (see skill 5.2, page 51)

OR
- First find 10%.
- Then multiply by the amount needed to make the required percentage.

To find $10\% = \frac{1}{10}$ ⇒ divide by 10
5% ⇒ half of 10%
$20\% = \frac{1}{5}$ ⇒ divide by 5
$25\% = \frac{1}{4}$ ⇒ divide by 4
$50\% = \frac{1}{2}$ ⇒ divide by 2

Q. 60% of 300 =

A. $\frac{60}{1\cancel{00}} \times \frac{3\cancel{00}}{1} =$ Simplify: ÷ 100
$= 60 \times 3$
$= \boldsymbol{180}$

OR **A.** $\frac{1}{1\cancel{0}} \times \frac{30\cancel{0}}{1} =$ Find 10%
$= 30$
30×6 Multiply by 6 to get 60%
$= \boldsymbol{180}$

a) 40% of 200 =
$\frac{40}{1\cancel{00}} \times \frac{2\cancel{00}}{1}$ Divide by 100
= 80

b) 10% of 500 =
$\frac{10}{100} \times \frac{500}{1}$
=

c) 20% of 300 = First find 10%
=

d) 3% of 700 =
=

e) 25% of 300 =
=

f) 8% of 400 =
=

g) 70% of \$600 =
= \$

h) 2% of \$700 =
= \$

i) 5% of \$300 =
= \$

j) 55% of \$10.00 =
= \$

k) 75% of \$20.00 =
= \$

l) 6% of \$30.00 =
= \$

m) 45% of \$20.00 =
= \$

n) 25% of \$900 =
= \$

o) 8% of \$5.00 =
= \$

Skill 6.4 Finding a percentage of any number (1).

- Change the percentage to a fraction out of 100.

  Example: $60\% = \frac{60}{100}$
- Rewrite the question as a multiplication (change “of” to “×”).
- Change the whole number to a fraction over of 1.

  Example: $3 = \frac{3}{1}$
- Cross simplify the fractions before multiplying.
  (see skill 5.2, page 51)

OR

- First find 10%.
- Then multiply by the amount needed to make the required percentage, i.e. multiply by 3 to get 30%.

To find $1\% = \frac{1}{100} \Rightarrow$ divide by 100

$12.5\% = \frac{1}{8} \Rightarrow$ divide by 8

$33\frac{1}{3}\% = \frac{1}{3} \Rightarrow$ divide by 3

$66\frac{2}{3}\% = \frac{2}{3} \Rightarrow$ divide by 3
multiply by 2

Q. 35% of 60 =

A. $\frac{35}{10\cancel{0}} \times \frac{6\cancel{0}}{1} =$ Simplify: ÷ 10

$= \frac{35}{10} \times \frac{6}{1}$

$= \frac{21\cancel{0}}{1\cancel{0}}$ Simplify: ÷ 10

$= \mathbf{21}$

OR **A.** $60 \div 10 =$ Find 10%

$= 6$

6×3

$= 18$ Multiply by 3 to get 30%

$\frac{1}{2} \times 6 = 3$ Half of 10% is 5%

$= 18 + 3$ Add 30% and 5%

$= \mathbf{21}$

a) 70% of 10 =

$= \frac{7\cancel{0}}{1\cancel{0}\cancel{0}} \times \frac{1\cancel{0}}{1}$ Simplify: Divide by 10, 2 times

$= 7 \times 1 = \boxed{7}$

b) 10% of 180 =

=

= =

c) 15% of 60 =

=

= =

d) 30% of 400 =

$400 \div 10 = 40$ First find 10%

$40 \times 3 =$

e) 20% of 10 =

=

f) 70% of 20 =

=

g) 5% of 180 =

=

h) 25% of 20 =

=

i) 75% of 56 =

=

Skill 6.4 Finding a percentage of any number (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

j) 12% of 125 =

=

= =

k) 24% of 50 =

=

= =

l) 80% of 16 =

=

= =

m) 14% of 50 =

=

=

n) 8% of 600 =

=

=

o) 13% of 300 =

=

=

p) 30% of 70 =

=

=

q) 15% of 50 =

=

=

r) 20% of 75 =

=

=

s) 25% of 180 =

=

=

t) 16% of 50 =

=

=

u) 2.5% of 800 =

=

=

v) 1% of 45 =

=

= =

w) 12.5% of 16 =

=

= =

x) $66\frac{2}{3}$% of 60 =

=

= =

y) 0.5% of 260 =

=

= =

z) $33\frac{1}{3}$% of 72 =

=

= =

A) 0.1% of 300 =

=

= =

Skill 6.5 Calculating percentages from word problems.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Write a number sentence from the information given.
- Calculate the percentage of the given amount. (see skills 6.3, page 61 and 6.4, page 62)

Q. Finn ate lunch at a restaurant. He left a 5% tip. If the bill was $52, what was the total cost of his lunch?

A. $Tip = \frac{5}{100} \times \frac{52}{1} =$

$= 5 \times 0.52$

$= \$2.60$

$Total\ cost = \$52 + \$2.60$

$= \mathbf{\$54.60}$

a) India won 50 medals at the Commonwealth Games in 2006. Gold medals made up 44% of India's medal tally. How many medals were gold?

=

b) Archie leaves an extra 5% of the restaurant bill as a tip. The bill was $150. How much was the tip?

= $

c) Petrol goes up by 5% tomorrow. If I can pay $1.40 per litre today, what will I pay tomorrow?

= $

d) The average life expectancy at birth in 1995 was 64 years. In 2025 it will grow by 12.5%. What will the life expectancy be in 2025?

=

e) In 2010 we consumed on average 500 g of carbohydrate per day. In 1980 we consumed 80% of this amount. How many grams of carbohydrate did we consume per day in 1980?

= g

f) The population of Whyalla in South Australia increased by 20% in the 20 years to 1981. If the population was 13 600 in 1961, what was it in 1981?

=

g) The global population reached 6 billion in 1999. What will the global population be in 2025 if it will grow by 30%?

= billion

h) You weigh 90 kg. If you lose 5% of your body weight, calculate your weight loss.

= kg

i) An island's population of 450 people increased by 2%. How many people live there now?

=

j) You weigh 70 kg. If you gain 5% of your body weight, calculate your weight gain.

= kg

Skill 6.6 Working with more than 100%.

To find 200% $= \frac{2}{1} \Rightarrow$ multiply by 2

300% $= \frac{3}{1} \Rightarrow$ multiply by 3

EITHER
- Change the percentage to a fraction out of 100.

  Example: $150\% = \frac{150}{100}$
- Rewrite the question as a multiplication (change “of” to “×”).
- Change the whole number to a fraction over 1.

  Example: $7 = \frac{7}{1}$
- Cross simplify the fractions before multiplying. (see skill 5.2, page 51)

OR
- First find 100% or other multiples of 100%.
- Then find the remaining percentage.
- Add the results.

Q. 200% of 45 =

A. $\frac{2\cancel{00}}{1\cancel{00}} \times \frac{45}{1} =$ Simplify: ÷ 100

$= 2 \times 45$

$= \mathbf{90}$

OR

A. *100% is 45.*
So 200% is double that or ***90****.*

a) 120% of 30 =

$= \frac{12\cancel{0}}{10\cancel{0}} \times \frac{3\cancel{0}}{1}$ Simplify: ÷ 10, twice

$= 12 \times 3$ = $\boxed{36}$

Add the 100% and 10%

b) 110% of 160 =

Find 10%

10% of 160 = 16

160 + 16 = $\boxed{}$

c) 200% of 70 =

= $\boxed{}$

d) 300% of 28 =

= $\boxed{}$

e) 120% of 30 =

= $\boxed{}$

f) How much is 400% of 34?

= $\boxed{}$

g) How much is 150% of 450?

= $\boxed{}$

h) 110% of 230 =

= $\boxed{}$

i) 105% of 32 =

= $\boxed{}$

j) How much is 101% of 400?

= $\boxed{}$

k) 125% of 80 =

= $\boxed{}$

l) 105% of 380 =

= $\boxed{}$

m) How much is 250% of 500?

= $\boxed{}$

n) 120% of 60 =

= $\boxed{}$

o) 110% of 250 =

= $\boxed{}$

Skill 6.7 Increasing an amount by a percentage.

- Calculate the percentage of the given amount. (see skills 6.3, page 61 and 6.4, page 62)
- Add this result to the given amount.

Hint: If an amount is increased by 20% it will become 120% of its original value.

Q. Increase 30 by 20%.

A. $\frac{2\cancel{0}}{1\cancel{0}\cancel{0}} \times \frac{3\cancel{0}}{1} =$ Simplify: ÷ 10, twice

$= 2 \times 3 = 6$

$6 + 30$ Add the 20% to 30

$= \mathbf{36}$

a) Increase 400 by 2%.

1% is 4 so 2% is 8

8 + 400 = 408

b) Increase 70 by 10%.

c) Increase 310 by 50%.

d) Increase 80 by 20%.

e) Increase 600 by 1%.

f) Increase 56 by 25%.

g) Increase 40 by 15%.

h) Increase 300 by 12%.

i) Increase 52 by 50%.

j) Increase 80 by 75%.

k) Increase 64 by 12.5%.

l) Increase 300 by 2%.

m) Increase 15 by 80%.

n) Increase 60 by 45%.

o) Increase 90 by 60%.

p) Increase 500 by 12%.

q) Increase 2500 by 8%.

r) Increase 800 by 5%.

s) Increase 750 by 30%.

t) Increase 90 by 40%.

u) Increase 240 by 12.5%.

Skill 6.8 Decreasing an amount by a percentage.

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 3 **3** 4 4

- Calculate the percentage of the given amount. (see skills 6.3, page 61 and 6.4, page 62)
- Subtract this result from the given amount.

Hint: If an amount is decreased by 20% it will become 80% of its original value.

Q. Decrease $35 by 5%.

A. $\frac{5}{100} \times \frac{35}{1} =$ Multiply

$= \frac{175}{100} = \$1.75$

$\$35 - \$1.75$ Subtract the 5% from $35

$= \boldsymbol{\$33.25}$

a) Reduce 700 by 1%.

1% of 700 is 7

700 − 7 = $\boxed{693}$

b) Reduce 500 by 10%.

c) Decrease 4000 by 11%.

d) Decrease 2300 by 4%.

e) Decrease 500 by 75%.

f) Reduce 20 by 15%.

g) Reduce 75 by 20%.

h) Decrease 120 by 5%.

i) Reduce 120 by 40%.

j) Decrease 350 by 2%.

k) Reduce 600 by 95%.

l) Decrease 25 by 4%.

m) Reduce 55 by 60%.

n) Reduce 800 by 9%.

o) Decrease 220 by 30%.

p) Reduce 150 by 8%.

q) Reduce 200 by 46%.

r) Decrease 500 by 6%.

s) Reduce 330 by 70%.

t) Decrease 800 by 21%.

u) Reduce 520 by 55%.

Skill 6.9 Calculating an amount given a percentage of that amount.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Write the words as an equation.

EITHER

- Bring the percentage to 100% by methods like doubling or first finding 1%, 5% or 10%.

OR

- Use algebra.

Q. If 75% or 24 students in the class are boys, how many students are in the class?

A. $\frac{75}{100} \times x = 24$ Write the words as an equation.

$x = \overset{8}{\cancel{24}} \times \frac{\overset{4}{\cancel{100}}}{\cancel{75}_{\cancel{3}\,1}}$ Get the unknown amount alone on one side of the equation.

Simplify: ÷ 25 then ÷ 3

$x = 8 \times 4$ Simplify by dividing both the top and the bottom of the fractions by common factors.

$x = \mathbf{32}$ Complete the multiplication.

a) If 20% of the cost is $13, what is the total cost?

If 20% of x is $13 then 10% is half or $6.50

Find 100%

$100\% = 10\% \times 10 = \$6.50 \times 10$ $\boxed{\$65}$

b) The tank is 5% full and has 800 L of water in it. How much water will the tank hold when full?

L

c) Maria's iPod has 400 songs, and only 20% of its memory is full. How many more songs can Maria load on her iPod?

d) In a bag of potatoes, 7 are rotten. If this is 25% of the bag, how many potatoes are in the bag altogether?

e) In a certain college 60% of all students are female. If 90 students are female, how many students are at the college altogether?

f) The pool has 15 000 L of water and it is 30% full. How many litres of water are in the pool when it is 100% full?

L

g) In the railway carriage there are 95 people. This is 25% more than the number of seats. How many seats are in this carriage?

$x + \frac{25}{100} \times x = 95$

$x =$

$x =$

h) Petrol has gone up 75% in the last 3 years to $1.40 cents per litre. How much per litre was petrol 3 years ago?

$x =$

$x =$ $

Skill 6.10 Finding a percentage change.

- Find the difference between the amounts (amount of change).
- Divide the amount of change by the original amount.
- Multiply by 100 to find the percentage.

$$\text{percentage change} = \frac{\text{amount of change}}{\text{original amount}} \times \frac{100}{1}\%$$

Q. Petrol prices have risen from $0.75 per litre in 2000 to $1.11 per litre in 2006. Find the percentage increase.

A. $\$1.11 - \$0.75 = \$0.36$ Subtract to find the amount of change

$$\frac{0.36}{0.75} \times \frac{100}{1}\% = \frac{36}{\cancel{75}_3} \times \frac{\cancel{100}^4}{1}\%$$ Simplify: ÷ 25

$= 144 \div 3 =$ ***48%***

a) Charlie bought a car for $24 000 and later sold it for $18 000. Find the percentage loss.

$24\,000 - 18\,000 = 6\,000$

$$\frac{600\cancel{0}}{24\cancel{000}} \times \frac{1\cancel{00}}{1}\%$$ Simplify: ÷ 100 then ÷ 10

$$= \frac{\overset{50}{\cancel{600}}}{{}_2\cancel{24}}\% = \frac{\overset{25}{\cancel{50}}}{{}_1\cancel{2}}\%$$ Simplify: ÷ 12 then ÷ 2

$= \boxed{25\%}$

b) Lou's wage increased from $90/week to $99/week. What is the percentage increase?

$99 - 90 =$

= ……… = ☐

c) Jeans usually sell for $88 but today they are discounted to $66. What is the percentage decrease?

= ……… = ☐

d) Mac bought an old chair for $80, repaired it and sold it for $200. What percentage profit did Mac make?

= ……… = ☐

e) Kate's gas bill decreased from $300 to $258. What percentage saving is this?

= ……… = ☐

f) The year 9 enrolments increased from 120 to 150 students. What is the percentage increase?

= ……… = ☐

g) Charles sold his $60 text book at the second hand book shop for $40. Calculate the loss as a percentage of the cost price.

= ……… = ☐

h) In Feb 2003 the value of a BHP share was $9. By Jan 2011 a BHP share was selling for $45/share. Find the percentage increase.

= ……… = ☐

Skill 6.11 Finding a number knowing a percentage of that number.

Mauve 11 22 33 44
Lime 11 22 33 4**4**

- Write the words as an equation.

EITHER

- Bring the percent to 100% by methods like doubling or first finding 1%, 5% or 10%.

OR

- Use algebra.

Q. 25% of ☐ = 145

A. $\frac{25}{100} \times x = 145$

Simplify: ÷ 25

$x = 145 \times \frac{\cancel{100}^{4}}{\cancel{25}_{1}}$

$x = 145 \times 4$

$x = \mathbf{580}$

Write the % as a fraction.

Get the unknown amount (x) alone on one side of the equation.

Simplify by dividing both the top and the bottom of the fraction by common factors.

Complete the multiplication.

a) 20% of $\boxed{450}$ = 90

If 20% of x is 90 then 10% is half, so 45

100% = 10% × 10 = 45 × 10 = 450

b) 5% of ☐ = 20

If 5% of x is 20 then 10% is

Find 100%

c) 6% of ☐ = 21

Use algebra

d) 60% of ☐ = 150

e) 11% of ☐ = 22

$x = 22 \times \frac{100}{11}$

$x =$

f) 12% of ☐ = 54

$x =$

$x =$

g) 80% of ☐ = 76

$x =$

$x =$

h) 75% of ☐ = 525

$x =$

$x =$

i) 30% of ☐ = 54

$x =$

$x =$

j) 15% of ☐ = 75

$x =$

$x =$

7. [Decimals / Fractions / Percentages]

Skill 7.1 Ordering decimal numbers.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Line up the decimal numbers at their decimal points.
- Compare digits in the same places, starting from the left, until you find the smallest digit.
  Hint: The number with the smallest digit will be the smallest number.
- Look for the second smallest number.
- Continue in this way until you find the largest number.

Q. Place in order from smallest to largest:
0.325, 0.025, 0.035, 0.235

units, tenths, hundredths, thousandths

	U	.	T	H	Th
(largest) 4th	0	.	3	2	5
(smallest) 1st	0	.	0	2	5
2nd	0	.	0	3	5
3rd	0	.	2	3	5

A. ***0.025, 0.035, 0.235, 0.325***

Find the smallest digits.
Work from left to right.

Units: all 0
Tenths: 0 < 2 < 3
either 0.025 or 0.035
is the smallest
Hundredths: 2 < 3
so 0.025 is the smallest and
0.035 is 2nd smallest
Tenths: 2 < 3
so 0.235 is 3rd smallest
0.325 is the largest

a) Place in order from smallest to largest:
0.606, 0.66, 0.066, 0.06

U	.	*T*	*H*	*Th*
0	.	*6*	*0*	*6*
0	.	*6*	*6*	
0	.	*0*	*6*	*6*
0	.	*0*	*6*	

the smallest number

b) Place in order from largest to smallest:
3.041, 3.04, 3.104, 3.014

U	.	*T*	*H*	*Th*

c) Write in ascending order:
0.263, 0.236, 0.326, 0.362

d) Write in descending order:
0.052, 0.025, 0.05, 0.205

e) Write in descending order:
0.075, 0.507, 0.570, 0.057

f) Write in ascending order:
1.264, 1.064, 1.24, 1.246

g) Write in ascending order:
0.617, 0.706, 0.076, 0.176

h) Write in descending order:
3.28, 3.892, 3.298, 3.928

Skill 7.2 Ordering fractions.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find the lowest common denominator of the fractions, which is the Lowest Common Multiple (LCM) of the denominators.
- Change the fractions to equivalent fractions with the lowest common denominator.
- Arrange the fractions in order of the numerators (the smallest fraction has the smallest numerator and so on).

smallest numerator = smallest fraction

$$\frac{1}{\boxed{6}} < \frac{3}{\boxed{6}} < \frac{5}{\boxed{6}}$$

same denominator

Hints: If unsure which is the LCM of the denominators, use their product as the common denominator.
When the smaller denominators divide evenly into the biggest denominator, this biggest number becomes the common denominator.

Q. Place in ascending order:

$\frac{4}{5}, \frac{21}{25}, \frac{83}{100}$

A. $\boldsymbol{\frac{4}{5}, \frac{83}{100}, \frac{21}{25}}$

$\frac{4}{5}, \frac{21}{25}, \frac{83}{100}$ LCM of 5, 25 and 100 is 100

$\frac{4^{\times 20}}{5_{\times 20}} = \frac{80}{100}$ $4 \times 20 = 80$ because $100 \div 5 = 20$

$\frac{21^{\times 4}}{25_{\times 4}} = \frac{84}{100}$ $21 \times 4 = 84$ because $100 \div 25 = 4$

$\frac{83}{100} = \frac{83}{100}$

$80 < 83 < 84$, so $\frac{80}{100} < \frac{83}{100} < \frac{84}{100}$ or $\boldsymbol{\frac{4}{5} < \frac{83}{100} < \frac{21}{25}}$

a) Place in ascending order:

$\frac{2}{3}, \frac{5}{6}, \frac{13}{18}$ LCM of 3, 6 and 18 is 18

$\frac{2^{\times 6}}{3_{\times 6}} = \frac{12}{18}$ $\frac{5^{\times 3}}{6_{\times 3}} = \frac{15}{18}$ $\frac{13}{18}$

$\frac{12}{18} < \frac{13}{18} < \frac{15}{18}$

$\boxed{\frac{2}{3}, \frac{13}{18}, \frac{5}{6}}$

b) Place in descending order:

$\frac{21}{50}, \frac{2}{5}, \frac{43}{100}$

c) Place in ascending order:

$\frac{3}{4}, \frac{7}{9}, \frac{23}{36}$

d) Place in descending order:

$\frac{7}{10}, \frac{31}{50}, \frac{71}{100}$

e) Place in ascending order:

$\frac{13}{40}, \frac{3}{8}, \frac{1}{4}$

f) Place in descending order:

$\frac{7}{18}, \frac{29}{54}, \frac{4}{9}$

Skill 7.3 Finding equivalent fractions.

- Check if you need to multiply or divide the numerator or denominator of the first fraction to reach the numerator or denominator of the second fraction.
- Do the same operation to the top and bottom of the fraction.
  Example: $\frac{2}{3} \underset{\times 6}{=} \frac{\boxed{?}}{18} \Rightarrow \frac{2^{\times 6}}{3_{\times 6}} = \frac{\boxed{12}}{18}$ So $\frac{2}{3}$ and $\frac{12}{18}$ are equivalent fractions.
- Multiply or divide the numerator and denominator of the first fraction by the same number until you reach the second fraction.

Q. Complete the equivalent fractions:

$$\frac{30}{180} = \frac{\boxed{}}{18} = \frac{1}{\boxed{}}$$

A. $\frac{30}{180} \underset{\div 10}{=} \frac{?}{18} \Rightarrow \frac{30^{\div 10}}{180_{\div 10}} = \frac{\mathbf{3}}{18}$

and $\frac{30}{180} \overset{\div 30}{=} \frac{1}{?} \Rightarrow \frac{30^{\div 30}}{180_{\div 30}} = \frac{1}{\mathbf{6}}$

$$\Rightarrow \frac{30}{180} = \frac{\boxed{\mathbf{3}}}{18} = \frac{1}{\boxed{\mathbf{6}}}$$

a) Complete the equivalent fractions:

$\frac{5}{6} = \frac{15}{\boxed{18}}$

$\frac{5}{6} \overset{\times 3}{=} \frac{15}{?} \Rightarrow \frac{5^{\times 3}}{6_{\times 3}} = \frac{15}{18}$

b) Complete the equivalent fractions:

$\frac{5}{8} = \frac{\boxed{}}{200}$

c) Complete the equivalent fractions:

$\frac{85}{100} = \frac{17}{\boxed{}}$

d) Complete the equivalent fractions:

$\frac{3}{4} = \frac{\boxed{}}{20} = \frac{75}{\boxed{}}$

e) Complete the equivalent fractions:

$\frac{64}{144} = \frac{16}{\boxed{}} = \frac{\boxed{}}{9}$

f) Complete the equivalent fractions:

$\frac{20}{70} = \frac{10}{\boxed{}} = \frac{\boxed{}}{7}$

g) Complete the equivalent fractions:

$\frac{2}{5} = \frac{10}{\boxed{}} = \frac{\boxed{}}{75}$

h) Complete the equivalent fractions:

$\frac{50}{80} = \frac{\boxed{}}{40} = \frac{5}{\boxed{}}$

i) Complete the equivalent fractions:

$\frac{4}{9} = \frac{12}{\boxed{}} = \frac{\boxed{}}{81}$

j) $\frac{11\times10}{12\times10} = \frac{11}{12}$

True or false? $\boxed{\text{true}}$

Simplify: Divide by 10

$\frac{11\times10}{12\times10} = \frac{11\cancel{0}}{12\cancel{0}} = \frac{11}{12}$

k) $\frac{4+8}{5+8} = \frac{4}{5}$

True or false? $\boxed{}$

l) $\frac{100-6}{200-6} = \frac{1}{2}$

True or false? $\boxed{}$

Skill 7.4 Writing a decimal number as a percentage.

Mauve 1 1 **2** 2 3 3 4 4
Lime 1 **1** 2 2 3 3 4 4

- Multiply the decimal number by 100, by moving the decimal point two places to the right.
- Add the percentage sign.

Hint: Zeros can be added at the end of any decimal number: 0.4 = 0.4000

Q. Write 0.125 as a percentage.

A. $0.125 = 0.125 \times 100\%$ ← 2 zeros, 2 places to the right

$= \mathbf{12.5\%}$

a) Write 0.03 as a percentage.

$0.03 = 0.03 \times 100\%$ = 3%

b) Write 0.2 as a percentage.

0.2 = ... = ☐

c) Write 0.35 as a percentage.

.... = ... = ☐

d) Write 0.88 as a percentage.

.... = ... = ☐

e) Write 0.08 as a percentage.

.... = ... = ☐

f) Write 0.1 as a percentage.

.... = ... = ☐

g) Write 0.02 as a percentage.

.... = ... = ☐

h) Write 0.4 as a percentage.

.... = ... = ☐

i) Write 0.463 as a percentage.

.... = ... = ☐

j) Write 0.055 as a percentage.

.... = ... = ☐

k) Write 0.015 as a percentage.

.... = ... = ☐

l) Write 0.071 as a percentage.

.... = ... = ☐

m) Write 1.2 as a percentage.

.... = ... = ☐

n) Write 2.5 as a percentage.

.... = ... = ☐

o) Write 2.3 as a percentage.

.... = ... = ☐

p) Write 3.1 as a percentage.

.... = ... = ☐

q) Write 0.343 as a percentage.

.... = ... = ☐

r) Write 0.214 as a percentage.

.... = ... = ☐

Skill 7.5 Writing a percentage as a decimal number.

- Write the percentage as a fraction out of 100.
- Divide the numerator of the fraction by 100, by moving the decimal point two places to the left.

Hint: Fractions are just divisions.

There is a decimal point and zeros which are not written, at the end of any whole number: $27 = 27.00$

Zeros can also be added before the number: $27 = 027.00$

Q. Write 2.45% as a decimal number.

A. $2.45\% = \dfrac{2.45}{100}$

$= 2.45 \div 100$

$= 002.45 \div 100$ ← 2 zeros, 2 places to the left

$= \mathbf{0.0245}$

a) Write 9% as a decimal number.

$9\% = \dfrac{9}{100} = 009.0 \div 100 = \boxed{0.09}$

b) Write 4% as a decimal number.

$4\% = \quad = \quad = \boxed{\quad}$

c) Write 70% as a decimal number.

$= \quad = \quad = \boxed{\quad}$

d) Write 86% as a decimal number..

$= \quad = \quad = \boxed{\quad}$

e) Write 40% as a decimal number.

$= \quad = \quad = \boxed{\quad}$

f) Write 63% as a decimal number.

$= \quad = \quad = \boxed{\quad}$

g) Write 2.5% as a decimal number.

$= \quad = \quad = \boxed{\quad}$

h) Write 4.15% as a decimal number.

$= \quad = \quad = \boxed{\quad}$

i) Write 11.5% as a decimal number.

$= \quad = \quad = \boxed{\quad}$

j) Write 3.25% as a decimal number.

$= \quad = \quad = \boxed{\quad}$

k) Income tax is 15% for an income between \$6001 and \$21 600. Write the percentage as a decimal.

$= \quad = \quad = \boxed{\quad}$

l) Approximately 60% of the waste at the tip is household waste. Write this as a decimal.

$= \quad = \quad = \boxed{\quad}$

m) The interest rate of a major credit card is 17.5%. Write this as a decimal.

$= \quad = \quad = \boxed{\quad}$

n) Maximum legal blood alcohol concentration for drivers in NSW is 0.05%. What is this as a decimal?

$= \quad = \quad = \boxed{\quad}$

Skill 7.6 Writing a decimal number as a fraction in simplest form.

- Write the decimal number as the numerator of the fraction.
- Ignore any zeros at the start of the number.
- Use the place value of the last digit of the decimal number to determine the size of the denominator.
  Example:

units	*tenths*	*hundredths*
0 .	***0***	***8***

$= 8 \text{ hundredths} = \frac{8}{100}$

Write the 8 as the numerator

8 in hundredths place, denominator = 100

- Write the fraction in simplest form. Divide both the numerator and the denominator by the same number.
  Example: $\frac{8^{\div 4}}{100_{\div 4}} = \frac{2}{25}$

Hint: For the denominator, write 1 followed by one zero for each digit after the decimal point.
Example: $0.\underline{08} = \frac{8}{1\underline{00}}$

Q. Write 0.92 as a fraction in simplest form.

A. $0.\underline{92} = \frac{92}{1\underline{00}}$

Write the 92 as the numerator

2 zeros for 2 decimal places

Simplify: ÷ 4

$= \frac{92^{\div 4}}{100_{\div 4}} = \boldsymbol{\frac{23}{25}}$

a) Write 0.6 as a fraction in simplest form.

$0.6 = \frac{6^{\div 2}}{10_{\div 2}}$ Simplify: ÷ 2 $= \boxed{\frac{3}{5}}$

b) Write 0.02 as a fraction in simplest form.

$0.02 = \qquad = \boxed{}$

c) Write 0.12 as a fraction in simplest form.

$0.12 = \qquad = \boxed{}$

d) Write 0.05 as a fraction in simplest form.

$= \qquad = \boxed{}$

e) Write 0.45 as a fraction in simplest form.

$= \boxed{}$

f) Write 0.8 as a fraction in simplest form.

$= \boxed{}$

g) Write 0.2 as a fraction in simplest form.

$= \boxed{}$

h) Write 0.68 as a fraction in simplest form.

$= \boxed{}$

i) Write 0.84 as a fraction in simplest form.

$= \boxed{}$

j) Write 0.04 as a fraction in simplest form.

$= \boxed{}$

Skill 7.7 Writing a fraction as a terminating decimal.

When the denominator **is** a power of 10:

- Divide the numerator by the power of 10 by moving the decimal point to the left.

Example: $\frac{27}{100} = 27 \div 100$

$= 027.0 \div 100$ 2 zeros, 2 places to the left

$= 0.27$

Hints: Fractions are just divisions.
There is a decimal point and zeros which are not written, at the end of any whole number: $27 = 27.00$
Zeros can also be added before the number: $27 = 027.0$
The number of zeros in the denominator shows the number of digits after the decimal point.

$$\frac{27}{1\underline{00}} = 0.\underline{27}$$

When the denominator **is not** a power of 10:

EITHER

- Multiply both the numerator and denominator by the same number to make the denominator a power of 10. (e.g. 10, 100 or 1000).

Example: $\frac{1}{4} = \frac{1^{\times 25}}{4_{\times 25}} = \frac{25}{100} = 0.25$ power of 10

OR

- Divide the numerator by the denominator.

Example: $\frac{1}{4} = 1 \div 4 = 1.00 \div 4 = 0.25$

$$\begin{array}{r} 0.25 \\ 4\overline{)\cancel{1}.{}^{1}0{}^{2}0} \end{array}$$

Hint: Fractions are just divisions.

Q. Write $\frac{2}{5}$ as a decimal.

A. $\frac{2}{5} = \frac{2^{\times 20}}{5_{\times 20}}$ Make denominator a power of 10

because $100 \div 5 = 20$, $2 \times 20 = 40$

$= \frac{40}{100}$

$= 40 \div 100$

$= 040.0 \div 100$ 2 zeros, 2 places to the left

$= 0.4\cancel{0}$

$= \mathbf{0.4}$

OR

A. $\frac{2}{5} = 2 \div 5$

$= 2.0 \div 5$

$= \mathbf{0.4}$

$$\begin{array}{r} 0.4 \\ 5\overline{)\cancel{2}.{}^{2}0} \end{array}$$

a) Write $\frac{3}{50}$ as a decimal.

$= \frac{3^{\times 2}}{50_{\times 2}} = \frac{6}{100}$

$= 006.0 \div 100 = \boxed{0.06}$

b) Write $\frac{9}{20}$ as a decimal.

$=$ $=$ ……………………

$=$ …………………… $=$ ☐

c) Write $\frac{1}{2}$ as a decimal.

$=$ $=$ ……………………

$=$ …………………… $=$ ☐

d) Write $\frac{17}{50}$ as a decimal.

$=$ $=$ ……………………

$=$ …………………… $=$ ☐

e) Write $\frac{14}{25}$ as a decimal.

$=$ $=$ ……………………

$=$ …………………… $=$ ☐

f) Write $\frac{3}{4}$ as a decimal.

$=$ $=$ ……………………

$=$ …………………… $=$ ☐

g) Write $\frac{4}{5}$ as a decimal.

$=$ $=$ ……………………

$=$ …………………… $=$ ☐

h) Write $\frac{11}{25}$ as a decimal.

$=$ $=$ ……………………

$=$ …………………… $=$ ☐

i) Write $\frac{11}{20}$ as a decimal.

$=$ $=$ ……………………

$=$ …………………… $=$ ☐

Skill 7.8 Writing a percentage as a fraction in simplest form.

Mauve 11 22 **3**3 44
Lime 11 2**2** 33 44

- Write the percentage as a fraction with the denominator of 100.
- Simplify the fraction by dividing both the numerator and the denominator by the same number.

Hints: Percent means “per hundred” or “out of a hundred”.
A percentage is another way of writing a fraction out of one hundred.

Example: 75% is said “75 percent” and means 75 out of 100.

Q. Write 8% as a fraction in simplest form.

A. $8\% = \frac{8^{\div 4}}{100_{\div 4}}$ Simplify: ÷ 4

$= \frac{\mathbf{2}}{\mathbf{25}}$

a) Write 36% as a fraction in simplest form.

$36\% = \frac{36^{\div 4}}{100_{\div 4}}$ Simplify: ÷ 4 $= \boxed{\frac{9}{25}}$

b) Write 6% as a fraction in simplest form.

$6\% =$ $= \boxed{\quad}$

c) Write 75% as a fraction in simplest form.

$= \boxed{\quad}$

d) Write 30% as a fraction in simplest form.

$= \boxed{\quad}$

e) Write 18% as a fraction in simplest form.

$= \boxed{\quad}$

f) Write 90% as a fraction in simplest form.

$= \boxed{\quad}$

g) Write 25% as a fraction in simplest form.

$= \boxed{\quad}$

h) Write 44% as a fraction in simplest form.

$= \boxed{\quad}$

i) Write 40% as a fraction in simplest form.

$= \boxed{\quad}$

j) Write 56% as a fraction in simplest form.

$= \boxed{\quad}$

k) If Australia’s Gross National Product grew 4% quarterly in 2006, what would the percentage be if written as a fraction in simplest form?

$= \boxed{\quad}$

l) A 2010 survey found that 74% of teenagers owned an MP3 player or an iPod. Write this percentage as a fraction in simplest form.

$= \boxed{\quad}$

m) If 26% of women have obtained a bachelor degree, what would the percentage be if written as a fraction in simplest form?

$= \boxed{\quad}$

n) A 2011 survey found that 35% of people give to charity once a month. Write this percentage as a fraction in simplest form.

$= \boxed{\quad}$

Skill 7.9 Writing a fraction as a percentage.

$$\frac{\text{Number}}{100} = \text{Number }\%$$

$$\text{Fraction} \times \frac{100}{1}\% = \text{Percentage}$$

EITHER

- Find the equivalent fraction which has a denominator of 100.
- The numerator of this fraction becomes the equivalent percentage.

Example: $\frac{3^{\times 10}}{10_{\times 10}} = \frac{30}{100} = 30\%$

OR

- Multiply the fraction by $\frac{100}{1}$ and include the % sign.

Example: $\frac{3}{10} = \frac{3}{1\cancel{0}} \times \frac{10\cancel{0}}{1}\%$ Simplify: ÷ 10

$= 30\%$

Q. What percentage is 3 out of 5?

A. $\frac{3}{5} = \frac{3^{\times 20}}{5_{\times 20}}$ because 100 ÷ 5 = 20 $\quad 3 \times 20 = 60$

$= \frac{60}{100}$

$= \mathbf{60\%}$

OR **A.** $\frac{3}{5} = \frac{3}{\underset{1}{\cancel{5}}} \times \frac{\overset{20}{\cancel{100}}}{1}\%$ Simplify: ÷ 5

$= 3 \times 20\%$

$= \mathbf{60\%}$

a) Write $\frac{3}{20}$ as a percentage.

$\frac{3}{20} = \frac{3^{\times 5}}{20_{\times 5}} = \frac{15}{100} =$ $\boxed{15\%}$

b) Write $\frac{7}{10}$ as a percentage.

$\frac{7}{10} = \qquad = \qquad =$ ☐

c) Write $\frac{3}{25}$ as a percentage.

$= \qquad = \qquad =$ ☐

d) Write $\frac{14}{70}$ as a percentage.

$= \qquad = \qquad =$ ☐

e) What percentage is 15 out of 150?

$= \qquad = \qquad =$ ☐

f) What percentage is 45 out of 50?

$\frac{45}{50} = \frac{45}{\underset{1}{\cancel{50}}} \times \frac{\overset{2}{\cancel{100}}}{1}\% = 45 \times 2\% =$ ☐

g) Ng receives \$50 commission on a \$1000 sale. What percentage is this?

$= \qquad = \qquad =$ ☐

h) In a class of 25 students, 10 play netball. What percentage is this?

$= \qquad = \qquad =$ ☐

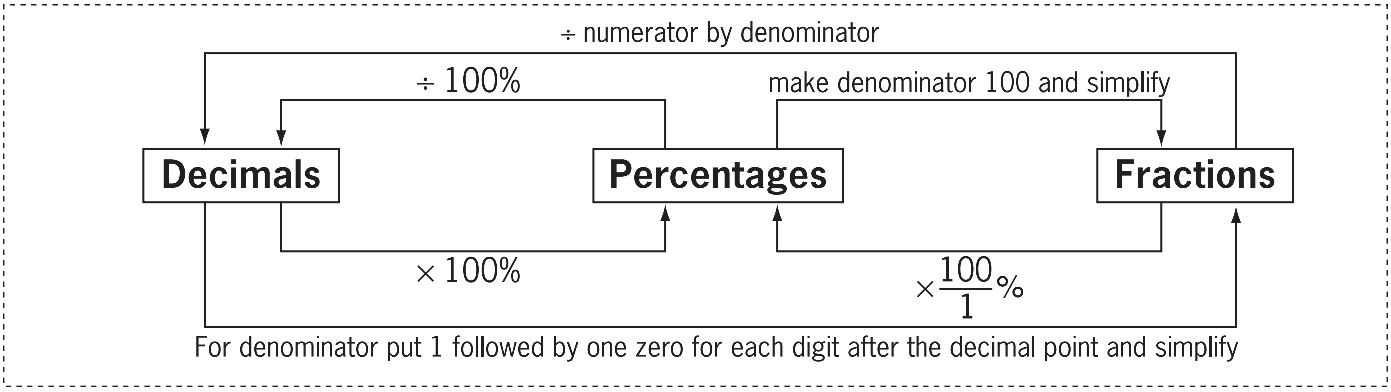
i) One quarter of the men surveyed enjoy walking as their main form of physical exercise. What percentage is this?

$= \qquad = \qquad =$ ☐

j) Three tenths of the women surveyed enjoy aerobics/fitness as their main form of physical exercise. What percentage is this?

$= \qquad = \qquad =$ ☐

Skill 7.10 Converting between decimals, fractions and percentages.



Q. Complete the table:

Decimal	Fraction	Percentage
		5%

A.

Fraction

$$5\% = \frac{5^{\div 5}}{100_{\div 5}}$$

Simplify: ÷ 5

$$= \boldsymbol{\frac{1}{20}}$$

Decimal

$$5\% = \frac{5}{100}$$
$$= 5 \div 100$$
$$= 005.0 \div 100$$

2 zeros, 2 places to the left

$$= \mathbf{0.05}$$

Decimal	Fraction	Percentage
0.05	$\frac{5}{100} = \frac{1}{20}$	5%

a) Complete the table:

Decimal	Fraction	Percentage
0.045		

$$0.045 = \frac{45^{\div 5}}{1000_{\div 5}} = \frac{9}{200}$$

Simplify: ÷ 5

$$0.045 = 0.045 \times 100\% = 4.5\%$$

b) Complete the table:

Decimal	Fraction	Percentage
0.75		

c) Complete the table:

Decimal	Fraction	Percentage
	$\frac{3}{5}$	

d) Complete the table:

Decimal	Fraction	Percentage
	$\frac{9}{50}$	

Skill 7.11 Finding a fraction of a whole number.

EITHER
- Change 'of' to '×'.
- Multiply the fraction by the whole number.
  (see skill 5.1, page 49)
- Cross simplify where possible before multiplying.
  (see skill 5.1, page 49)

OR
- Divide the whole number by the denominator of the fraction.
- Multiply the result by the numerator of the fraction.

Example: Three fifths of 20?
First find one fifth of 20 by dividing 20 by 5:
20 ÷ 5 = 4
Then find three fifths of 20 by multiplying 4 by 3.
4 + 4 + 4 = 4 × 3 = 12
So three fifths of 20 is 12.

To find $\frac{1}{2}$ of a number ⇒ ÷ 2
$\frac{1}{3}$ of a number ⇒ ÷ 3
$\frac{1}{4}$ of a number ⇒ ÷ 4
$\frac{1}{5}$ of a number ⇒ ÷ 5
$\frac{1}{6}$ of a number ⇒ ÷ 6
$\frac{1}{10}$ of a number ⇒ ÷ 10
and so on.

Q. Of the 1500 seats in the Opera Theatre at Sydney Opera House in Sydney, five sixths were occupied. How many spectators were in the Opera Theatre?

A. $\frac{5}{6}$ of $1500 = \frac{5}{\cancel{6}_{1}} \times \overset{250}{\cancel{1500}}$ Simplify: ÷ 6

$= \frac{5 \times 250}{1}$

$= \mathbf{1250}$

OR **A.** 1500 ÷ 6 = 250 Find $\frac{1}{6}$

250 × 5 = **1250** Find $\frac{5}{6}$

a) Find $\frac{3}{10}$ of 160.

$\frac{3}{10}$ of $160 = \frac{3}{\cancel{10}} \times 16\cancel{0} = 3 \times 16 =$ $\boxed{}$ Simplify: ÷ 10

b) Find four sevenths of 280.

= $\boxed{}$

c) Find $\frac{2}{9}$ of 360.

= $\boxed{}$

d) Find five eighths of 3200.

= $\boxed{}$

e) Of the 50 European countries, $\frac{3}{25}$ have German as their official language. How many European countries have German as their official language?

= $\boxed{}$

f) The Western Bulldogs won four elevenths of the 22 games played in the 2013 AFL season. How many games did the Western Bulldogs win?

= $\boxed{}$

g) Of the 36 medals won by New Zealand at the 2010 Commonwealth Games, one sixth were gold. How many gold medals did New Zealand win?

= $\boxed{}$

h) Only one tenth of the 120 qualifiers at the *American Idol* are chosen for the finals. How many will sing in the finals?

= $\boxed{}$

i) Two fifths of the $1200 raised at the Fireworks Frenzy were from the entry tickets. How much money was raised from the tickets?

= $\boxed{\$}$

j) Maria paid one twentieth of $350 000 as a deposit for a house. How much did she pay up-front?

= $\boxed{\$}$

Skill 7.12 Comparing and ordering decimals, fractions and percentages.

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 3 **3** 4 4

- Convert the decimals, fractions and percentages to the same form, by writing all as decimals, or as fractions, or as percentages. (see skill 7.11, page 81)
- Compare and order the decimals, or the fractions, or the percentages.

Hint: The most convenient form is the decimal form. Write the fractions and percentages as decimals.

Q. Write in ascending order:

$\frac{17}{100}$, 0.7, 7%

A.

Fraction

$$\frac{17}{100} = 17 \div 100$$

Write the fraction as a decimal

$$= 017.0 \div 100$$

2 zeros, 2 places to the left

$$= \mathbf{0.17}$$

Percentage

$$7\% = \frac{7}{100}$$

Write the percentage as a decimal

$$= 7 \div 100$$

$$= 007.0 \div 100$$

$$= \mathbf{0.07}$$

The order from smallest to largest is:

0.07, 0.17, 0.7 OR ***7%, $\frac{17}{100}$, 0.7***

a) Which is greater?
0.09 or 90%

$90\% = \frac{90}{100} = 090.0 \div 100 = 0.9$

$0.9 > 0.09$

90%

b) Which is greater?
0.8 or 75%

c) Place in ascending order:
$\frac{1}{3}$, 0.31, 30%

Fraction

$\frac{1}{3} = 1 \div 3 = 1.0 \div 3 = 0.33.. = 0.\dot{3}$

Percentage

$30\% = \frac{30}{100} = 030.0 \div 100 = 0.3$

$0.3 < 0.31 < 0.\dot{3}$

d) Place in descending order:
0.66, 6%, $\frac{6}{10}$

Fraction

Percentage

e) Place in ascending order:
$\frac{1}{4}$, 0.14, 41%

Fraction

Percentage

f) Place in descending order:
$\frac{4}{5}$, 0.83, 81%

Fraction

Percentage

Skill 7.13 Recognising expanded form for recurring decimals.

If the dot is above one digit
- Write the digit repeatedly.

If the dots are above two digits side by side
- Write the group of 2 digits repeatedly.

If the dots are above two digits which are not side by side
- Write the group of all the digits in between the dots repeatedly.

Q. $0.4\dot{9}\dot{5}$ is the notation for:
A) 0.4995599.....
B) 0.4959595.....
C) 0.4999555.....

A. *B*

The dots above 95 (side by side) mean that 95 must be repeated indefinitely after 4.

a) $0.\dot{2}$ is the notation for:
A) 0.02222.....
B) 0.2222.....
C) 2.2222.....

B

b) $5.\dot{3}$ is the notation for:
A) 5.03333.....
B) 0.3333.....
C) 5.3333.....

c) $9.\dot{5}$ is the notation for:
A) 9.9955.....
B) 9.5555.....
C) 0.9595.....

d) $0.7\dot{3}$ is the notation for:
A) 0.7333.....
B) 0.737373.....
C) 0.773377.....

e) $0.2\dot{7}$ is the notation for:
A) 0.222777.....
B) 0.272727.....
C) 0.2777.....

f) $0.86\dot{1}$ is the notation for:
A) 0.86111.....
B) 0.861861.....
C) 0.868686.....

g) $0.\dot{2}\dot{3}$ is the notation for:
A) 0.222333.....
B) 0.232323.....
C) 0.2333.....

h) $0.\dot{5}\dot{4}$ is the notation for:
A) 0.55445544.....
B) 0.555444.....
C) 0.545454.....

i) $0.5\dot{9}\dot{3}$ is the notation for:
A) 0.5939393.....
B) 0.5993399.....
C) 0.593593.....

j) $0.\dot{3}7\dot{6}$ is the notation for:
A) 0.3337666.....
B) 0.376376.....
C) 0.3766666.....

Skill 7.14 Writing a fraction as a recurring decimal.

- Divide the numerator by the denominator.
- Write a decimal point and zeros at the end of the numerator to complete the division.
- In the result, when a single digit is repeating after the decimal point, write the digit only once with a dot on top.
- In the result, when a pattern of digits is repeating after the decimal point, write the pattern only once, with a dot over the first and last digit of it.

Examples:

$\frac{5}{9} = 5 \div 9 = 5.0000... \div 9 = 0.5555...$ OR $= 0.\dot{5}$

$$\begin{array}{r} 0.5\,5\,5\,5 \\ 9\overline{)\cancel{5}.{}^{5}0\,{}^{5}0\,{}^{5}0\,{}^{5}0} \end{array}$$

$\frac{1}{6} = 1 \div 6 = 1.0000... \div 6 = 0.1666...$ OR $= 0.\dot{1}\dot{6}$

$$\begin{array}{r} 0.1\,6\,6\,6 \\ 6\overline{)\cancel{1}.{}^{1}0\,{}^{4}0\,{}^{4}0\,{}^{4}0} \end{array}$$

$\frac{3}{11} = 3 \div 11 = 3.0000... \div 11 = 0.2727...$ OR $= 0.\dot{2}\dot{7}$

$$\begin{array}{r} 0.2\,7\,2\,7 \\ 11\overline{)\cancel{3}.{}^{3}0\,{}^{8}0\,{}^{3}0\,{}^{8}0} \end{array}$$

$\frac{3}{7} = 3 \div 7 = 3.0000... \div 7 = 0.428571428571...$ OR $= 0.\dot{4}2857\dot{1}$

$$\begin{array}{r} 0.4\,2\,8\,5\,7\,1\,4\,2\,8\,5\,7\,1 \\ 7\overline{)\cancel{3}.{}^{3}0\,{}^{2}0\,{}^{6}0\,{}^{4}0\,{}^{5}0\,{}^{1}0\,{}^{3}0\,{}^{2}0\,{}^{6}0\,{}^{4}0\,{}^{5}0\,{}^{1}0} \end{array}$$

Q. Write $\frac{2}{9}$ as a recurring decimal.

A. $\frac{2}{9} = 2 \div 9$

$= 2.0000... \div 9$

$= 0.2222...$

$= \mathbf{0.\dot{2}}$

$$\begin{array}{r} 0.2\,2\,2\,2 \\ 9\overline{)\cancel{2}.{}^{2}0\,{}^{2}0\,{}^{2}0\,{}^{2}0} \end{array}$$

a) Write $\frac{1}{11}$ as a recurring decimal.

$\frac{1}{11} = 1 \div 11 = 1.0000... \div 11 =$ $\boxed{0.\dot{0}\dot{9}}$

$$\begin{array}{r} 0.0\,9\,0\,9 \\ 11\overline{)\cancel{1}.{}^{1}0\,{}^{10}0\,{}^{1}0\,{}^{10}0} \end{array}$$

b) Write $\frac{4}{11}$ as a recurring decimal.

$$\begin{array}{r} 0.3\,6 \\ 11\overline{)\cancel{4}.{}^{4}0\,{}^{7}0\,0\,0} \end{array}$$

c) Write $\frac{2}{3}$ as a recurring decimal.

$$3\overline{)2.0\,0\,0\,0}$$

d) Write $\frac{4}{9}$ as a recurring decimal.

$$9\overline{)4.0\,0\,0\,0}$$

e) Write $\frac{11}{15}$ as a recurring decimal.

$$15\overline{)11.0\,0\,0\,0}$$

f) Write $\frac{5}{12}$ as a recurring decimal.

$$12\overline{)5.0\,0\,0\,0}$$

Skill 7.15 Writing a recurring decimal as a fraction.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 **4**

- Convert each fraction to a decimal to find the answer. (see skill 7.14, page 84)

Q. Which fraction does $0.2\dot{6}$ equal?

A) $\frac{1}{6}$ B) $\frac{26}{100}$ C) $\frac{4}{15}$

A. *A)* $\frac{1}{6} = 1 \div 6$

$= 1.0000... \div 6$

$= 0.1666...$

$= 0.1\dot{6}$

$$\begin{array}{r} 0.1666 \\ 6\overline{)\cancel{1}.{}^{4}0{}^{4}0{}^{4}0{}^{4}0} \end{array}$$

B) $\frac{26}{100} = 26 \div 100 = 0.26$

C) $\frac{4}{15} = 4 \div 15$

$= 4.0000... \div 15$

$= 0.2666...$

$= 0.2\dot{6}$

$$\begin{array}{r} 0.2666 \\ 15\overline{)\cancel{4}.{}^{4}0{}^{10}0{}^{10}0{}^{10}0} \end{array}$$

The answer is ***C****.*

a) Which fraction is approximately equal to 0.4444?

A) $\frac{1}{4}$ B) $\frac{4}{9}$ C) $\frac{1}{2}$

$\frac{1}{4} = 1 \div 4 = 0.25$ $\frac{4}{9} = 4 \div 9 = 0.4444$

$\frac{1}{2} = 1 \div 2 = 0.5$

$$\begin{array}{r} 0.4444 \\ 9\overline{)\cancel{4}.{}^{4}0{}^{4}0{}^{4}0{}^{4}0} \end{array}$$

b) Which fraction does $0.\dot{5}$ equal?

A) $\frac{1}{5}$ B) $\frac{5}{10}$ C) $\frac{5}{9}$

$\frac{1}{5} = 1 \div 5 = 0.2$

$$\begin{array}{r} 0.5 \\ 9\overline{)\cancel{5}.{}^{5}0{}^{5}0\,0\,0} \end{array}$$

c) Which fraction does $0.\dot{3}$ equal?

A) $\frac{3}{5}$ B) $\frac{1}{3}$ C) $\frac{3}{10}$

$$3\overline{)1.0\,0\,0\,0}$$

d) Which fraction does $0.0\dot{8}$ equal?

A) $\frac{8}{90}$ B) $\frac{8}{10}$ C) $\frac{8}{100}$

$$90\overline{)8.0\,0\,0\,0}$$

e) Which fraction does $0.\dot{2}\dot{3}$ equal?

A) $\frac{23}{10}$ B) $\frac{23}{100}$ C) $\frac{23}{99}$

f) Which fraction does $0.\dot{7}\dot{2}$ equal?

A) $\frac{72}{100}$ B) $\frac{72}{99}$ C) $\frac{72}{10}$

8. [Integer +,−]

Skill 8.1 Adding integers.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

Hint: Every number has a sign attached to it, so if there is no sign, the number is positive. These signs should not be confused with the operations of addition and subtraction.

Using a number line

- Start at 0.
- When the number is "+" move that many to the right.
- When the number is "–" move that many to the left.

Using Addition Rules

Addition Rules
same signs:
Add the numbers, ignoring their signs.
Keep that sign.

Addition Rules
different signs:
Subtract the numbers, ignoring their signs.
Keep the sign of the larger number.

Example:

$(+4) + (+3) = +(4 + 3) = +7 = 7$ or simply

$4 + 3 = 7$

$(-5) + (-8) = -(5 + 8) = -13$ or simply

$-5 + -8 = -13$

Example:

$(-9) + (+3) = -(9 - 3) = -6$ or simply

$-9 + 3 = -6$

$(-4) + (+11) = +(11 - 4) = +7 = 7$ or simply

$-4 + 11 = 7$

Q. $(-7) + (+9) =$

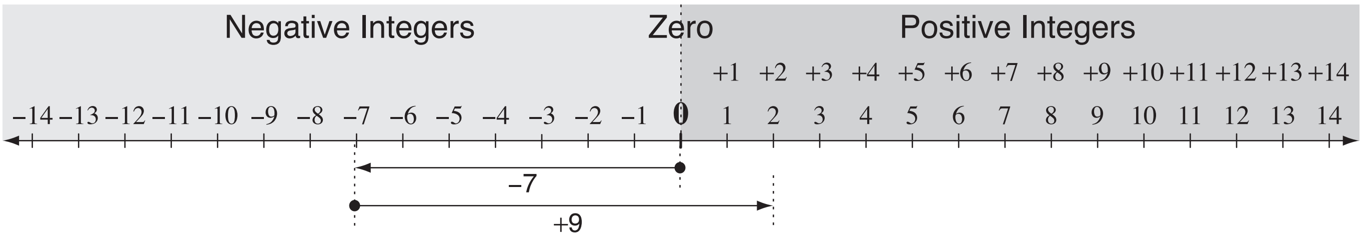
A. $(-7) + (+9)$

$= -7 + 9$

$= 2$

+ + = +

start at −7, move forward 9



a) $(+5) + (-7) =$

+ − = −

subtract, keep "−"

$= 5 - 7$ .... $= \boxed{-2}$

b) $(-4) + (-8) =$

$= -4$ .... $= \boxed{}$

c) $(-5) + (-3) =$

= .... = ☐

d) $(+2) + (-8) =$

= .... = ☐

e) $(+4) + (-6) =$

= .... = ☐

f) $(-7) + (+4) =$

= .... = ☐

g) $(-3) + (+6) =$

= .... = ☐

h) $(+5) + (-8) =$

= .... = ☐

i) $(+2) + (-14) =$

= .... = ☐

j) $(-16) + (+9) =$

= .... = ☐

k) $(-15) + (-8) =$

= .... = ☐

l) $2 + -7 =$

= .... = ☐

Skill 8.2 Subtracting integers.

Hint: Every number has a sign attached to it, so if there is no sign, the number is positive. These signs should not be confused with the operations of addition and subtraction.

Using a number line

- Start at 0.
- When the number is "+" move that many to the right.
- When the number is "–" move that many to the left.

Using Addition Rules

- Consider subtracting an integer as adding its opposite.
  So change the number to be subtracted to its opposite. Example: $8 - (-2) = 8 + (+2)$
- Then apply the addition rules.

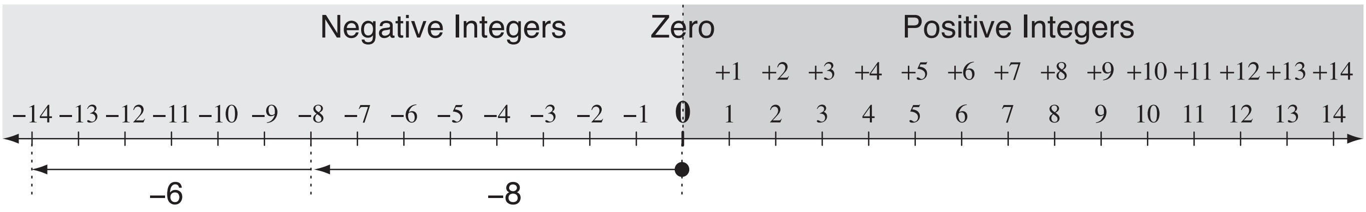
Addition Rules	**Addition Rules**
same signs: Add the numbers, ignoring their signs. Keep that sign.	different signs: Subtract the numbers, ignoring their signs. Keep the sign of the larger number.
Example: $(-9) - (-3) = (-9) + (+3) = -(9 - 3) = -6$ or simply $-9 - (-3) = -9 + 3 = -6$ $(-4) - (-11) = (-4) + (+11) = +(11 - 4)$ $= +7 = 7$ or simply $-4 - (-11) = -4 + 11 = 7$	Example: $(-5) - (+8) = (-5) + (-8) = -(5 + 8) = -13$ or simply $-5 - 8 = -13$ $(+4) - (-3) = (+4) + (+3) = +(4 + 3) = +7 = 7$ or simply $4 - (-3) = 4 + 3 = 7$

Q. $(-8) - (+6) =$

A. $(-8) - (+6)$ + – = –

$= -8 - 6$

$= \mathbf{-14}$ start at –8, move backward 6



a) $(-5) - (-6) =$ – – = +

$= -5 + 6 = \boxed{1}$

b) $(+3) - (+9) =$ subtract, keep "–"

$= 3 - 9 =$

c) $(+7) - (+8) =$

= .... =

d) $(+7) - (-7) =$

= .... =

e) $(-3) - (-2) =$

= .... =

f) $(-4) - (-8) =$

= .... =

g) $(+6) - (-7) =$

= .... =

h) $(+4) - (-9) =$

= .... =

i) $(-19) - (+11) =$

= .... =

j) $(-16) - (+9) =$

= .... =

k) $(-12) - (-15) =$

= .... =

l) $-6 - -3 =$

= .... =

Skill 8.3 Adding and subtracting integers.

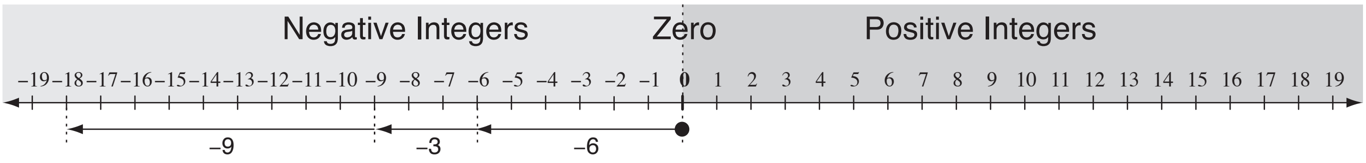
- Use the sign rules. (see skills 8.1, page 87 and 8.2, page 88)

Addition Rules
same signs:
Add the numbers, ignoring their signs.
Keep that sign.

Addition Rules
different signs:
Subtract the numbers, ignoring their signs.
Keep the sign of the larger number.

Q. (−6) − (+3) − (+9) =

A. *(−6) − (+3) − (+9)* − + = −
= *−6 − 3 − 9* work from left to right
= *−9 − 9* start at −9, move backward 9 more
= ***−18***



a) (−5) + (−6) + (+9) = + − = − + + = +
= *−5 − 6 + 9* subtract, keep "−"
= *−11 + 9* = $\boxed{-2}$

b) (+1) − (−7) − (−7) = − − = +
= *1 +* ……………
= …………… = $\boxed{}$

c) (+9) + (−6) − (−2) =
= ……………
= …………… = $\boxed{}$

d) (−8) − (−5) + (+4) =
= ……………
= …………… = $\boxed{}$

e) (−2) + (−6) − (−9) =
= ……………
= …………… = $\boxed{}$

f) (+5) − (+7) − (−8) =
= ……………
= …………… = $\boxed{}$

g) (+3) − (−6) + (−8) =
= ……………
= …………… = $\boxed{}$

h) (+5) + (−4) − (+3) =
= ……………
= …………… = $\boxed{}$

i) (−2) − (−6) − (+7) =
= ……………
= …………… = $\boxed{}$

j) (+7) + (+15) + (−19) =
= ……………
= …………… = $\boxed{}$

k) (−12) − (−13) + (+15) =
= ……………
= …………… = $\boxed{}$

l) (−14) − (+16) + (+18) =
= ……………
= …………… = $\boxed{}$

m) 8 − 2 − −7 =
= …………… = $\boxed{}$

n) 5 + −7 + −9 =
= …………… = $\boxed{}$

o) −6 + 5 + −8 =
= …………… = $\boxed{}$

p) −9 − 2 + −4 =
= …………… = $\boxed{}$

q) 10 + −5 + −6 =
= …………… = $\boxed{}$

r) −5 + −10 + 12 =
= …………… = $\boxed{}$

Skill 8.4 Adding and subtracting integers using order of operations.

Mauve 11 22 33 44
Lime 11 22 **33** 44

- Complete the operations in the correct order.
  1. Simplify within brackets.
  2. Add and/or subtract from left to right.
- Use the sign rules. (see skills 8.1, page 87 and 8.2, page 88)

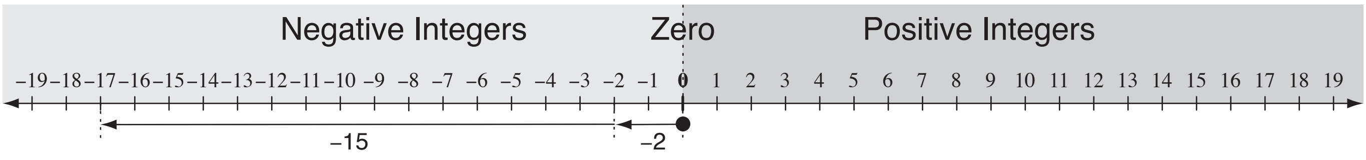
Q. $(5-7)-(6+9)=$

A. $(5-7)-(6+9)$ complete the brackets first

$=(-2)-(+15)$ − + = −

$=-2-15$ start at −2, move backward 15 more

$=\mathbf{-17}$



a) $4+(-6+3)=$ brackets first

+ − = −

$=4+(-3)$

$=4-3$ $=\boxed{1}$

b) $2+(4-9)=$

$=2+(-5)$

$=$ $=$

c) $7+(3-8)=$

$=$

$=$ $=$

d) $4-(9-7)=$

$=$

$=$ $=$

e) $5-(-8+6)=$

$=$

$=$ $=$

f) $6+(-5-4)=$

$=$

$=$ $=$

g) $7-(3-4)=$

$=$

$=$ $=$

h) $10+(-2-5)=$

$=$

$=$ $=$

i) $8-(-3+9)=$

$=$

$=$ $=$

j) $(2-5)-(3+4)=$ brackets first

− + = −

$=(-3)-(+7)$ add, keep "−"

$=-3-7$ $=$

k) $(8-4)+(3-9)=$

$=$

$=$ $=$

l) $(5-9)-(9-5)=$

$=$

$=$ $=$

m) $(5+6)-(4-11)=$

$=$

$=$ $=$

n) $(3-8)+(9-14)=$

$=$

$=$ $=$

o) $(-8-6)-(7-13)=$

$=$

$=$ $=$

Skill 8.5 Finding missing integers using addition and subtraction.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 **4**

- Circle the positive integer (no sign) or negative integer ('–' sign) that is on the side of the unknown.
  Hint: Don't confuse the sign with the operation. (see skill 8.1, page 87)
- Use the inverse operations of addition or subtraction to remove the circled integer from the side of the unknown.
  Hint: e.g. +6 added to –6 will cancel each other and leave zero as the result.
- Perform the same operation on the other side of the equation.
- If the unknown has a negative sign attached, multiply both sides of the equation by another negative sign.
  Hint: '– – = +' i.e. The sign of the unknown will become its inverse, a '+'.

Q. $-6 - \boxed{} = 8$

A. $-6 - x = 8$ Use inverse of –6

$\cancel{-6} - x \cancel{+6} = 8 + 6$ +6 to both sides

cancel

$-x = 8 + 6$

Use inverse of '–' – – = +

$- -x = -14$

$x = \mathbf{-14}$

a) $\boxed{-8} + -4 = -12$

$x + \cancel{-4} \cancel{+4} = -12 + 4$

$x = -8$

b) $\boxed{} - 3 = -5$

$x - 3 + 3 = -5 + 3$

c) $4 + \boxed{} = -3$

$4 + x$

d) $\boxed{} - -6 = -9$

e) $-5 + \boxed{} = 13$

f) $-8 + \boxed{} = -3$

g) $\boxed{} + -4 = -8$

h) $\boxed{} - -6 = 1$

i) $\boxed{} + 7 = -4$

j) $9 - \boxed{11} = -2$

$\cancel{9} - x \cancel{-9} = -2 - 9$

$-x = -11 \Rightarrow - -x = - -11$ – – = +

k) $-6 - \boxed{} = 7$

l) $-9 - \boxed{} = -3$

m) $- \boxed{} + 4 = -6$

n) $8 - \boxed{} = 5$

o) $- \boxed{} - 7 = 3$

9. [Integer ×,÷]

Skill 9.1 Multiplying integers.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Multiply the integers ignoring the signs.
- Determine the sign of the result using the multiplication rules.

Multiplcation Rules
same signs: positive × positive = positive
negative × negative = positive

Example: $-9 \times (-3)$ − − = +
$= 27$

Multiplcation Rules
different signs: positive × negative = negative
negative × positive = negative

Example: $9 \times (-3)$
$+9 \times (-3)$
$= -27$ + − = −

Q. $(+2) \times (-9) =$

A. $(+2) \times (-9)$
$= 2 \times -9$ + × − = −
$= \mathbf{-18}$

a) $(-3) \times (+8) =$ − × + = −
$= -3 \times 8 = \boxed{-24}$

b) $(-3) \times (-4) =$
$=$ ………… $=$

c) $(+5) \times (-9) =$
$=$ ………… $=$

d) $(-10) \times (+10) =$
$=$ ………… $=$

e) $(-2) \times (+6) =$
$=$ ………… $=$

f) $(-4) \times (-7) =$
$=$ ………… $=$

g) $(+7) \times (-3) =$
$=$ ………… $=$

h) $(+4) \times (-5) =$
$=$ ………… $=$

i) $(+8) \times (+8) =$
$=$ ………… $=$

j) $(+2) \times (-17) =$
$=$ ………… $=$

k) $(-3) \times (-15) =$
$=$ ………… $=$

l) $-21 \times -2 =$
$=$ ………… $=$

m) $(-5) \times (-2) \times (+7) =$
− × − = +
$= -5 \times -2 \times 7$
$= 10 \times 7 = \boxed{70}$

n) $(+3) \times (-4) \times (-2) =$
$=$ …………
$=$ ………… $=$

o) $(-5) \times (+3) \times (+3) =$
$=$ …………
$=$ ………… $=$

p) $(-4) \times (+4) \times (-2) =$
$=$ …………
$=$ ………… $=$

q) $(-6) \times (-6) \times (-10) =$
$=$ …………
$=$ ………… $=$

r) $20 \times -5 \times 3 =$
$=$ …………
$=$ ………… $=$

Skill 9.2 Dividing integers.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Divide the integers ignoring the signs.
- Determine the sign of the result using the division rules.

Division Rules

same signs: positive ÷ positive = positive
negative ÷ negative = positive

Division Rules

different signs: positive ÷ negative = negative
negative ÷ positive = negative

Example: $-9 \div (-3)$ – – = +
$= 3$

Example: $9 \div (-3)$
$+9 \div (-3)$ + – = –
$= -3$

Q. $(+12) \div (-3) =$

A. $(+12) \div (-3)$ + ÷ – = –
$= 12 \div -3$
$= -4$

a) $(-18) \div (+9) =$ – ÷ + = –
$= -18 \div 9$ ……………… $= \boxed{-2}$

b) $(-6) \div (+1) =$
= ……………… =

c) $(+12) \div (-4) =$
= ……………… =

d) $(-15) \div (-3) =$
= ……………… =

e) $(-24) \div (+6) =$
= ……………… =

f) $(+9) \div (+9) =$
= ……………… =

g) $(+35) \div (-5) =$
= ……………… =

h) $(-27) \div (+3) =$
= ……………… =

i) $(-28) \div (-7) =$
= ……………… =

j) $\frac{32}{-4} =$ division
$= 32 \div -4$ ……………… $= \boxed{-8}$

k) $\frac{-15}{-3} =$
= ……………… =

l) $\frac{-42}{7} =$
= ……………… =

m) $\frac{24}{-6} =$
= ……………… =

n) $\frac{-18}{-2} =$
= ……………… =

o) $\frac{-40}{5} =$
= ……………… =

p) $\frac{56}{-4} =$
= ……………… =

q) $\frac{-36}{-9} =$
= ……………… =

r) $\frac{-75}{15} =$
= ……………… =

s) $\frac{80}{-8} =$
= ……………… =

t) $\frac{-64}{-8} =$
= ……………… =

u) $\frac{-84}{12} =$
= ……………… =

Skill 9.3 Multiplying integers involving powers of 10.

Mauve 11 22 33 44
Lime 11 22 33 44

- Multiply the integers ignoring the signs.
- Determine the sign of the result using the multiplication rules.

Multiplcation Rules

same signs: positive × positive = positive
negative × negative = positive

Multiplcation Rules

different signs: positive × negative = negative
negative × positive = negative

Example: $-9 \times (-3)$ — – = +
$= 27$

Example: $9 \times (-3)$
$+9 \times (-3)$ + – = –
$= -27$

- Consider the zeros as making groups of 10's or 100's and place them last.
(see skill 1.3, page 4)

Q. $(+200) \times (-2) =$

A. $(+200) \times (-2)$ + × – = –
$= 200 \times -2$
$= \mathbf{-400}$

a) $(-3) \times (+100) =$ – × + = –
$= -3 \times 100 = \boxed{-300}$

b) $(-20) \times (-4) =$
= .... = ☐

c) $(+50) \times (-2) =$
= .... = ☐

d) $(-4) \times (-100) =$
= .... = ☐

e) $(+100) \times (-8) =$
= .... = ☐

f) $(-700) \times (+6) =$
= .... = ☐

g) $(-100) \times (+10) =$
= .... = ☐

h) $(+20) \times (+100) =$
= .... = ☐

i) $(-10) \times (-40) =$
= .... = ☐

j) $(+300) \times (-3) =$
= .... = ☐

k) $(+80) \times (-10) =$
= .... = ☐

l) $(+4) \times (+300) =$
= .... = ☐

m) $(+600) \times (-1) =$
= .... = ☐

n) $(-40) \times (-50) =$
= .... = ☐

o) $(-500) \times (-3) =$
= .... = ☐

p) $(+6) \times (-200) =$
= .... = ☐

q) $(-300) \times (-5) =$
= .... = ☐

r) $(-700) \times (-7) =$
= .... = ☐

Skill 9.4 Multiplying and dividing integers.

- Multiply and/or divide from left to right. (see skills 9.1, page 93 and 9.2, page 94)
- When multiplying and dividing integers use the multiplication and division rules. (see skills 9.1, page 93 and 9.2, page 94)

Q. $(+10) \div (-2) \times (-7) =$

A. $(+10) \div (-2) \times (-7)$

$= 10 \div -2 \times -7$ work from left to right

+ ÷ − = −

$= -5 \times -7$

$= 35$

− × − = +

a) $(-4) \times (+5) \div (+5) =$

− × + = −

$= -4 \times 5 \div 5$

$= -20 \div 5 = \boxed{-4}$

b) $(+10) \times (-3) \div (-5) =$

=

= =

c) $(+15) \div (+3) \times (-3) =$

=

= =

d) $(-8) \times (-2) \div (+4) =$

=

= =

e) $(+24) \div (-6) \times (-2) =$

=

= =

f) $(-5) \times (-4) \div (-10) =$

=

= =

g) $(+30) \div (-10) \times (+3) =$

=

= =

h) $(+28) \div (-14) \times (-7) =$

=

= =

i) $(-2) \times (-150) \div (+20) =$

=

= =

j) $(+7) \times (+6) \div (-21) =$

=

= =

k) $(-2) \times (+32) \div (+8) =$

=

= =

l) $(-35) \div (-7) \times (+9) =$

=

= =

m) $10 \times 3 \div -5 =$

= =

n) $24 \div -4 \times -4 =$

= =

o) $-6 \times 8 \div -12 =$

= =

p) $8 \times -4 \times -5 =$

= =

q) $-4 \times 5 \div -10 =$

= =

r) $-6 \times 9 \div -3 =$

= =

s) $30 \div -5 \times -2 =$

= =

t) $-44 \div 11 \times 12 =$

= =

u) $45 \div -9 \times -4 =$

= =

Skill 9.5 Multiplying and dividing integers using order of operations.

- Complete the operations in the correct order.
  1. Simplify within brackets.
  2. Multiply or divide the results.
- When multiplying and dividing integers use the multiplication and division rules. (see skills 9.1, page 93 and 9.2, page 94)

Q. $(6+4)\times(-6-4)=$

A. $(6+4)\times(-6-4)$ brackets first
$= 10\times -10$ $+\times-=-$
$= \mathbf{-100}$

a) $(3+3)\times(-4+9)=$

$= 6\times 5$ $= \boxed{30}$

b) $(2+4)\times(-6+4)=$

$=$ ………… $= \boxed{}$

c) $(8-4)\times(6-9)=$

$=$ ………… $= \boxed{}$

d) $(7-4)\times(-8+3)=$

$=$ ………… $= \boxed{}$

e) $(1-8)\times(4-5)=$

$=$ ………… $= \boxed{}$

f) $(5+3)\times(3-5)=$

$=$ ………… $= \boxed{}$

g) $(-3-1)\times(-3+5)=$

$=$ ………… $= \boxed{}$

h) $(6+6)\times(-2+8)=$

$=$ ………… $= \boxed{}$

i) $(8-5)\times(5-8)=$

$=$ ………… $= \boxed{}$

j) $(-1-7)\times(3-9)=$

$=$ ………… $= \boxed{}$

k) $(5+4)\times(-5-4)=$

$=$ ………… $= \boxed{}$

l) $(-4-3)\times(-1+4)=$

$=$ ………… $= \boxed{}$

m) $(-5+2)\times(-6+9)=$

$=$ ………… $= \boxed{}$

n) $(2-8)\times(-1+2)=$

$=$ ………… $= \boxed{}$

o) $(5-1)\times(-3-2)=$

$=$ ………… $= \boxed{}$

p) $\dfrac{7-1}{2-5}=$ division

$= \dfrac{6}{-3}$

$= 6\div -3$ $= \boxed{}$

q) $\dfrac{5-8}{-5+8}=$

$=$ …………

$=$ ………… $= \boxed{}$

r) $\dfrac{-40}{-2\times 5}=$

$=$ …………

$=$ ………… $= \boxed{}$

s) $\dfrac{8-2}{2-5}=$

$=$ …………

$=$ ………… $= \boxed{}$

t) $\dfrac{2-9}{-2+9}=$

$=$ …………

$=$ ………… $= \boxed{}$

u) $\dfrac{36}{-3\times 4}=$

$=$ …………

$=$ ………… $= \boxed{}$

Skill 9.6 Finding missing integers using multiplication and division.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 **4**

To find a missing integer using multiplication

- Circle the integer, including its sign, that is on the side of the unknown.
- Use division, the inverse operation of multiplication, to remove the circled integer from the side of the unknown.
  Hint: e.g. × −6 and ÷ −6 will cancel leaving 1.
- Perform the same operation on the other side of the equation.

To find a missing integer using division

When dividing a number by an unknown -

- Divide the number by the result to determine the unknown.

OR When dividing an unknown by a number

- Circle the integer, including its sign, that is on the side of the unknown.
- Use multiplication, the inverse operation of division to remove the circled integer from the side of the unknown.
  Hint: e.g. ÷ −6 and × −6 will cancel leaving 1.
- Perform the same operation on the other side of the equation.

Q. $-96 \div \boxed{} = 8$

A. $-96 \div x = 8$
$-96 \div 8 = x$ ÷ by result
$-96 \div 8 = -12$
$x = \mathbf{-12}$

OR

A. $-96 \cancel{\div x} \cancel{\times x} = 8 \times x$
$-96 = 8x$
$\dfrac{\cancel{8}x}{\cancel{8}} = \dfrac{-96}{8} \begin{matrix}\div 8\\ \div 8\end{matrix}$
$x = \mathbf{-12}$

a) $\boxed{-5} \times \enclose{circle}{-7} = 35$
$x \times \cancel{-7} \div \cancel{-7} = 35 \div -7$
$x = -5$

b) $54 \div \boxed{} = -9$
$54 \div -9 =$

c) $\boxed{} \div \enclose{circle}{3} = -7$
$x \div \cancel{3} \times \cancel{3} = -7 \times 3$

d) $\boxed{} \times \enclose{circle}{-2} = -10$

e) $-7 \times \boxed{} = 63$

f) $-48 \div \boxed{} = -6$

g) $\boxed{} \times -12 = -120$

h) $\boxed{} \div -6 = 11$

i) $\boxed{} \times 8 = -24$

j) $-6 \times \boxed{} = 54$

k) $-121 \div \boxed{} = -11$

l) $\boxed{} \div 8 = -7$

m) $-8 \times \boxed{} = 72$

n) $-450 \div \boxed{} = 30$

o) $\boxed{} \div -6 = 7$

10. [Rates / Ratios]

Skill 10.1 Finding the unit rate and the unit price.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

To find the unit rate

- Divide the two quantities to show how many of the first quantity correspond to one of the second quantity.

To find the unit price

- Divide the total amount by the number of items.

Q. 200 litres flow in 5 minutes = ☐ litres per minute

A. *200 litres in 5 minutes*

$$= \frac{200\ L}{5\ min}$$

$$= \frac{200}{5}\ \frac{L}{min}$$

$$= \mathbf{40} \text{ litres per minute (L/min)}$$

Simplify the fraction by 5.

a) 300 cars sold in 30 days = ☐ cars per day

300 cars in 30 days

$$= \frac{300\ cars}{30\ days}$$ Simplify: ÷ 30

= *10 cars per day*

b) 420 m walked in 5 minutes = ☐ m per minute

420 m in 5 minutes

= ....

= ....

c) 12°C drop in 6 hours = ☐ °C per hour

....

= ....

= ....

d) 600 beats in 10 minutes = ☐ beats per minute

....

= ....

= ....

e) If 8 kg of apples are sold for $10, what is the cost per kilogram?

....

= .... $ /kg

f) The plumber charged $231 for a 3-hour job. What was the charge per hour?

....

= .... /h

g) Nina earns $100 for an 8-hour shift. How much does she earn per hour?

....

= .... /h

h) A 6-pack of exercise books cost $9.60. What is the price per exercise book?

....

= .... $

Skill 10.2 Simplifying ratios.

- Write the quantities of the ratio with the same unit of measurement.

EITHER

- Find the largest number that divides evenly into each quantity of the ratio (Highest Common Factor).
- Divide each quantity by the HCF.

Hints: The order of the quantities in a ratio matters.
‘:’ means ‘to’.

Examples: The ratio of legs to ears in a dog is 4 : 2 = 2 : 1
The ratio of ears to legs in a dog is 2 : 4 = 1 : 2

OR

- Divide each quantity of the ratio by any factor until the ratio is reduced to its simplest form.

$a : b = \frac{a}{b}$ Ratio

Q. Simplify 80 min : 3 h

A. *3 h = 3 × 60 min = 180 min* 1 h = 60 min

80 min : 3 h

$= \overset{4}{\cancel{80}}\text{ min} : \overset{9}{\cancel{180}}\text{ min}$ HCF of 80 and 180 is 20 so ÷ 20

$= \mathbf{4 : 9}$ Ignore the units

OR **A.** *3 h = 3 × 60 min = 180 min*

80 min : 3 h

$= 8\cancel{0}\text{ min} : 18\cancel{0}\text{ min}$

$= \overset{4}{\cancel{8}} : \overset{9}{\cancel{18}}$ Simplify: ÷ 10

$= \mathbf{4 : 9}$ Simplify: ÷ 2

a) Simplify 600 mL : 0.2 L 1 L = 1000 mL 3 zeros, 3 places right

0.2 L = 0.200 × 1000 mL = 200 mL

$6\cancel{0}\cancel{0}\text{ mL} : 2\cancel{0}\cancel{0}\text{ mL}$ Simplify: ÷ 100

$= \overset{3}{\cancel{6}} : \overset{1}{\cancel{2}}$ Simplify: ÷ 2 = 3 : 1

b) Simplify 2 m : 50 cm

= =

c) Simplify 750 g : 1 kg

= =

d) Simplify 6 months : 4 years

= =

e) Simplify 2 km : 3200 m

= =

f) Simplify $24 : 300¢

= =

g) Simplify 0.5 kg : 2000 g : 4 kg

= =

h) Simplify 50¢ : $4.00 : $2.50

= =

Skill 10.3 Finding the ratio of two or more quantities as a set : set comparison.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Write the ratio in words.
- Replace the words with numbers.
- Simplify the ratio:

EITHER

- Find the largest number that divides evenly into each quantity of the ratio (Highest Common Factor) and divide each quantity by the HCF.

Hint: The order of the quantities in a ratio matters.

OR

- Divide each quantity of the ratio by any factor until the ratio is reduced to simplest form.

Q. Ocean water represents 70% of the earth's surface and the rest is land. Find the ratio of water to land.

A. *land = 100% – 70% = 30%*

water : land

$= 7\cancel{0}\% : 3\cancel{0}\%$ Simplify: ÷ 10

$= \mathbf{7 : 3}$ Ignore the % sign

a) An orchestra has 60 strings, 12 brass and 9 woodwinds instruments. What is the ratio of strings to brass to woodwinds instruments?

strings : brass: woodwind

$60 : 12 : 9$

Simplify: ÷ 3 $\overset{20}{\cancel{60}} : \overset{4}{\cancel{12}} : \overset{3}{\cancel{9}}$ = $\boxed{20 : 4 : 3}$

b) Tin foil is made up of 88% tin, 4% copper and the rest lead. What is the ratio of tin to copper to lead in the foil?

: :

: :

=

c) Find the ratio of the average weight of a Blue Whale (120 t) to the average weight of a Humpback Whale (30 t).

:

:

=

d) Find the ratio of the Boeing 747 wingspan (64 m) to the Airbus A380 wingspan (80 m).

:

:

=

e) A ticket to the musical "The Boy from Oz" is \$90. A movie ticket to the Village cinemas is \$15. What is the ratio of musical to movie ticket prices?

:

:

=

f) To purchase a "Two Day Park Hopper" to Disnelyland, Florida, costs \$96 for a child and \$116 for an adult. What is the ratio of an adult to child ticket prices?

:

:

=

g) Commercial butter is approximately 80% milk fat and 20% other components. What is the ratio of milk fat to other components?

:

:

=

h) Of the 76 seats in the Senate, 28 belong to the Labor Party. What is the ratio of Labor Party to other parties seats in the Senate?

:

:

=

Skill 10.4 Finding the ratio of two quantities as a subset : set comparison.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Write the ratio in words.
- Replace the words with numbers.
- Simplify the ratio:

EITHER

- Find the largest number that divides evenly into each quantity of the ratio (Highest Common Factor) and divide each quantity by the HCF.

Hint: The order of the quantities in a ratio matters.

OR

- Divide each quantity of the ratio by any factor until the ratio is reduced to simplest form.

Q. Of the 22 million Australian people, approximately 4 million live in Sydney. What is the ratio of Sydney's population to Australian population?

A. *Sydney's population : Australian population*

$= 4 \text{ million} : 22 \text{ million}$ Ignore the units

$= \overset{2}{\cancel{4}} : \overset{11}{\cancel{22}}$ Simplify: ÷ 2

$= \mathbf{2 : 11}$

a) Of the 226 seats in the Australian Parliament, 76 are in the Senate. What is the ratio of Senate to Parliament seats?

Senate seats : Parliament seats

76 : 226

Simplify: ÷ 2 $\overset{38}{\cancel{76}} : \overset{113}{\cancel{226}}$ $= \boxed{38 : 113}$

b) Land represents 30% of the earth's surface. Find the ratio of land to earth's surface.

:

:

=

c) What is the ratio of carbon to the total number of atoms in the ethane formula C_2H_6?

:

:

=

d) What is the ratio of hydrogen to the total number of atoms in the ethylene formula C_2H_4?

:

:

=

e) A viscose/polyester blouse has 44% polyester. What is the ratio of viscose to total composition?

:

:

=

f) Of the 2 L of cordial drink, 250 mL is concentrated cordial. Find the ratio of concentrated cordial to cordial drink.

:

:

=

g) Of the $500 000 paid for the property, $150 000 was for the block of land, and the rest was for building the house. Find the ratio of land to total property price.

:

:

=

h) Blood plasma makes up 55% of the human blood composition. Find the ratio of plasma to total blood components.

:

:

=

Skill 10.5 Finding the average speed.

$$\text{average speed } (v) = \frac{\text{distance travelled } (d)}{\text{time taken } (t)} \quad \text{OR} \quad v = \frac{d}{t}$$

- Write the formula for the average speed.
- Convert the given units into the required units if necessary. (see Maths Facts, page 455)
  Hints: If the speed must be calculated in km/h, convert the units for distance to km and convert the units for time to h.
  Changing from smaller units into larger units, always divide by the conversion factor.
  Changing from larger units into smaller units, always multiply by the conversion factor.
- Substitute the values for distance and time into the formula.
- Evaluate and simplify.

Q. The marine green turtle was recorded swimming 480 km in 10 days. What was its average speed in km/h?

A. $v = \frac{d}{t}$

where $t = 10 \text{ days} = 10 \times 24 \text{ h} = 240 \text{ h}$

$v = \frac{480 \text{ km}}{240 \text{ h}}$ Substitute into the formula

$= \frac{{}^{2}\cancel{480}}{{}_{1}\cancel{240}} \text{ km/h}$ Simplify: ÷ 240

$= \mathbf{2 \text{ km/h}}$

a) A garden snail named Archie covered a 33 cm course in 2 minutes at the 1995 World Snail Racing Championships, held in England. What was Archie's average speed?

$t = 2 \text{ min} = \frac{{}^{1}\cancel{2}}{{}_{30}\cancel{60}} \text{ h} = \frac{1}{30} \text{ h}$ Simplify: ÷ 2

$v = \frac{33 \text{ cm}}{\frac{1}{30} \text{ h}} = 33 \div \frac{1}{30} \text{ cm/h}$

$= 33 \times \frac{30}{1} \text{ cm/h}$ = $\boxed{\quad\quad \text{cm/h}}$

b) The Gentoo penguin in the Antarctic Islands can swim 20 km in half an hour. What is its average speed?

$t = 30 \text{ min} =$

$v =$

= $\boxed{\quad\quad \text{km/h}}$

c) The Suzuki Hayabusa is the world's fastest motorbike. It can travel 100 km in 20 minutes. What is its average speed?

$t = 20 \text{ min} =$

$v =$

= $\boxed{\quad\quad \text{km/h}}$

d) Some species of dolphins can swim 9 km in 10 minutes. What is their average speed in kilometres per hour?

$t =$

$v =$

= $\boxed{\quad\quad \text{km/h}}$

e) A boat crosses a 3 km lake in 20 minutes. What is its average speed?

$t = 20 \text{ min} =$

$v =$

= $\boxed{\quad\quad \text{km/h}}$

f) Johann jogs 12 km in one and a half hours. What is his average speed?

$t =$

$v =$

= $\boxed{\quad\quad \text{km/h}}$

Skill 10.6 Finding the distance travelled.

Mauve 1 1 **2** 2 3 3 4 4
Lime 1 **1** 2 2 3 3 4 4

distance travelled (d) = speed (v) × time taken (t)   OR $d = vt$

- Write the formula for distance travelled.
- Convert the given units into the required units if necessary. (see Maths Facts, page 455)
  Hints: Changing from smaller units into larger units, always divide by the conversion factor.
  Changing from larger units into smaller units, always multiply by the conversion factor.
- Substitute the values for average speed and time into the formula.
- Evaluate and simplify.

Q. The ride duration of the Disneyland monorail in California is approximately 15 minutes. What is the length of the ride if the train's average speed is 16 km/h?
[Give the answer in km.]

A. $d = vt$

where $t = 15\ min = \frac{{}^{1}\cancel{15}}{{}_{4}\cancel{60}}\ h = \frac{1}{4}\ h$

$d = 16\ km/h \times \frac{1}{4} h$ Substitute into the formula

$= {}^{4}\cancel{16} \times \frac{1}{\cancel{4}_{1}}\ km$ Simplify: ÷ 4

$= \mathbf{4\ km}$

a) The Russian Alpha class nuclear-powered submarine has a maximum speed of 75 km/h, or 40 knots. At this speed, what distance can it cover in 12 hours?

$d = 75\ km/h \times 12\ h$

$= 75 \times 12\ km$ = $\boxed{\quad\quad}$ km

b) The Japanese Bullet train is the fastest scheduled train service in the world with an average speed of 260 km/h. At this speed, what distance can it cover in 3 hours?

$d =$

$=$ = $\boxed{\quad\quad}$ km

c) The Nile flows at an average speed of around 7.5 km/h during inundation season. At this speed, what distance might a boat floating on the Nile travel in 6 hours?

$d =$

$=$ = $\boxed{\quad\quad}$ km

d) The Eurostar trains operate from London to Brussels and run at an average speed of 140 km/h. If the trip takes two and half hours, what is the distance from London to Brussels?

$d =$

$=$ = $\boxed{\quad\quad}$ km

e) A garden snail can travel at 0.012 m/s. At this speed, what distance can it cover in 10 minutes?

$t = 10\ min = 10 \times 60\ s = 600\ s$

Use d = vt

$d =$ = $\boxed{\quad\quad}$ m

f) The F-16 Falcon fighter aircraft can fly at a speed of 2400 km per hour at sea level. At this speed, what distance can it cover in 20 min?

$t =$

$d =$ = $\boxed{\quad\quad}$ km

Skill 10.7 Finding the time taken to travel a distance.

$$\text{time taken } (t) = \frac{\text{distance travelled } (d)}{\text{speed } (v)} \qquad \text{OR} \qquad t = \frac{d}{v}$$

- Write the formula for the time taken.
- Convert the given units into the required units if necessary. (see Maths Facts, page 455)
  Hints: Changing from smaller units into larger units, always divide by the conversion factor.
  Changing from larger units into smaller units, always multiply by the conversion factor.
- Substitute the values for the distance travelled and the average speed into the formula.
- Evaluate and simplify.

Q. Donghai Bridge China is the longest cross-sea bridge in the world, with a length of 32 km. How long will it take a car travelling at 80 km/h to cross the bridge? [Give the answer in hours.]

A. $t = \frac{d}{v}$

$= \frac{32 \text{ km}}{80 \text{ km/h}}$ Substitute into the formula

$= \frac{\overset{2}{\cancel{32}}}{\underset{5}{\cancel{80}}} \text{ h}$ Simplify: ÷ 16

$= 2 \div 5 \text{ h}$

$= \mathbf{0.4 \text{ h}}$

a) The speed of long distance jogging for an average person is around 10 km/h. At this speed, how long will it take a person to run 15 km?

$t = \frac{15 \text{ km}}{10 \text{ km/h}}$

$= 15.0 \div 10 \text{ h}$ = ______ h

b) A golf ball leaves the tee and flies at an average speed of 40 m/s till it reaches the 200 m mark. How long did it take the ball to fly this distance?

$t =$

= ______ = ______ s

c) Top athletes can sprint at a speed around 10 m/s within a short distance. How long will it take an athlete to sprint 200 m?

$t =$

= ______ = ______ s

d) The average speed of a space shuttle in orbit is 8000 m/s. At this speed, how long will it take a space shuttle to travel 1000 km?

$t =$

= ______ = ______ s

e) The average walking speed for adults is 5 km/h. At this speed, how long will it take an adult to walk 7 km?

$t =$

= ______ = ______ min

f) The Metro monorail in Sydney is 3.6 km long. At an average speed of 24 km/h, how long would it take the train to complete a loop?

$t =$

= ______ = ______ min

Skill 10.8 Deciding if two ratios are in proportion.

Mauve 1 1 2 2 **3** 3 4 4
Lime 1 1 2 **2** 3 3 4 4

- Write the two ratios as equal fractions side by side.
- Cross multiply the numerators and the denominators of the fractions.
- If the two products are equal, then the two ratios are equivalent (or form a proportion).

A proportion $a:b = c:d$ 2 equal ratios

$\frac{a}{b} \times \frac{c}{d}$ Cross product

$a \times d = b \times c$

$ad = bc$

Q. 5 : 2 is in proportion with 25 : 10
True or false?

A. $\frac{5}{2} \times \frac{25}{10}$ Cross multiply

$5 \times 10 = 2 \times 25$

$50 = 50$

true

a) 5 : 6 is in proportion with 3 : 5
True or false?

$\frac{5}{6} \times \frac{3}{5} \Rightarrow 5 \times 5 = 6 \times 3$

$25 = 18$

false

b) 8 : 12 is in proportion with 6 : 8
True or false?

c) 2 : 12 is in proportion with 3 : 18
True or false?

d) 9 : 15 is in proportion with 30 : 50
True or false?

e) 12 : 15 is in proportion with 4 : 5
True or false?

f) 8 : 20 is in proportion with 20 : 50
True or false?

g) 15 : 150 is in proportion with 6 : 20
True or false?

h) 8 : 10 is in proportion with 20 : 25
True or false?

i) 5 : 6 is in proportion with 10 : 18
True or false?

j) 2 : 3 is in proportion with 3 : 5
True or false?

Skill 10.9 Finding the missing term in a proportion.

- Write the proportion as two equal fractions.
- Cross multiply the numerators and the denominators of the fractions.
- Equate the products.
- Solve the equation to find the missing number (x).

A proportion $a:b = c:d$ 2 equal ratios

$$\frac{a}{b} \times \frac{c}{d}$$

$$a \times d = b \times c$$

$$ad = bc$$

Q. Complete the missing term in the proportion:

10 is to ☐ = 5 is to 25

A. $\frac{10}{x} = \frac{5}{25}$

$\frac{10}{x} \times \frac{5}{25}$ Cross multiply

$10 \times 25 = x \times 5$

$5x = 250$

$\frac{{}^{1}\cancel{5}x}{{}_{1}\cancel{5}} = \frac{\cancel{250}^{50}}{{}_{1}\cancel{5}}$ Simplify: ÷ 5

$x = \mathbf{50}$

a) Complete the missing term in the proportion:

$4 : 6 = 16 : \boxed{24}$

$\frac{4}{6} \times \frac{16}{x}$ Cross multiply

$4x = 96$

$\frac{{}^{1}\cancel{4}x}{{}_{1}\cancel{4}} = \frac{\cancel{96}^{24}}{\cancel{4}_{1}}$ Simplify: ÷ 4 $\Rightarrow x = 24$

b) Complete the missing term in the proportion:

5 : ☐ = 50 : 100

c) Complete the missing term in the proportion:

20 is to 15 = 8 is to ☐

d) Complete the missing term in the proportion:

8 : ☐ = 10 : 15

e) Find the missing term in the proportion:

$\frac{4}{12} = \frac{y}{9}$

$4 \times 9 = 12 \times y$ Cross multiply

$12y = 36$

$y =$

f) Find the missing term in the proportion:

$\frac{6}{x} = \frac{2}{3}$

$x =$

Skill 10.10 Dividing a quantity into a given ratio.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find the total number of equal parts, by adding the numbers in the ratio.
- Calculate what fraction each share represents out of the total number of parts.
- Multiply this fraction by the original quantity.

Q. The ratio of cement to sand to gravel in a concrete mix is 1 : 2 : 3. How much sand is in a 24 kg concrete mix?

A. *equal parts* $= 1 + 2 + 3 = 6$

sand share $=$ *2 out of 6* $= \frac{2^{\div 2}}{6_{\div 2}} = \frac{1}{3}$

sand in 24 kg $= \frac{1}{3}$ *of 24 kg*

$= \frac{1}{{}_1\cancel{3}} \times \overset{8}{\cancel{24}}$ *kg* Simplify: ÷ 3

$=$ ***8 kg***

a) The 18-carat gold is used for jewellery in a ratio of 3 : 1 pure gold to other metals. How many grams of pure gold are needed for a 12 g necklace?

equal parts $= 3 + 1 = 4$

pure gold share $= \frac{3}{4}$

pure gold in 12 g $= \frac{3}{4}$ *of 12 g*

$= \frac{3}{{}_1\cancel{4}} \times \overset{3}{\cancel{12}}$ *g* Simplify: ÷ 4 = ☐ g

b) The ratio of marriages to divorces in Australia in 2006 was 2 : 1. How many divorces would be likely in 1500 couples?

equal parts =

divorce share =

divorces in 1500 couples =

= …………………………………… = ☐

c) The ratio of vowels to consonants in the English language is 5 : 21. How many vowels are likely to be in a 52 000 letter article?

equal parts =

vowels share =

vowels in 52 000 letters =

= …………………………………… = ☐

d) The fuel mix for a chainsaw is 4 parts oil to 21 parts petrol. How much petrol is in a 1500 mL chainsaw tank?

……………………………………

……………………………………

……………………………………

= …………………………………… = ☐ mL

e) The ratio of gold to silver to bronze medals won by Switzerland at the 2010 Vancouver Winter Olympics is 2 : 0 : 1. If Switzerland won 9 medals in total, how many gold medals did they win?

……………………………………

……………………………………

= …………………………………… = ☐

f) The combined monthly bill for the phone, mobile phone and internet is $180. If the ratio of phone to mobile phone to internet costs is 5 : 6 : 4, how much is the phone cost?

……………………………………

……………………………………

……………………………………

= …………………………………… = ☐ $

Skill 10.11 Solving proportions.

- Write the proportion using words.
- Replace the words with numbers:
  - First the given ratio.
  - Then the ratio of the given quantity to the unknown quantity (x).
- Rewrite the proportion as two equal fractions.
- Cross multiply the numerators and the denominators of the fractions.
- Equate the products.
- Solve the equation to find x.

A proportion $a:b = c:d$ 2 equal ratios

$$\frac{a}{b} \times \frac{c}{d}$$

$$a \times d = b \times c$$

$$ad = bc$$

Q. The ratio of concentrated cordial to water is 1 : 4. How much water is needed to dilute 1.5 L of concentrated cordial?

A. *cordial : water = cordial : water*

$1 : 4 = 1.5 : x$

$$\frac{1}{4} \times \frac{1.5}{x}$$

Cross multiply

$1 \times x = 4 \times 1.5$

$x =$ **6 L**

a) A risotto recipe uses a ratio of 2 cups of rice to 6 cups of water. How many cups of water have to be added to 6 cups of rice?

rice : water = rice : water

$2 : 6 = 6 : x$

$$\frac{2}{6} \times \frac{6}{x}$$

Cross multiply

$2x = 36$ Simplify: ÷ 2

So $x =$ ☐

b) The fuel mix for a chainsaw is 4 parts oil to 21 parts petrol. How much petrol needs to be added to 240 mL of oil?

=

=

So $x =$ ☐ mL

c) The ratio of bronze to silver medals won by England at the 2010 Commonwealth games is 3 : 4. If England won 45 bronze medals, how many silver medals did it win?

=

=

So $x =$ ☐

d) To make a 25% saline solution, 1 part of salt is used for every 3 parts of water. If you use 120 g of salt, how much water do you need to make the saline solution?

=

=

So $x =$ ☐ g

e) To mix concrete, 2 buckets of sand are needed for every 3 buckets of gravel. How many buckets of gravel are needed for 10 buckets of sand?

sand : gravel =

=

So $x =$ ☐

f) The superjumbo jet Airbus A380 has a length of 73 m and a wingspan of 80 m. A model of this plane has a wingspan of 160 cm. How long is the model?

=

=

So $x =$ ☐ cm

Skill 10.12 Finding other rates.

$\text{rate} = \dfrac{\text{amount}}{\text{time}}$

Rate of change
(**amount** and **time** are given)

- Convert the given units to the required units.
  (see Maths Facts, page 451)
- Divide the amount by the time taken.
  Example: A 300 L bathtub can be filled in 10 minutes.

  $\text{Rate} = \dfrac{30\cancel{0}\text{ L}}{1\cancel{0}\text{ min}} = 30\text{ L/min}$

$\text{amount} = \text{rate} \times \text{time}$

Amount
(**rate** and **time** are given)

- Convert the given units to the required units.
  (see Maths Facts, page 451)
- Multiply the rate by the time taken.
  Example: Sam worked 7 hours and was paid at a rate of $16/h.

  $\text{Amount (pay)} = 16 \times 7 = \$112$

$\text{time} = \dfrac{\text{amount}}{\text{rate}}$

Time taken
(**amount** and **rate** are given)

- Convert the given units to the required units.
  (see Maths Facts, page 451)
- Divide the amount by the rate.
  Example: A Lexmark E232 prints 990 pages at a rate of 22 pages/min (ppm).

  $\text{Time} = \dfrac{990\text{ p}}{22\text{ ppm}} = 45\text{ min}$

Q. The average pulse for a new born baby is around 130 beats per minute. How many beats in 3 hours is this?

A. *amount = rate × time*

rate = 130 beats/min

time = 3 h = 3 × 60 min = 180 min

amount = 130 beats/min × 180 min

= ***23 400 beats***

a) The ruby throated hummingbird can beat its wings around 21 000 times in 5 minutes. What is its wing beat rate in beats per second?

time = 5 × 60 s = 300 s

Simplify: ÷ 100 then ÷ 3

$\text{rate} = \dfrac{\overset{7}{\cancel{21}}\,\cancel{000}\text{ beats}}{{}_{1}\cancel{300}\text{ s}} =$ beats/s

b) The heart of an unborn baby beats at around 9000 times in an hour. What is the heart rate of an unborn baby in beats per minute?

time = 1 × 60 min = 60 min

rate = = beats/min

c) Find the time taken to print 875 sheets of paper, if a Lexmark T640 printer can print 35 pages per minute.

$\text{time} = \dfrac{\text{amount}}{\text{rate}}$

= = min

d) An adult's air intake adds up to 7500 L per day. How many litres of air does an adult breathe in a week?

amount =

= = L

e) Victoria has an average population density of around 22 people per square kilometre. If Victoria has an area of approximately 228 000 km^2, what is its population?

population = area × density rate

= =

f) Bangladesh is the most densely populated country, with around 1050 people per km^2 in 2007. If in 2007 Bangladesh had a population of 151 200 000, what is its area in km^2?

area = population ÷ density rate

= = km^2

Skill 10.13 Completing equivalent rates.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Write the given rate as a fraction.
- Show the respective units in the fraction.
- Convert the units to match to the new rate.
- Use the conversion factors for units of time, weight and length.
- Multiply by the conversion factor to change from larger units to smaller units.
  Example: To change cm to mm
  × by 10
- Divide by the conversion factor to change from smaller units to larger units.
  Example: To change m to km
  ÷ by 1000

Q. 2 km/min = ☐ km/h

A. *2 km per minute*

$$= \frac{2\ km}{1\ min}$$ 1 h = 60 min

$$= \frac{2\ km}{\frac{1}{60}\ h}$$ $1\ \text{min} = \frac{1}{60}\ \text{h}$

$$= 2 \times \frac{60}{1}\frac{km}{h}$$

$$= \mathbf{120}\ km/h$$

a) 144 L/h = ☐ mL/h

144 L per hour Convert L to mL

$$= \frac{144\ L}{1\ h} = \frac{144\,000\ mL}{1\ h} = 144\,000\ mL/h$$

b) 10 km/h = ☐ m/h

= = =

c) 3000 g/min = ☐ kg/min

= = =

d) \$5/kg = ☐ cents/kg

= = =

e) 60 mm/s = ☐ mm/min

= = =

f) 5 answers/min = ☐ answers/h

= = =

g) 2000 mm/s = ☐ m/s

= = =

h) 480 beats/h = ☐ beats/min

= = =

Skill 10.14 Identifying direct proportion in real life situations.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 3 4 4

- Write the rates of the two quantities as fractions.
  - The rate of the units (e.g. km per hour = km/h).
  - The first rate of the given quantities.
  - The second rate containing the unknown quantity (*x*).
- Write the proportion as two equal fractions.
- Cross multiply the numerators and the denominators of the fractions.
- Equate the products.
- Solve the equation to find *x*.

A proportion

$a:b = c:d$

2 equal ratios

$\frac{a}{b} \times \frac{c}{d}$

$a \times d = b \times c$

$ad = bc$

Q. Lin can write 7 pages in 24 minutes. Assuming her writing rate is directly proportional, how many pages can she write in 120 minutes?

A. *rate of units:* $\frac{pages}{minute}$

Let *x* be the number of pages Lin can write in 120 minutes.

1st rate: $\frac{7}{24}$

2nd rate: $\frac{x}{120}$

Cross multiply

$\frac{7}{24} \times \frac{x}{120}$

Write the proportion. Cross multiply.

$7 \times 120 = 24 \times x$

$24x = 840$

$x = \mathbf{35}$

Solve for *x*.

a) I can buy eight pens for \$4.50. How many pens can I buy for \$18?

rate of units: $\frac{pens}{\$}$ *1st rate:* $\frac{8}{4.50}$

2nd rate: $\frac{x}{18}$ $\Rightarrow$ $\frac{8}{4.50} = \frac{x}{18}$

$\Rightarrow 8 \times 18 = 4.5 \times x$ $x =$

b) If 6 m of fabric cost \$75, what is the cost of 4 m of the same fabric?

rate of units: $\frac{m}{\$}$ *1st rate:* $\frac{6}{75}$

2nd rate: $\Rightarrow$

$\Rightarrow$ $x =$ \$

c) It took Jon 40 minutes to run 6 laps of the oval. Assuming his running rate is directly proportional, how long it would take him to run 9 laps?

rate of units: *1st rate:*

2nd rate: $\Rightarrow$

$\Rightarrow$ $x =$ min

d) Paper costs \$4.50 for 500 sheets. What is the cost for 200 sheets?

rate of units: *1st rate:*

2nd rate: $\Rightarrow$

$\Rightarrow$ $x =$ \$

e) Fred's respiration rate is 62 breaths in 4 minutes. At this rate, how many breaths will he take in 20 minutes?

rate of units: *1st rate:*

2nd rate: $\Rightarrow$

$\Rightarrow$ $x =$

f) Tony eats 5 fruits in 2 days. Assuming his eating rate is directly proportional, how many fruits does Tony eat in 16 days?

rate of units: *1st rate:*

2nd rate: $\Rightarrow$

$\Rightarrow$ $x =$

Skill 10.15 Converting units of speed.

$$\text{speed} = \frac{\text{distance}}{\text{time}} \quad \text{OR} \quad v = \frac{d}{t}$$

Convert km/h to m/s

- Write the speed as distance over time.
- Change km to m by multiplying by 1000.
- Change h to s by multiplying by $60 \times 60 = 3600$.
- Evaluate and simplify.

Convert m/s to km/h

- Write the speed as distance over time.
- Change m to km by dividing by 1000.
- Change s to h by dividing by 60 to get to mins and by 60 again to get to hours.
- Evaluate and simplify.

Q. The fastest combat jet is the former Soviet MIG 25, which reached a speed of 940 m/s. What is the speed in km/h?

A. $v = \frac{d}{t}$

$= \frac{940\ m}{1\ s}$

change m to km: ÷ 1000

change s to h: ÷ 3600

$= \frac{940 \div 1000}{1 \div 3600}$

fraction line means division

$= \frac{940}{1000} \div \frac{1}{3600}$ km/h

$= \frac{94\cancel{0}}{1\cancel{0}\cancel{0}\cancel{0}} \times \frac{36\cancel{0}\cancel{0}}{1}$ km/h

Simplify: ÷ 100 then ÷ 10

$=$ **3384 km/h**

a) Shoaib Akhtar, Pakistan, has bowled the fastest ball in Test cricket with a speed of nearly 162 km/h. How many m/s is this?

$v = \frac{162\ km}{1\ h}$

$= \frac{\overset{27}{\cancel{162}} \times 10\cancel{0}\cancel{0}\ m}{1 \times \underset{6}{\cancel{36}\cancel{0}\cancel{0}}\ s} = \frac{270}{6} =$ ☐ m/s

Simplify: ÷ 100 then ÷ 6

b) The fastest propelled aircraft is the former Soviet Tupolev Tu-95/142, which reached a speed of 250 m/s. How many km/h is this?

$v = 250\ m/s = \frac{250 \div 1000\ km}{1 \div 3600\ h}$

$=$ .... $=$ ☐ km/h

c) The average speed of a space shuttle in orbit is 28 800 km/h. How many m/s is this?

$v =$ ....

$=$ .... $=$ ☐ m/s

d) The earth is moving around the sun at a speed of about 30 000 m/s. How many km/h is this?

$v =$ ....

$=$ .... $=$ ☐ km/h

e) The fastest electric vehicle reached 396 km/h in Utah, USA, in 1999. What is this speed in m/s?

$v =$ ....

$=$ .... $=$ ☐ m/s

f) While crossing the Eurotunnel the Eurostar train can reach speeds of up to 85 m/s. How many km/h is this?

$v =$ ....

$=$ .... $=$ ☐ km/h

Skill 10.16 Working with ratio scales (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

RATIO SCALE

Refers to the map — 1 : 4 000 000 — Refers to real life

scale factor

Hint: 1 unit on the map represents 4 000 000 of the same units in real life.
The real distance is 4 000 000 times bigger than the map distance.

Finding the real life distance (ratio scale and distance on the map are given)	**Finding the distance on the map (ratio scale and real life distance are given)**	**Finding the scale factor (real life distance and distance on the map are given)**
• Multiply the distance on the map (usually in cm) by the scale factor. • Convert the result from cm to km, by dividing by 100 000.	• Convert the real distance from km to cm by multiplying by 100 000. • Divide the real distance in cm by the scale factor.	• Convert the real distance from km to cm by multiplying by 100 000. • Divide the real distance in cm by the map distance in cm.

Q. On a map the scale is 1 : 10 000 000. What is the distance on the map between Brisbane and Adelaide, if they are 2000 km apart?

A. *real life distance = 2000 km*

scale factor = 10 000 000

map distance = ?

Convert km to cm

2000 km = 2000 × 100 000 cm =

= 200 000 000 cm

Cross off respective 0's

$20\cancel{0}\,\cancel{0}\cancel{0}\cancel{0}\,\cancel{0}\cancel{0}\cancel{0}\text{ cm} \div 1\cancel{0}\,\cancel{0}\cancel{0}\cancel{0}\,\cancel{0}\cancel{0}\cancel{0} =$

= 20 cm

a) On a map the scale ratio is 1 : 20 000 000. What is the real distance between Tokyo and Geneva, if they are 80 cm apart on the map?

real distance = 80 cm × 20 000 000

= 1 600 000 000 cm

Cross off respective 0's

$1\,600\,0\cancel{0}\cancel{0}\,\cancel{0}\cancel{0}\cancel{0}\text{ cm} \div 1\cancel{0}\cancel{0}\,\cancel{0}\cancel{0}\cancel{0} =$ ☐ km

b) On a map the scale ratio is 1 : 30 000 000. What is the real distance between Sydney and London, if they are 70 cm apart on the map?

real distance = ...

= ....

.... = ☐ km

c) On a map the scale ratio is 1 : 25 000 000. What is the real distance between Broome and Melbourne, if they are 20 cm apart on the map?

real distance = ...

= ....

.... = ☐ km

d) On a map the scale ratio is 1 : 3 000 000. What is the real distance between Sydney and Canberra, if they are 10 cm apart on the map?

....

....

.... ☐ km

Skill 10.16 Working with ratio scales (2).

e) The Empire State Building is 380 m high. If you are drawing a model using a scale of 1 : 2000, how high is the model?

real height = 380 m = 38 000 cm

model height = 38 000 cm ÷ 2000

= ☐ cm

f) Taipei 101 is the world's tallest building, with a height of 500 m. If you are building a model using a scale ratio of 1 : 1000, how tall is the model?

real height =

model height=

= ☐ cm

g) On a map the scale ratio is 1 : 10 000 000. What is the distance on the map between Sydney and Singapore, if the real life distance is 4000 km?

real distance =

map distance =

= ☐ cm

h) On a town plan the scale ratio is 1 : 100 000. What is the distance on the map between the airport and the post office, if they are 15 km apart?

real distance =

map distance =

= ☐ cm

i) On a map the distance between Perth and Cairns is 50 cm. What is the scale factor of the map, if the real life distance is 6000 km?

real distance = 6000 × 100 000 cm

= 600 000 000 cm

scale factor = 600 000 000 cm ÷ 50 cm

= 12 000 000 = 1 :12 000 000

j) The Walt Disney studios' main theatre screen dimensions are 14 m wide by 5 m high. If you make a model 2.5 cm high, what is the scale factor used to build the model?

real height =

=

scale factor =

= = 1 :

k) On a map the distance between Darwin and Perth is 20 cm. What is the scale factor of the map, if the real life distance is 4000 km?

real distance =

=

scale factor =

= = 1 :

l) Petronas Towers in Kuala Lumpur are 450 m tall. A model has a height of 30 cm. What scale factor was used to build the model?

real height =

=

scale factor =

= = 1 :

Skill 10.17 Comparing rates.

Mauve 11 22 33 44
Lime 11 22 33 44

- Write the rates as fractions.
- Bring the rates to the same unit. (see Maths Facts, page 455)
  Hint: It is easier to change from larger units into smaller units, because you have to multiply by the conversion factor.
- Evaluate and simplify.
- Compare the numbers.

Q. Which density is lower?

A) 670 kg/m³ (gasoline)
B) 1.025 g/cm³ (seawater)

A. *kg/m³ to g/cm³: kg to g ⇒ × 1000*

m³ to cm³ ⇒ × 1 000 000

$$A)\ density = \frac{670\ kg}{1\ m^3}$$

$$= \frac{67\cancel{0} \times 1\cancel{000}\ g}{1 \times 1\,00\cancel{0}\,\cancel{000}\ cm^3}$$

cross off respective 0's

$$= \frac{67}{100}\ g/cm^3$$

$$= 0.67\ g/cm^3$$

$$B)\ density = 1.025\ g/cm^3$$

0.67 < 1.025, so the answer is ***A***

a) Which heartbeat rate is higher?

A) 70 beats/min (adult)
B) 2 beats/s (baby)

$$B)\ rate = \frac{2\ beats}{1\ s} = \frac{2\ beats}{\frac{1}{60}\ min}$$

$$= 2 \div \frac{1}{60} = 2 \times \frac{60}{1} = 120\ beats/min$$

120 beats/min > 70 beats/min

B

b) Which heartbeat rate is lower?

A) 1.5 beats/s (child)
B) 70 beats/min (adult)

$$A)\ rate =$$

c) Which birth rate is lower?

A) 12.1 per 1000 people (Australia)
B) 82 per 10 000 people (Germany)

$$A)\ rate = \frac{12.1}{1000} =$$

d) Which birth rate is higher?

A) 14.1 per 1000 people (USA)
B) 94 per 10 000 people (Japan)

e) Which density is lower?

A) 800 kg/m³ (petrol)
B) 1 g/cm³ (water)

f) Which density is higher?

A) 2.2 g/cm³ (graphite)
B) 1300 kg/m³ (PVC)

11. [Indices]

Skill 11.1 Evaluating whole numbers in index form.

Mauve **11** 22 33 44
Lime **11** 22 33 44

- Multiply the base by itself the same number of times as indicated by the index (exponent).

power

5^4 Read as: 5 to the power of 4 $\Rightarrow$ base — 5^4 — index $\Rightarrow$ $5^4 = 5 \times 5 \times 5 \times 5$

5 multiplied by itself 4 times

$6^0 =$ **1** Any number raised to the power of zero equals one.	$3^1 =$ **3** Any number raised to the power of one equals itself.	3^2 = three squared $= 3 \times 3$ = **9**	2^3 = two cubed $= 2 \times 2 \times 2$ = **8**

Q.	**A.**
$2^5 =$	$2^5 =$ $= 2 \times 2 \times 2 \times 2 \times 2$ — 2 multiplied by itself 5 times $=$ ***32***

a) $3^4 =$

$= 3 \times 3 \times 3 \times 3 = \boxed{81}$

b) $2^3 =$

$= 2 \times 2 \times 2 = \boxed{}$

c) $2^6 =$

$=$ .... $= \boxed{}$

d) $5^2 =$

$=$ .... $= \boxed{}$

e) $1^7 =$

$=$ .... $= \boxed{}$

f) $4^2 =$

$=$ .... $= \boxed{}$

g) $7^2 =$

$=$ .... $= \boxed{}$

h) $6^3 =$

$=$ .... $= \boxed{}$

i) $10^3 =$

$=$ .... $= \boxed{}$

j) $3^5 =$

$=$ .... $= \boxed{}$

k) $7^3 =$

$=$ .... $= \boxed{}$

l) $9^2 =$

$=$ .... $= \boxed{}$

m) $8^1 =$

$=$ .... $= \boxed{}$

n) $9^0 =$

$=$ .... $= \boxed{}$

o) $0^7 =$

$=$ .... $= \boxed{}$

p) $4^3 =$

$=$ .... $= \boxed{}$

q) $1^{10} =$

$=$ .... $= \boxed{}$

r) $10^4 =$

$=$ .... $= \boxed{}$

Skill 11.2 Evaluating powers with fraction bases.

- Observe the index. The index tells you how many times to multiply the numerator by itself and the denominator by itself.

numerator, 2 to the power of 3

Base $\left(\frac{2}{5}\right)^3$ Index

denominator, 5 to the power of 3

2 multiplied by itself 3 times

$$\left(\frac{2}{5}\right)^3 = \frac{2\times 2\times 2}{5\times 5\times 5} = \frac{8}{125}$$

5 multiplied by itself 3 times

Q. $\left(\frac{3}{7}\right)^2 =$

A. $\left(\frac{3}{7}\right)^2 =$

$= \frac{3\times 3}{7\times 7}$

3 multiplied by itself 2 times

7 multiplied by itself 2 times

$= \frac{\mathbf{9}}{\mathbf{49}}$

a) $\left(\frac{3}{10}\right)^3 =$

$= \frac{3\times 3\times 3}{10\times 10\times 10} = \boxed{\frac{27}{1000}}$

b) $\left(\frac{1}{5}\right)^3 =$

$=$ …………………… $= \boxed{}$

c) $\left(\frac{2}{7}\right)^2 =$

$=$ …………………… $= \boxed{}$

d) $\left(\frac{1}{10}\right)^2 =$

$=$ …………………… $= \boxed{}$

e) $\left(\frac{2}{3}\right)^3 =$

$=$ …………………… $= \boxed{}$

f) $\left(\frac{3}{8}\right)^2 =$

$=$ …………………… $= \boxed{}$

g) $\left(\frac{6}{11}\right)^2 =$

$=$ …………………… $= \boxed{}$

h) $\left(\frac{4}{9}\right)^2 =$

$=$ …………………… $= \boxed{}$

i) $\left(\frac{1}{4}\right)^4 =$

$=$ …………………… $= \boxed{}$

j) $\left(\frac{2}{3}\right)^5 =$

$=$ …………………… $= \boxed{}$

k) $\left(\frac{4}{5}\right)^3 =$

$=$ …………………… $= \boxed{}$

l) $\left(\frac{7}{10}\right)^3 =$

$=$ …………………… $= \boxed{}$

m) $\left(\frac{3}{4}\right)^3 =$

$=$ …………………… $= \boxed{}$

n) $\left(\frac{5}{8}\right)^2 =$

$=$ …………………… $= \boxed{}$

o) $\left(\frac{5}{12}\right)^2 =$

$=$ …………………… $= \boxed{}$

p) $\left(\frac{3}{4}\right)^4 =$

$=$ …………………… $= \boxed{}$

q) $\left(\frac{3}{10}\right)^4 =$

$=$ …………………… $= \boxed{}$

r) $\left(\frac{9}{13}\right)^2 =$

$=$ …………………… $= \boxed{}$

Skill 11.3 Multiplying powers with the same base.

$$a^m \times a^n = a^{m+n}$$

- Add the indices of like numbers or like variables.

Example: $8^3 \times 8^4 = (8 \times 8 \times 8) \times (8 \times 8 \times 8 \times 8)$
$= 8^{3+4}$
$= 8^7$

- The size of the new index tells you how many times to multiply the base by itself.

Q. Evaluate 6×6^2

A. 6×6^2
$= 6^{1+2}$ add the indices
$= 6^3$
$= 6 \times 6 \times 6$
$= \mathbf{216}$

a) Evaluate $2^3 \times 2$
$= 2^{3+1} = 2^4$
$= 2 \times 2 \times 2 \times 2 = \boxed{16}$

b) Evaluate $3^2 \times 3^3$
= ……… = ………
= ……… = ☐

c) Evaluate 2×2^2
= ……… = ………
= ……… = ☐

d) Evaluate $5^3 \times 5$
= ……… = ………
= ……… = ☐

e) Evaluate 4×4^2
= ……… = ………
= ……… = ☐

f) Evaluate $3^2 \times 3^2$
= ……… = ………
= ……… = ☐

g) Evaluate 3×3^4
= ……… = ………
= ……… = ☐

h) Evaluate $5^2 \times 5$
= ……… = ………
= ……… = ☐

i) Evaluate $6^3 \times 6$
= ……… = ………
= ……… = ☐

j) Simplify $x^3 \times x^6$
= ……… = ☐

k) Simplify $z \times z^3$
= ……… = ☐

l) Simplify $y^3 \times y^2$
= ……… = ☐

m) Simplify $d \times d^6$
= ……… = ☐

n) Simplify $ab^2 \times a^4b^2$
$= a^{1+4} \times b^{2+2} = \boxed{a^5b^4}$

o) Simplify $fg^2 \times f^4g$
= ……… = ☐

p) Simplify $cd^3 \times c^2d^2$
= ……… = ☐

q) Simplify $bc \times b^3c^2$
= ……… = ☐

r) Simplify $u^3v^2 \times uv^3$
= ……… = ☐

s) Simplify $l^2m^3 \times lm^4$
= ……… = ☐

t) Simplify $a^3b \times ab$
= ……… = ☐

u) Simplify $g^2h^2 \times gh^2$
= ……… = ☐

Skill 11.4 Dividing powers with the same base.

$$a^m \div a^n = a^{m-n}$$

- Subtract the indices of like numbers or like variables.

  Example: $8^5 \div 8^3 = \frac{8^5}{8^3} = \frac{8\times 8\times \cancel{8}\times \cancel{8}\times \cancel{8}}{\cancel{8}\times \cancel{8}\times \cancel{8}}$

  $= 8^{5-3} = 8^2 = 64$

- The size of the new index tells you how many times to multiply the base by itself.

  Hint: A number or a variable without an index actually is to the power of 1.

Q. Evaluate $2^9 \div 2^3$

A. $2^9 \div 2^3$

$= 2^{9-3}$ subtract the exponents

$= 2^6$

$= 2\times 2\times 2\times 2\times 2\times 2$

$= \mathbf{64}$

a) Evaluate $4^3 \div 4$

$= 4^{3-1} \quad = 4^2$

$= 4 \times 4 \quad = \boxed{16}$

b) Evaluate $3^8 \div 3^4$

= =

= =

c) Evaluate $2^7 \div 2^2$

= =

= =

d) Evaluate $6^4 \div 6$

= =

= =

e) Evaluate $9^6 \div 9^4$

= =

= =

f) Evaluate $5^8 \div 5^5$

= =

= =

g) Evaluate $\frac{8^8}{8^5}$

$= 8^{8-5} \quad = 8^3$

= =

h) Evaluate $\frac{7^4}{7^2}$

= =

= =

i) Evaluate $\frac{10^9}{10^6}$

= =

= =

j) Simplify $t^4 \div t^3$

= =

k) Simplify $p^8 \div p^2$

= =

l) Simplify $r^9 \div r^2$

= =

m) Simplify $\frac{j^8}{j^3}$

= =

n) Simplify $\frac{q^9}{q^4}$

= =

o) Simplify $\frac{y^7}{y^5}$

= =

p) Simplify $\frac{p^6q^4}{p^3q^2}$

$= (p^6 \div p^3) \times (q^4 \div q^2)$

$= p^{6-3} \times q^{4-2} \quad = \boxed{p^3q^2}$

q) Simplify $\frac{a^5b^3}{ab}$

=

= =

r) Simplify $\frac{t^4u^6}{tu^2}$

=

= =

Skill 11.5 Multiplying powers with coefficients and with the same base.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 **2** 2 3 3 4 4

- Multiply the coefficients.
- Add the indices of like variables.

Example: $2a^3 \times 3a^2 = \boxed{2 \times 3} \times \boxed{a \times a \times a} \times \boxed{a \times a}$
$= 6a^{3+2} = 6a^5$

Hint: A number or a variable without an index actually is to the power of 1.

Q. Simplify $7a \times a^3$

A. $7a \times a^3$
multiply the coefficients $= (7 \times 1) \times (a^1 \times a^3)$
$= 7a^{1+3}$ add the indices
$= \mathbf{7a^4}$

a) Simplify $3t^4 \times 3t$
$= (3 \times 3) \times (t^4 \times t^1)$
$= 9t^{4+1} = \boxed{9t^5}$

b) Simplify $x^3 \times 2x$
=
= =

c) Simplify $2p^2 \times 2p$
=
= =

d) Simplify $2b \times 3b^2$
=
= =

e) Simplify $2d^2 \times 4d^4$
=
= =

f) Simplify $3m^3 \times 5m^5$
=
= =

g) Simplify $4s^2 \times 6s^3$
=
= =

h) Simplify $5a^4 \times 2a^6$
=
= =

i) Simplify $7k^2 \times k^7$
=
= =

j) Simplify $6c^6 \times 3c^5$
=
= =

k) Simplify $2y \times y^7$
=
= =

l) Simplify $7w^8 \times 4w$
=
= =

m) Simplify $2r \times 8r$
=
= =

n) Simplify $5g^4 \times 5g^4$
=
= =

o) Simplify $y^8 \times 6y^5$
=
= =

p) Simplify $10a^2 \times 2a^4$
=
= =

q) Simplify $7p^3 \times 5p^6$
=
= =

r) Simplify $3d^7 \times 12d$
=
= =

Skill 11.6 Dividing powers with coefficients and with the same base.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 **2** 3 3 4 4

- Divide the coefficients.
- Subtract the indices of like variables.

Example: $12a^5 \div 4a^2 = (12 \div 4) \times (a^5 \div a^2)$
$= 3 \times a^{5-2}$
$= 3a^3$

OR

$$\frac{12a^5}{4a^2} = \frac{12 \times a \times a \times a \times \cancel{a} \times \cancel{a}}{4 \times \cancel{a} \times \cancel{a}} = \frac{12a^3}{4} = 3a^3$$

Hint: A number or a variable without an index actually is to the power of 1.

Q. Simplify $10j^6 \div 5j^3$

A. $10j^6 \div 5j^3 =$
$= (10 \div 5) \times (j^6 \div j^3)$ subtract the indices
$= 2 \times j^{6-3}$
$= \mathbf{2j^3}$

a) Simplify $8c^4 \div 2c^3$
$= (8 \div 2) \times (c^4 \div c^3)$
$= 4 \times c$ $= \boxed{4c}$

b) Simplify $6a^5 \div 2a^2$
= ……………………
= …………… = $\boxed{\quad}$

c) Simplify $8h^7 \div 2h^3$
= ……………………
= …………… = $\boxed{\quad}$

d) Simplify $10m^9 \div 2m$
= ……………………
= …………… = $\boxed{\quad}$

e) Simplify $5z^8 \div 5z^4$
= ……………………
= …………… = $\boxed{\quad}$

f) Simplify $12f^7 \div 2f^2$
= ……………………
= …………… = $\boxed{\quad}$

g) Simplify $\frac{8u^{11}}{4u^7}$
$= (8 \div 4) \times (u^{11} \div u^7)$
$= 2 \times u^{11-7}$ = $\boxed{\quad}$

h) Simplify $\frac{12b^3}{6b}$
= ……………………
= …………… = $\boxed{\quad}$

i) Simplify $\frac{6w^5}{2w^2}$
= ……………………
= …………… = $\boxed{\quad}$

j) Simplify $\frac{18v^{12}}{9v^9}$
= ……………………
= …………… = $\boxed{\quad}$

k) Simplify $\frac{25x^{13}}{5x}$
= ……………………
= …………… = $\boxed{\quad}$

l) Simplify $\frac{16n^7}{2n^2}$
= ……………………
= …………… = $\boxed{\quad}$

m) Simplify $\frac{7e^{10}}{e^6}$
= ……………………
= …………… = $\boxed{\quad}$

n) Simplify $\frac{14q^6}{7q^6}$
= ……………………
= …………… = $\boxed{\quad}$

o) Simplify $\frac{9w^6}{3w^2}$
= ……………………
= …………… = $\boxed{\quad}$

Skill 11.7 Raising a product to a power.

$$(ab)^m = a^m \times b^m$$

- Raise each number or variable in the product to the index.
  Example: $(ab)^3 = a^3 \times b^3 = a^3b^3$
- Multiply from left to right.
  Hint: A number or a variable without an index actually is to the power of 1.

Q. Simplify $5(2x)^3$

A. $5(2x)^3$
$= 5 \times 2^3 \times x^3$
$= 5 \times 8 \times x^3$
$= \mathbf{40x^3}$

a) Simplify $(2x)^4$
$= 2^4 \times x^4$
$= 16 \times x^4 = \boxed{16x^4}$

b) Simplify $(5y)^3$
=
= =

c) Simplify $(2v)^6$
=
= =

d) Simplify $(tu)^4$
= =

e) Simplify $(fg)^2$
= =

f) Simplify $(de)^f$
= =

g) Simplify $(6m)^3$
=
= =

h) Simplify $(7r)^2$
=
= =

i) Simplify $(3p)^4$
=
= =

j) Simplify $(2b)^2$
=
= =

k) Simplify $(5y)^3$
=
= =

l) Simplify $(4k)^3$
=
= =

m) Simplify $6(2y)^3$
$= 6 \times 2^3 \times y^3$
$= 6 \times 8 \times y^3 = \boxed{48y^3}$

n) Simplify $2(2q)^2$
=
= =

o) Simplify $4(2n)^3$
=
= =

p) Simplify $5(2v)^5$
=
= =

q) Simplify $3(4s)^2$
=
= =

r) Simplify $8(3h)^2$
=
= =

Skill 11.8 Raising a power to another power.

- Multiply the indices of the variable or pronumeral.

Example: $(a^2)^4 = a^2 \times a^2 \times a^2 \times a^2 = a^{2+2+2+2}$
$= a^{2\times 4}$
$= a^8$

$$(a^m)^n = a^{m\times n}$$

Hint: A number or a variable without an index actually is to the power of 1.

Q. Simplify $(m^y)^z$

A. $(m^y)^z$
$= m^{y\times z}$ multiply the exponents
$= \boldsymbol{m^{yz}}$

a) Simplify $(y^4)^2$
$= y^4 \times y^4 = y^{4+4}$
$= y^{4\times 2} = \boxed{y^8}$

b) Simplify $(r^4)^4$
= ……………………
= …………… = $\boxed{\quad}$

c) Simplify $(x^2)^5$
= ……………………
= …………… = $\boxed{\quad}$

d) Simplify $(a^e)^f$
= …………… = $\boxed{\quad}$

e) Simplify $(p^q)^r$
= …………… = $\boxed{\quad}$

f) Simplify $(t^u)^v$
= …………… = $\boxed{\quad}$

g) Simplify $(d^2)^2$
= ……………………
= …………… = $\boxed{\quad}$

h) Simplify $(h^3)^2$
= ……………………
= …………… = $\boxed{\quad}$

i) Simplify $(n^4)^3$
= ……………………
= …………… = $\boxed{\quad}$

j) Simplify $(w^5)^0$
= ……………………
= …………… = $\boxed{\quad}$

k) Simplify $(a^4)^5$
= ……………………
= …………… = $\boxed{\quad}$

l) Simplify $(g^2)^5$
= ……………………
= …………… = $\boxed{\quad}$

m) Simplify $2(b^3)^2$
$= 2 \times (b^3)^2$
$= 2 \times b^{3\times 2} = \boxed{2b^6}$

n) Simplify $4(q^3)^3$
= ……………………
= …………… = $\boxed{\quad}$

o) Simplify $5(z^3)^2$
= ……………………
= …………… = $\boxed{\quad}$

p) Simplify $6(c^4)^3$
= ……………………
= …………… = $\boxed{\quad}$

q) Simplify $8(w^4)^2$
= ……………………
= …………… = $\boxed{\quad}$

r) Simplify $7(k^3)^5$
= ……………………
= …………… = $\boxed{\quad}$

Skill 11.9 Raising a negative number to a power.

- Observe the index.
- Multiply the number (base) the same number of times by itself as the index shows.
- Give the result a sign.

A negative number raised to an **even index** gives a **positive result**

$(-5)^{2} = -5 \times (-5)$
$= +25$

A negative number raised to an **odd index** gives a **negative result**

$(-5)^{3} = -5 \times (-5) \times (-5)$
$= +25 \times (-5)$
$= -125$

Q. Evaluate $(-4)^3$

A. $(-4)^3$ odd index
$= -4 \times -4 \times -4$
$= \mathbf{-64}$ negative result

a) Evaluate $(-9)^2$ even index
$= -9 \times -9$ .... $= \boxed{81}$
positive result

b) Evaluate $(-2)^2$
= .... =

c) Evaluate $(-1)^5$
= .... =

d) Evaluate $(-4)^2$
= .... =

e) Evaluate $(-8)^2$
= .... =

f) Evaluate $(-2)^4$
= .... =

g) Evaluate $(-1)^7$
= .... =

h) Evaluate $(-2)^6$
= .... =

i) Evaluate $(-3)^3$
= .... =

j) Evaluate $(-5)^1$
= .... =

k) Evaluate $(-4)^4$
= .... =

l) Evaluate $(-7)^3$
= .... =

m) Evaluate $(-6)^3$
= .... =

n) Evaluate $(-7)^2$
= .... =

o) Evaluate $(-5)^4$
= .... =

p) Evaluate $(-3)^4$
= .... =

q) Evaluate $(-5)^3$
= .... =

r) Evaluate $(-10)^4$
= .... =

s) Evaluate $(-10)^3$
= .... =

t) Evaluate $(-12)^2$
= .... =

u) Evaluate $(-1)^{12}$
= .... =

v) Evaluate $(-15)^2$
= .... =

w) Evaluate $(-11)^3$
= .... =

x) Evaluate $(-1)^{123}$
= .... =

Skill 11.10 Raising a number to a negative power.

- Write the inverse of the same number with the positive index (exponent).

$$a^{-n} = \frac{1}{a^n}$$

Negative index

Base

$$a^{\textcircled{-2}} = \frac{1}{a^{\textcircled{2}}}$$

Positive index

Q. Evaluate 4^{-2}

A. 4^{-2}

$= \frac{1}{4^2}$

$= \frac{1}{4 \times 4}$ Multiply 4 by itself 2 times

$= \mathbf{\frac{1}{16}}$

a) Evaluate 10^{-3}

$= \frac{1}{10^3} = \frac{1}{10 \times 10 \times 10} = \boxed{\frac{1}{1000}}$

b) Evaluate 3^{-3}

= = ... = ☐

c) Evaluate 2^{-2}

= = ... = ☐

d) Evaluate 4^{-3}

= = ... = ☐

e) Evaluate 7^{-1}

= = ... = ☐

f) Evaluate 8^{-2}

= = ... = ☐

g) Evaluate 9^{-2}

= .... = ☐

h) Evaluate 2^{-4}

= .... = ☐

i) Evaluate 5^{-4}

= .... = ☐

j) Evaluate 6^{-3}

= .... = ☐

k) Evaluate 5^{-2}

= .... = ☐

l) Evaluate 3^{-5}

= .... = ☐

m) Evaluate 2^{-5}

= .... = ☐

n) Evaluate 10^{-4}

= .... = ☐

o) Evaluate 4^{-4}

= .... = ☐

p) Evaluate 2^{-6}

= .... = ☐

q) Evaluate 3^{-3}

= .... = ☐

r) Evaluate 6^{-2}

= .... = ☐

s) Evaluate 10^{-2}

= .... = ☐

t) Evaluate 5^{-3}

= .... = ☐

u) Evaluate 1^{-25}

= .... = ☐

12. [Square Roots]

Skill 12.1 Calculating square roots of perfect squares.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Estimate which number, multiplied by itself, produces the number under the square root sign.
- Check your estimation by multiplying your guess by itself.

Hint: Calculating the square root is the opposite to squaring.

Q. $\sqrt{2500} =$

A. $\sqrt{2500} =$
$= \sqrt{50 \times 50}$
$= \mathbf{50}$

a) $\sqrt{25} =$
$= \sqrt{5 \times 5}$ $= \boxed{5}$

b) $\sqrt{4} =$
= .... = ☐

c) $\sqrt{9} =$
= .... = ☐

d) $\sqrt{49} =$
= .... = ☐

e) $\sqrt{81} =$
= .... = ☐

f) $\sqrt{64} =$
= .... = ☐

g) $\sqrt{121} =$
= .... = ☐

h) $\sqrt{1} =$
= .... = ☐

i) $\sqrt{169} =$
= .... = ☐

j) $\sqrt{16} =$
= .... = ☐

k) $\sqrt{36} =$
= .... = ☐

l) $\sqrt{400} =$
= .... = ☐

m) $\sqrt{100} =$
= .... = ☐

n) $\sqrt{196} =$
= .... = ☐

o) $\sqrt{144} =$
= .... = ☐

p) $\sqrt{225} =$
= .... = ☐

q) $\sqrt{256} =$
= .... = ☐

r) $\sqrt{10\,000} =$
= .... = ☐

s) $\sqrt{3600} =$
= .... = ☐

t) $\sqrt{8100} =$
= .... = ☐

u) $\sqrt{4900} =$
= .... = ☐

Skill 12.2 Calculating square roots of perfect squares in fraction form.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- If the number is a mixed number, convert it to an improper fraction first.
- Estimate which number multiplied by itself produces the numerator.
- Estimate which number multiplied by itself produces the denominator.
- Check your estimation by multiplying your guess by itself.

Hint: Calculating the square root is the opposite to squaring.

Q. $\sqrt{5\frac{4}{9}} =$

A. $\sqrt{5\frac{4}{9}} =$

$= \sqrt{\frac{5\times 9+4}{9}}$

Change the mixed number to an improper fraction

$= \sqrt{\frac{49}{9}} = \frac{\sqrt{49}}{\sqrt{9}} = \frac{\sqrt{7\times 7}}{\sqrt{3\times 3}}$

Find the square root of the numerator and the denominator

$= \frac{7}{3} = \mathbf{2\frac{1}{3}}$

a) $\sqrt{\frac{1}{36}} =$

$= \sqrt{\frac{1}{6}\times\frac{1}{6}}$ …………………… $= \boxed{\frac{1}{6}}$

b) $\sqrt{\frac{1}{16}} =$

$=$ …………………… $= \boxed{}$

c) $\sqrt{\frac{4}{25}} =$

$=$ …………………… $= \boxed{}$

d) $\sqrt{\frac{100}{144}} =$

$=$ ………… $=$ ………… $= \boxed{}$

e) $\sqrt{\frac{16}{121}} =$

$=$ …………………… $= \boxed{}$

f) $\sqrt{\frac{25}{81}} =$

$=$ …………………… $= \boxed{}$

g) $\sqrt{1\frac{7}{9}} =$

$=$ ………… $=$ ………… $= \boxed{}$

h) $\sqrt{2\frac{1}{4}} =$

$=$ ………… $=$ ………… $= \boxed{}$

i) $\sqrt{1\frac{11}{25}} =$

$=$ ………… $=$ ………… $= \boxed{}$

j) $\sqrt{20\frac{1}{4}} =$

$=$ ………… $=$ ………… $= \boxed{}$

k) $\sqrt{1\frac{17}{64}} =$

$=$ ………… $=$ ………… $= \boxed{}$

l) $\sqrt{3\frac{1}{16}} =$

$=$ ………… $=$ ………… $= \boxed{}$

m) $\sqrt{11\frac{1}{9}} =$

$=$ ………… $=$ ………… $= \boxed{}$

n) $\sqrt{1\frac{15}{49}} =$

$=$ ………… $=$ ………… $= \boxed{}$

o) $\sqrt{4\frac{21}{25}} =$

$=$ ………… $=$ ………… $= \boxed{}$

Skill 12.3 Calculating square roots of perfect squares in decimal form.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Estimate which number, multiplied by itself, produces the number under the square root sign.
- Check your estimation by multiplying your guess by itself.
- Given the number is a decimal number consider the position of the decimal point.

Hint: Calculating the square root is the opposite to squaring.

Q. $\sqrt{0.04} =$

A. $\sqrt{0.04} =$

$= \sqrt{0.2 \times 0.2}$

$= \mathbf{0.2}$

a) $\sqrt{0.01} =$

= .... = ☐

b) $\sqrt{0.16} =$

= .... = ☐

c) $\sqrt{0.25} =$

= .... = ☐

d) $\sqrt{0.36} =$

= .... = ☐

e) $\sqrt{1.69} =$

= .... = ☐

f) $\sqrt{0.64} =$

= .... = ☐

g) $\sqrt{1.96} =$

= .... = ☐

h) $\sqrt{0.09} =$

= .... = ☐

i) $\sqrt{2.56} =$

= .... = ☐

j) $\sqrt{0.49} =$

= .... = ☐

k) $\sqrt{6.25} =$

= .... = ☐

l) $\sqrt{0.81} =$

= .... = ☐

m) $\sqrt{1.44} =$

= .... = ☐

n) $\sqrt{1.21} =$

= .... = ☐

o) $\sqrt{2.25} =$

= .... = ☐

p) $\sqrt{4.41} =$

= .... = ☐

q) $\sqrt{5.76} =$

= .... = ☐

r) $\sqrt{6.76} =$

= .... = ☐

Skill 12.4 Calculating multiples of square roots.

- Find the square root. (see skill 12.1, page 127)
- Multiply the whole numbers.

Q. $3\sqrt{81} =$

A. $3\sqrt{81} =$
$= 3 \times \sqrt{9 \times 9}$
$= 3 \times 9$
$= \mathbf{27}$

a) $2\sqrt{64} =$
$= 2 \times \sqrt{8 \times 8}$
$= 2 \times 8$ = ☐

b) $4\sqrt{9} =$
= ……………………
= …………… = ☐

c) $3\sqrt{16} =$
= ……………………
= …………… = ☐

d) $2\sqrt{49} =$
= ……………………
= …………… = ☐

e) $4\sqrt{25} =$
= ……………………
= …………… = ☐

f) $2\sqrt{144} =$
= ……………………
= …………… = ☐

g) $6\sqrt{100} =$
= ……………………
= …………… = ☐

h) $7\sqrt{36} =$
= ……………………
= …………… = ☐

i) $4\sqrt{64} =$
= ……………………
= …………… = ☐

j) $2\sqrt{169} =$
= ……………………
= …………… = ☐

k) $2\sqrt{900} =$
= ……………………
= …………… = ☐

l) $5\sqrt{121} =$
= ……………………
= …………… = ☐

m) $4\sqrt{225} =$
= ……………………
= …………… = ☐

n) $2\sqrt{625} =$
= ……………………
= …………… = ☐

o) $5\sqrt{1600} =$
= ……………………
= …………… = ☐

p) $3\sqrt{400} =$
= ……………………
= …………… = ☐

q) $6\sqrt{2500} =$
= ……………………
= …………… = ☐

r) $10\sqrt{0.25} =$
= ……………………
= …………… = ☐

Skill 12.5 Multiplying square roots of perfect squares.

- Find the square roots.
  (see skill 12.1, page 127)
- Multiply the whole numbers.

The square root of any number multiplied by itself equals the number

$$\sqrt{9} \times \sqrt{9} = \sqrt{9 \times 9} = 9$$

Q. $2\sqrt{49} \times \sqrt{49} =$

A. $2\sqrt{49} \times \sqrt{49} =$
$= 2 \times 49$
$= \mathbf{98}$

a) $\sqrt{36} \times \sqrt{36} =$ $\boxed{36}$

b) $\sqrt{7} \times \sqrt{7} =$ ☐

c) $\sqrt{25} \times \sqrt{25} =$ ☐

d) $\sqrt{81} \times \sqrt{25} =$
$= 9 \times 5$ $=$ $\boxed{45}$

e) $\sqrt{16} \times \sqrt{9} =$
= .... = ☐

f) $\sqrt{49} \times \sqrt{64} =$
= .... = ☐

g) $\sqrt{121} \times 3\sqrt{9} =$
= .... = ☐

h) $3\sqrt{16} \times \sqrt{100} =$
= .... = ☐

i) $\sqrt{36} \times 4\sqrt{25} =$
= .... = ☐

j) $\sqrt{9} \times 2\sqrt{49} =$
= .... = ☐

k) $\sqrt{144} \times 4\sqrt{4} =$
= .... = ☐

l) $2\sqrt{36} \times \sqrt{25} =$
= .... = ☐

m) $2\sqrt{25} \times \sqrt{64} =$
= .... = ☐

n) $\sqrt{169} \times 2\sqrt{9} =$
= .... = ☐

o) $2\sqrt{4} \times 3\sqrt{121} =$
= .... = ☐

p) $4\sqrt{9} \times 2\sqrt{100} =$
= .... = ☐

q) $3\sqrt{64} \times 2\sqrt{49} =$
= .... = ☐

r) $5\sqrt{36} \times 2\sqrt{144} =$
= .... = ☐

Skill 12.6 Dividing square roots of perfect squares.

Mauve 1 1 2 2 3 **3** 4 **4**
Lime 1 1 2 2 3 **3** 4 4

- Find the square roots.
  (see skill 12.1, page 127)
- Divide the whole numbers.

The square root of any number divided by itself equals 1

$$\frac{\sqrt{9}}{\sqrt{9}} = \sqrt{9} \div \sqrt{9} = 1$$

Q. $\frac{\sqrt{81}}{\sqrt{9}} =$

A. $\frac{\sqrt{81}}{\sqrt{9}} =$ Find the square root of the numerator and the denominator

$= \frac{9}{3}$

$= 3$

a) $\sqrt{64} \div \sqrt{16} =$

$= 8 \div 4$ .... $= \boxed{2}$

b) $\sqrt{100} \div \sqrt{25} =$

$=$ .... $= \boxed{}$

c) $\sqrt{36} \div \sqrt{9} =$

$=$ .... $= \boxed{}$

d) $\sqrt{900} \div \sqrt{36} =$

$=$ .... $= \boxed{}$

e) $\sqrt{144} \div \sqrt{4} =$

$=$ .... $= \boxed{}$

f) $\sqrt{196} \div \sqrt{49} =$

$=$ .... $= \boxed{}$

g) $\frac{\sqrt{400}}{\sqrt{16}} =$

$=$ .... $= \boxed{}$

h) $\frac{\sqrt{144}}{\sqrt{9}} =$

$=$ .... $= \boxed{}$

i) $\frac{\sqrt{900}}{\sqrt{25}} =$

$=$ .... $= \boxed{}$

j) $8\sqrt{4} \div 2\sqrt{4} =$

$= (8 \times 2) \div (2 \times 2)$

$= 16 \div 4$ .... $= \boxed{}$

k) $4\sqrt{100} \div 2\sqrt{25} =$

$=$ ....

$=$ .... $= \boxed{}$

l) $4\sqrt{900} \div 2\sqrt{9} =$

$=$ ....

$=$ .... $= \boxed{}$

m) $\frac{4\sqrt{9}}{\sqrt{9}} =$

$=$ .... $= \boxed{}$

n) $\frac{2\sqrt{400}}{\sqrt{4}} =$

$=$ .... $= \boxed{}$

o) $\frac{3\sqrt{25}}{\sqrt{9}} =$

$=$ .... $= \boxed{}$

p) $\frac{2\sqrt{100}}{\sqrt{25}} =$

$=$ .... $= \boxed{}$

q) $\frac{5\sqrt{64}}{\sqrt{16}} =$

$=$ .... $= \boxed{}$

r) $\frac{4\sqrt{36}}{2\sqrt{4}} =$

$=$ .... $= \boxed{}$

Skill 12.7 Adding and subtracting square roots of perfect squares.

- Find the square roots. (see skill 12.1, page 127)
- Add or subtract the whole numbers.

Hint: Square roots that are not alike cannot be added or subtracted.

Example $\sqrt{16}+\sqrt{4}\neq\sqrt{16+4}$ and $\sqrt{16}-\sqrt{4}\neq\sqrt{16-4}$

$4+2\neq\sqrt{20}$ $\quad$ $4-2\neq\sqrt{12}$

Q.	A.
$\sqrt{25}+\sqrt{25}+\sqrt{25}+\sqrt{25}=\sqrt{100}$ True or false?	$\sqrt{25}+\sqrt{25}+\sqrt{25}+\sqrt{25}=\sqrt{100}$ $5+5+5+5=10$ $20=10$ ***False***

a) $\sqrt{64}-\sqrt{9}=$

$= 8 - 3$ …………………… $= \boxed{5}$

b) $\sqrt{100}+\sqrt{36}=$

= …………………… = $\boxed{}$

c) $\sqrt{25}+\sqrt{49}=$

= …………………… = $\boxed{}$

d) $\sqrt{196}-\sqrt{49}=$

= …………………… = $\boxed{}$

e) $\sqrt{144}-\sqrt{64}=$

= …………………… = $\boxed{}$

f) $\sqrt{81}+\sqrt{121}=$

= …………………… = $\boxed{}$

g) $\sqrt{9}+\sqrt{16}=$

= …………………… = $\boxed{}$

h) $\sqrt{36}+\sqrt{64}=$

= …………………… = $\boxed{}$

i) $\sqrt{49}-\sqrt{25}=$

= …………………… = $\boxed{}$

j) $\sqrt{25}+\sqrt{100}=$

= …………………… = $\boxed{}$

k) $\sqrt{144}+\sqrt{49}=$

= …………………… = $\boxed{}$

l) $\sqrt{144}+\sqrt{256}=$

= …………………… = $\boxed{}$

m) $\sqrt{400}-\sqrt{81}=$

= …………………… = $\boxed{}$

n) $\sqrt{121}-\sqrt{100}=$

= …………………… = $\boxed{}$

o) $\sqrt{169}-\sqrt{144}=$

= …………………… = $\boxed{}$

p) $\sqrt{169}-\sqrt{25}=\sqrt{144}$
True or false?

……………………………………

…………………… $\boxed{}$

q) $\sqrt{4}+\sqrt{4}=\sqrt{9}$
True or false?

……………………………………

…………………… $\boxed{}$

r) $\sqrt{64}+\sqrt{36}=\sqrt{100}$
True or false?

……………………………………

…………………… $\boxed{}$

s) $\sqrt{64}-\sqrt{25}=\sqrt{9}$
True or false?

……………………………………

…………………… $\boxed{}$

t) $\sqrt{100}-\sqrt{36}=\sqrt{64}$
True or false?

……………………………………

…………………… $\boxed{}$

u) $\sqrt{9}+\sqrt{9}+\sqrt{9}+\sqrt{9}=\sqrt{36}$
True or false?

……………………………………

…………………… $\boxed{}$

Skill 12.8 Estimating square roots.

- Find the perfect squares greater than (>) and less than (<) the number.

Q. Between which two consecutive whole numbers does $\sqrt{8}$ lie?

A. $4 < 8 < 9$

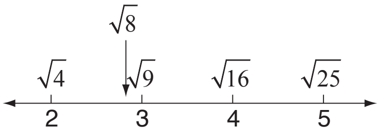
$\sqrt{4} < \sqrt{8} < \sqrt{9}$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$2 < \sqrt{8} < 3$

The answer is ***2 & 3***



a) Between which two consecutive whole numbers does $\sqrt{72}$ lie?

$\sqrt{64} = 8 \qquad \sqrt{81} = 9$

8 & 9

b) Between which two consecutive whole numbers does $\sqrt{10}$ lie?

&

c) Between which two consecutive whole numbers does $\sqrt{5}$ lie?

&

d) Between which two consecutive whole numbers does $\sqrt{60}$ lie?

&

e) Between which two consecutive whole numbers does $\sqrt{34}$ lie?

&

f) Between which two consecutive whole numbers does $\sqrt{24}$ lie?

&

g) Between which two consecutive whole numbers does $\sqrt{80}$ lie?

&

h) Between which two consecutive whole numbers does $\sqrt{145}$ lie?

&

i) Between which two consecutive whole numbers does $\sqrt{56}$ lie?

&

j) Between which two consecutive whole numbers does $\sqrt{150}$ lie?

&

k) Between which two consecutive whole numbers does $\sqrt{99}$ lie?

&

l) Between which two consecutive whole numbers does $\sqrt{138}$ lie?

&

13. [Exploring Number]

Skill 13.1 Using ‘order of operations’ involving a mix of ( ), ×, ÷, + or –

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Simplify inside the brackets.
- Multiply (×) and/or divide (÷) in order from left to right.
- Add (+) and/or subtract (–) in order from left to right.

Q. $18 \div (9 - 3) + 2 =$

A. $18 \div (9 - 3) + 2 =$

$= 18 \div 6 + 2$ subtract inside the brackets

$= 3 + 2$ division before addition

$= \mathbf{5}$

a) $6 + 12 \div 4 \times 3 =$

$= 6 + 3 \times 3$

$= 6 + 9$ $=$

b) $6 \times 15 - 8 \times 3 =$

$=$

$=$ $=$

c) $5 + 12 \div 6 \times 3 =$

$=$

$=$ $=$

d) $3 \times (5 - 3) \times 8 =$

$=$

$=$ $=$

e) $(15 + 8) - (7 + 6) =$

$=$

$=$ $=$

f) $120 \div 5 - 6 \times 3 =$

$=$

$=$ $=$

g) $22 - 8 - (11 - 4) =$

$=$

$=$ $=$

h) $20 - (15 - 9) + 6 =$

$=$

$=$ $=$

i) $6 \times (14 + 7) =$

$=$

$=$ $=$

j) $8 \times 5 \div (7 - 3) =$

$=$

$=$ $=$

k) $4 + (9 - 4) \times 7 =$

$=$

$=$ $=$

l) $36 - 2 \times (12 + 5) =$

$=$

$=$ $=$

m) $144 \div 6 - 4 \times 5 + 18 \div 3 =$

$=$

$=$ $=$

n) $40 - 3 \times (13 + 5) \div 3 + 12 =$

$=$

$=$ $=$

Skill 13.2 Using ‘order of operations’ involving powers and ( ), ×, ÷, + or –

Mauve 1 1 2 2 3 3 4 4
Like 1 1 2 2 3 3 4 4

- Simplify inside the brackets.
- Simplify the power.
- Always multiply (×) and/or divide (÷) in order from left to right.
- Always add (+) and/or subtract (–) in order from left to right.

Q. $(6 + 2 \times 5)^2 =$

A. $(6 + 2 \times 5)^2 =$ multiply within brackets first

$= (6 + 10)^2$ add inside the brackets

$= 16^2$

$= \mathbf{256}$

a) $(3 \times 5)^2 =$

$= 15^2$ .... $= \boxed{}$

b) $(2 \times 7)^2 =$

= .... $= \boxed{}$

c) $(5 + 5 \times 3)^2 =$

= ....

= .... $= \boxed{}$

d) $(2 \times 4 + 6)^2 =$

= ....

= .... $= \boxed{}$

e) $(2 + 8)^2 \div 4 =$

= ....

= .... $= \boxed{}$

f) $(7 + 5)^2 \div 8 =$

= ....

= .... $= \boxed{}$

g) $5 + (12 - 6)^2 =$

= ....

= .... $= \boxed{}$

h) $8 + (13 - 8)^2 =$

= ....

= .... $= \boxed{}$

i) $(4 \times 2 + 2)^2 =$

= ....

= .... $= \boxed{}$

j) $(3 \times 4 + 8)^2 =$

= ....

= .... $= \boxed{}$

k) $3 + (1 + 8)^2 =$

= ....

= .... $= \boxed{}$

l) $6 + (7 + 1)^2 =$

= ....

= .... $= \boxed{}$

m) $(10 - 1)^2 \div (30 - 3) =$

= ....

= .... $= \boxed{}$

n) $(10 - 3)^2 \div (12 - 5) =$

= ....

= .... $= \boxed{}$

Skill 13.3 Rounding decimal numbers to a given place.

Rounding terminating decimals to a given place

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
  5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

Rounding recurring decimals to a given place

- Write the first 4 digits after the decimal point. (see skill 7.14, page 84)
- Apply the procedure described above for terminating decimals.

Q. Round $0.4\dot{6}$ to 3 decimal places.

A. $0.4\dot{6} = 0.4666...$ 6 is repeating indefinitely

$0.466\textcircled{6}...$ circle the fourth digit

$\approx \mathbf{0.467}$ 6 ≥ 5 ⇒ round up by adding 1 to 6

a) Round 0.13 to 1 decimal place.

$0.1\textcircled{3}$ 3 < 5 round down by keeping 1 $\approx$ $\boxed{0.1}$

b) Round 7.89 to 1 decimal place.

$\approx$ $\boxed{\quad}$

c) Round 12.45 to 1 decimal place.

$\approx$ $\boxed{\quad}$

d) Round 31.5841 to 2 decimal places.

$31.58\textcircled{4}1$ 4 < 5 round down by keeping 8 $\approx$ $\boxed{31.58}$

e) Round 24.793 to 2 decimal places.

$\approx$ $\boxed{\quad}$

f) Round 4.231 to 2 decimal places.

$\approx$ $\boxed{\quad}$

g) Round 3.859 to 1 decimal place.

$\approx$ $\boxed{\quad}$

h) Round 50.296 to 2 decimal places.

$\approx$ $\boxed{\quad}$

i) Round $4.\dot{7}$ to 2 decimal places.

$4.\dot{7} = 4.77\textcircled{7}7...$ 7 ≥ 5 round up by adding 1 to 7 $\approx$ $\boxed{4.78}$

j) Round $3.\dot{4}\dot{2}$ to 2 decimal places.

$\approx$ $\boxed{\quad}$

k) Round $0.\dot{6}$ to 2 decimal places.

$\approx$ $\boxed{\quad}$

l) Round $1.7\dot{3}$ to 3 decimal places.

$\approx$ $\boxed{\quad}$

m) Round $4.2\dot{8}$ to 3 decimal places.

$\approx$ $\boxed{\quad}$

n) Round $0.\dot{1}\dot{6}$ to 3 decimal places.

$\approx$ $\boxed{\quad}$

Skill 13.4 Writing rational approximations of simple irrational numbers.

Mauve 1 1 **2** 2 3 3 4 4
Lime 1 **1** 2 2 3 3 4 4

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
  5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

Hint: To write a decimal number correct to two decimal places is the same thing as rounding off to the the nearest hundredth.

Irrational number $\sqrt{2} \approx 1.41421356$ Rational approximation

Q. cos 45° ≈ 0.70711
Write the rational approximation of cos 45° correct to two decimal places.

A. *0.70⑦11*
circle the third digit

≈ ***0.71***
7 ≥ 5
round up by
adding 1 to 0

a) $\sqrt{12} \approx 3.46410162$
Write the rational approximation of $\sqrt{12}$ correct to two decimal places.

3.46④10162
4 < 5
round down
by keeping 6
≈ $\boxed{3.46}$

b) $\sqrt{20} \approx 4.47213595$
Write the rational approximation of $\sqrt{20}$ correct to two decimal places.

≈

c) $\sqrt{24} \approx 4.89897949$
Write the rational approximation of $\sqrt{24}$ correct to two decimal places.

≈

d) $\sqrt{30} \approx 5.47722558$
Write the rational approximation of $\sqrt{30}$ correct to two decimal places.

≈

e) $\pi \approx 3.14159265$
Write the rational approximation of π correct to three decimal places.

≈

f) $\phi \approx 1.61803398$ (the golden ratio)
Write the rational approximation of ϕ correct to three decimal places.

≈

g) sine 15° ≈ 0.25882
Write the rational approximation of sine 15° correct to three decimal places.

≈

h) tan 60° ≈ 1.73205
Write the rational approximation of tan 60° correct to three decimal places.

≈

i) e ≈ 2.71828182 (Euler's number)
Write the rational approximation of e correct to two decimal places.

≈

j) $\sqrt{10} \approx 3.16227766$
Write the rational approximation of $\sqrt{10}$ correct to three decimal places.

≈

Skill 13.5 Writing very large and very small numbers in scientific notation.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

$243\,000 = 2.43 \times 10^5$

basic numeral — Very large

scientific notaion — Product of: Number ≥ 1 and < 10
Power of 10 with positive index

$0.000802 = 8.02 \times 10^{-4}$

basic numeral — Very small

scientific notaion — Product of: Number ≥ 1 and < 10
Power of 10 with negative index

To write a number greater than or equal to 1 and less than 10 in scientific notation

- Move the decimal point after the first non-zero digit and count the places moved.
- Write this number ignoring the zeros at either end.

To find the power of 10

- Write base 10 with an index equal to the number of places moved.
- Check the sign of the index: Decimal point moves left ⇒ positive index.
  Decimal point moves right ⇒ negative index.

Q. Write 384 000 000 m, the distance from earth to moon, in scientific notation.

A. $384\,000\,000$ 8 places

$= 3.84 \times 100\,000\,000$ 8 zeros

number ≥ 1 and < 10

$= \mathbf{3.84 \times 10^8}$ Index = +8 because of the 8 places and large original number (> 3.84)

a) 73 000 written in scientific notation is:
A) 7.3×10^{-4} B) 0.73×10^5 C) 7.3×10^4

$73\,000$ 4 places

Index = +4

4 zeros

$= 7.3 \times 10\,000 = 7.3 \times 10^4$ = $\boxed{\quad}$

b) 5 010 000 written in scientific notation is:
A) 5.01×10^6 B) 5.1×10^5 C) 5.01×10^5

$5\,010\,000$

= .... = $\boxed{\quad}$

c) Some people have as many as 150 000 hair follicles on their head. Write this in scientific notation.

.... = $\boxed{\quad}$

d) By 2010 the world's population is estimated to reach 6.84 billion people. Write this in scientific notation.

.... = $\boxed{\quad}$

e) 0.00304 written in scientific notation is:
A) 3.4×10^{-4} B) 3.04×10^{-3} C) 3.04×10^{-4}

0.00304 3 places

index = −3

3 zeros

$= 3.04 \times 1000 = 3.04 \times 10^{-3}$ = $\boxed{\quad}$

f) 0.000051 written in scientific notation is:
A) 5.1×10^{-5} B) 5.1×10^5 C) 5.1×10^{-4}

....

= .... = $\boxed{\quad}$

g) Write 0.00000024 m, the diameter of a human hair strand, in scientific notation.

.... = $\boxed{\quad}$

h) In diamonds the distance between two Carbon atoms is 0.000000142 m. Write this in scientific notation.

.... = $\boxed{\quad}$

Skill 13.6 Writing a number in scientific notation as a basic numeral.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

2.43×10^5	$= 243\,000$
scientific notation	**basic numeral**
Product of: Number ≥ 1 and < 10 Power of 10 with positive index	Very large

8.02×10^{-4}	$= 0.000802$
scientific notation	**basic numeral**
Product of: Number ≥ 1 and < 10 Power of 10 with negative index	Very small

If the power of 10 is **positive:**

- Move the decimal point to the right as many places as the power of 10.
- Add zeros as place holders if necessary.
  Example: 3.1 = 3.1000
  Hint: By convention 37 = 37. = 37.0

If the power of 10 is **negative:**

- Move the decimal point to the left as many places as the power of 10.
- Add zeros as place holders if necessary.
  Example: 4.5 = 00004.5
- If the result is less than 1, write a zero in the units place.
  Hint: By convention 0.37 not .37

Q. Write 3.5×10^{-4} m, the diameter of optical fibre, as a basic numeral.

A. $3.5 \times 10^{-4} =$ index = –4

move decimal point 4 places left

$= 00003.5 \times 10^{-4}$

add zeros as place holders

$= \mathbf{0.00035}$

a) 6.2×10^5 is the scientific notation for:
A) 6200 B) 620 000 C) 6.20000

$6.2 \times 10^5 =$ index = +5

$= 620\,000.$ 5 places right

B

b) 4.12×10^6 is the scientific notation for:
A) 4 120 000 B) 412 000 C) 4.120000

....

= ....

c) 2.15×10^3 is the scientific notation for:
A) 2.15000 B) 215 000 C) 2150

....

= ....

d) 1.8×10^7 is the scientific notation for:
A) 1 800 000 B) 18 000 000 C) 180 000

....

= ....

e) Earth's atmosphere extends upward for 9.65×10^5 m. Write this as a basic numeral.

....

f) Write 1.4×10^9, China's population in 2010, as a basic numeral.

....

g) The size of a red blood cell, 8.0×10^{-3} mm, is scientific notation for:
A) 0.0008 B) 8000 C) 0.008

....

= ....

h) The size of a virus, 2.5×10^{-5} mm, is scientific notation for:
A) 0.00025 B) 0.000025 C) 250 000

....

= ....

i) Write 2.5×10^{-11} m, the radius of a hydrogen atom, as a decimal number.

....

j) Write 5×10^{-7} m, the size of a speck of dust, as a decimal number.

....

Skill 13.7 Using ‘order of operations’ involving negative numbers.

Mauve 1 1 2 2 **3** 3 4 4
Lime 1 1 2 **2** 3 3 4 4

- Simplify within the brackets.
- Simplify the power.
- Always multiply (×) and/or divide (÷) in order from left to right.
- Always add (+) and/or subtract (−) in order from left to right.
- Use the addition, subtraction, multiplication and division rules of negative numbers.

Q. −48 ÷ (−8 − 4) + 23 =

A. *−48 ÷ (−8 − 4) + 23 =*
subtract inside the brackets
= *−48 ÷ (−12) + 23*
− ÷ − = +
= *4 + 23*
division before addition
= ***27***

a) −6 × (24 + 6) =
= *−6 × 30* .... = −180

b) −4 × (7 + 9) =
= .... =

c) (10 − 2) ÷ (3 − 7) =
= .... =

d) (11 − 3) ÷ (1 − 9) =
= .... =

e) (6 − 10) × (−4 − 8) =
= .... =

f) (−5 − 4) × (2 − 5) =
= .... =

g) (−9 − 2) × (12 − 7) =
= .... =

h) (−8 + 3) × (6 − 12) =
= .... =

i) 8 × 12 ÷ (2 − 6) =
= ....
= .... =

j) 5 × 14 ÷ (3 − 10) =
= ....
= .... =

k) −5 − 4 × (11 − 9) =
= ....
= .... =

l) −7 + 2 × (15 − 4) =
= ....
= .... =

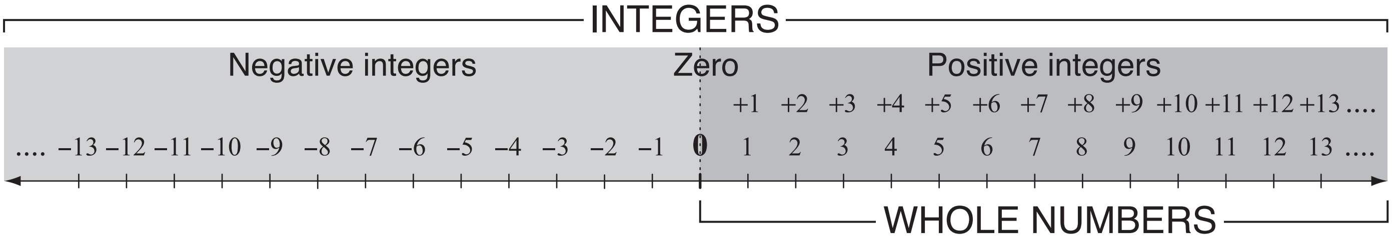
m) −24 ÷ (−3 − 3) + 17 =
= ....
= .... =

n) −12 × 5 − 45 ÷ 9 =
= ....
= .... =

o) −18 + 4 × (7 + 8) ÷ 10 − 2 =
= ....
= .... =

p) 5 − 3 × (6 + 2) ÷ 2 − 8 =
= ....
= .... =

Skill 13.8 Recognising whole numbers and integers.



Whole Numbers

Negative integers, fractions, terminating decimals, recurring decimals and infinite non-recurring decimals are **not** whole numbers apart from these 2 situations:

1) Any positive fraction whose numerator is divisible by the denominator. $\frac{12}{4} = 3$
2) Any positive decimal with only zeros after the decimal point. $8.00 = 8$
3) Any square root of a perfect square. $\sqrt{36} = 6$

Integers

Fractions, terminating decimals, recurring decimals and infinite non-recurring decimals are **not** integers, apart from these 3 situations:

1) Any fraction whose numerator is divisible by the denominator. $-\frac{5}{1} = -5$
2) Any decimal with only zeros after the decimal point. $-3.00 = -3$
3) Any square root of a perfect square. $-\sqrt{16} = -4$

Q. Choose the whole numbers from this list:
$-7, \frac{8}{2}, -\frac{1}{3}, 0, -3.6, 50$

A. *−7 is negative, so not a whole number*
$\frac{8}{2} = 8 \div 2 = 4$ *is a whole number*
$-\frac{1}{3}$ *is a fraction, so not a whole number*
−3.6 is a decimal, so not a whole number
So $\boldsymbol{\frac{8}{2}}$***, 0, 50*** *are whole numbers.*

a) Choose the whole numbers from this list:
7.43, [✓ 89], -5, $3\frac{1}{5}$, [✓ 14], 0.6

b) Choose the whole numbers from this list:
$567, 0.73, -4, \frac{3}{10}, 12, 0$

c) Choose the whole numbers from this list:
$1.4142, 18, -5.\dot{9}, \frac{4}{11}, -5, 143$

d) Choose the whole numbers from this list:
$-25, 0.6666..., 34, \frac{5}{7}, -1, 8.93567, 2$

e) Choose the integers from this list:
$-3.5, 11, 2.\dot{1}\dot{4}, -1, 3\frac{2}{7}, 2$

f) Choose the integers from this list:
$3.14, \frac{16}{4}, -3, -0.\dot{7}\dot{2}, \sqrt{25}$

g) Choose the integers from this list:
$-75, 2.23607, -\frac{8}{2}, \sqrt{90}, 10.00$

h) Choose the integers from this list:
$-\sqrt{4}, \frac{\pi}{4}, 0.5252, 18, 0$

Skill 13.9 Recognising rational and irrational numbers.

Mauve 11 22 33 **44**
Lime 11 22 **3**3 44

A number is **rational** if:

- It can be written as a fraction of 2 integers.

Hint: All integers are rational numbers.

$$-2, 700, \sqrt{16}, \frac{5}{1}, \frac{25}{5}$$

All terminating decimals are rational.

$$2.16, -5.753469$$

All recurring decimals are rational.

$$0.57575757... = 0.\dot{5}\dot{7}$$

A number is **irrational** (not rational) if:

- It can be written as a decimal, but not as a fraction.
- It has infinite non-recurring digits after the decimal point.

Example: 2.52849302953...

Hint: Square roots of prime numbers and rational numbers that are not perfect squares are irrational numbers.

$$\sqrt{5}, \sqrt{18}$$

Q. Which numbers are rational?

A) $-\sqrt{\frac{3}{5}}$

B) 0.999...

C) π

D) $\frac{11}{2}$

A. *$-\sqrt{\frac{3}{5}}$ is irrational, because $\frac{3}{5}$ is not a perfect square.*

0.999... is rational, because it is a recurring decimal.

π is irrational, because it has infinite non-recurring digits after the decimal point ($\pi \approx 3.1415926535....$)

$\frac{11}{2}$ is rational, because it is a fraction.

*So **B and D** are rational.*

a) Is $\sqrt{7}$ a rational or an irrational number?

irrational

b) Is $4.12\dot{6}\dot{3}$ a rational or an irrational number?

c) Is $\frac{48}{25}$ a rational or irrational number?

d) Is 1.72430982... a rational or irrational number?

e) Is −60 a whole number, an integer or an irrational number?

f) Is 2.676767... a whole number, an integer or a rational number?

g) Is $-\frac{7}{2}$ a whole number, an integer or a rational number?

h) Is −12 000 a whole number, an integer or an irrational number?

i) Which is **not** a rational number?

A) −6 B) 0.18952

C) π D) $-\sqrt{9}$

j) Which is **not** a rational number?

A) $-0.3\dot{1}$ B) $\sqrt{3}$

C) 2.135135135... D) $\frac{11}{49}$

k) Which is an irrational number?

A) 3 B) −2.5

C) $\sqrt{4}$ D) $-\sqrt{2}$

l) Which is an irrational number?

A) $2.\dot{6}$ B) 6.15

C) $\sqrt{7}$ D) $5\frac{3}{10}$

m) Which numbers are rational?

A) $\sqrt{8}$ B) $6.5\dot{9}$

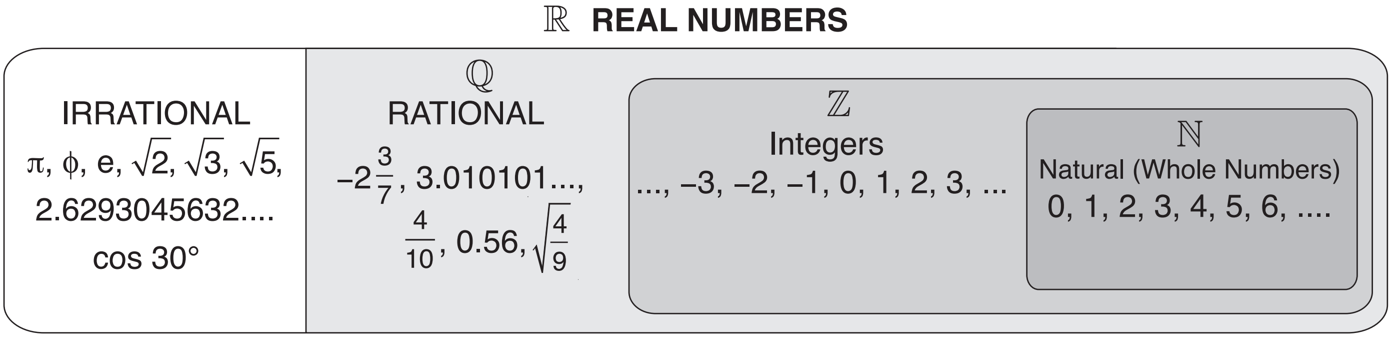
C) −4.131133111333... D) 3.161616...

n) Choose the rational numbers from this list:

$$-2012, \pi, 0, \frac{28}{11}, -\sqrt{6}$$

Skill 13.10 Recognising classes of numbers.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 **4** 4



Hint: Rational numbers include integers, terminating decimals and recurring decimals.
Irrational numbers include infinite non-recurring decimals.
ℕ included in ℤ ,ℤ included in ℚ, ℚ included in ℝ
Irrational number included in ℝ

Q. Which classification describes $-\sqrt{81}$?
A) integer and irrational
B) rational and real
C) irrational and rational
D) real and natural

A. $-\sqrt{81} = -9$

integer	✔
rational	✔
real number	✔
natural	✘
irrational	✘

*So **B** is the correct description.*

a) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
4.327	false	true	false	true

b) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
–500				

c) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
π				

d) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
$\frac{3}{14}$				

e) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
$\sqrt{26}$				

f) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
$\frac{36}{9}$				

g) Which classification describes 0.65291...?
A) integer and rational
B) rational and real
C) integer and irrational
D) irrational and real

h) Which classification describes $-\sqrt{49}$?
A) integer and rational
B) irrational and real
C) integer and irrational
D) rational and irrational

i) Which classification describes $0.\dot{1}5384\dot{6}$?
A) integer and irrational
B) irrational and real
C) integer and rational
D) rational and real

j) Which classification describes $\frac{257}{43}$?
A) integer and rational
B) irrational and real
C) rational and real
D) rational and irrational

Skill 13.11 Comparing and ordering rational and irrational numbers.

Mauve 11 22 33 44
Lime 11 22 33 44

- Express the numbers as decimals.
- Round the decimal numbers to one or two decimal places, as needed in the question.
  Examples: $\pi = 3.1415926... \approx 3.14$ correct to 2 decimal places
  $\frac{5}{3} = 1.6666... \approx 1.67$ correct to 2 decimal places
  $\sqrt{3} = 1.732050808... \approx 1.73$ correct to 2 decimal places
- Find the perfect squares greater than (>) and less than (<) the number under the square root.
- Compare and order the terminating decimals.

Q. Place in ascending order:
$\sqrt{10}, \frac{10}{3}, 3.21, \pi, \sqrt{12}$

A.
$\sqrt{10} = 3.16227766... \approx 3.16$
$\frac{10}{3} = 3.33333... \approx 3.33$ round to 2 decimal places
$\pi = 3.1415... \approx 3.14$
$\sqrt{12} = 3.46410161... \approx 3.46$

express numbers as decimals

$\Rightarrow 3.14 < 3.16 < 3.21 < 3.33 < 3.46$

OR $\pi < \sqrt{10} < 3.21 < \frac{10}{3} < \sqrt{12}$

The answer is $\boldsymbol{\pi, \sqrt{10}, 3.21, \frac{10}{3}, \sqrt{12}}$

a) Which number is greater?
$\sqrt{21}$ or 5
$16 < 21 < 25 \Rightarrow 4 < \sqrt{21} < 5 \Rightarrow$ 5

b) Which number is greater?
$\sqrt{72}$ or 8
$\Rightarrow$

c) Which number is smaller?
1.41 or $\sqrt{2}$
$\Rightarrow$

d) Which number is greater?
$\sqrt{8}$ or 3
$\Rightarrow$

e) Which number is greater?
π or $\sqrt{9}$
$\Rightarrow$

f) Which number is smaller?
$\frac{5}{2}$ or $\sqrt{6}$
$\Rightarrow$

g) Which number is greater?
$\sqrt{18}$ or 4
$\Rightarrow$

h) Which number is smaller?
$\sqrt{25}$ or $5.\dot{1}$
$\Rightarrow$

i) Place in ascending order:
$\sqrt{5}, \frac{7}{3}, 2.\dot{2}, \frac{5}{2}, 2.4$

j) Place in ascending order:
$\sqrt{8}, \frac{8}{3}, \frac{10}{4}, 2.76, \sqrt{7}$

14. [Financial Mathematics]

Skill 14.1 Minimising expenses - saving.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Write number sentences from the information given.

Q. Kai's home content insurance direct debit is \$33.50 per month. How much does he save if he pays the up-front annual amount of \$340?

A. $\$33.50 \times 12 = \$402$ Calculate the total direct debit for 12 months.

$\$402 - \$340 = \$\mathbf{62}$ Subtract the annual up-front amount from the total direct debit.

a) Litia saves \$5 per day for January. How much does she save the whole month?

January = 31 days

31×5 = \$

b) Gerardo saves \$15 per day for November. How much does he save the whole month?

= \$

c) How much can I save in 4 weeks if my pocket money is \$20 per week, and my expenses for 2 fortnights are as shown?

Expense	Cost
Sport	\$28
Entertainment	\$30
Clothes	\$15

= \$

d) Which company has the cheapest car hire over 8 days?

Co.	Rates	Cost
A	Hire fee	\$75
	Daily rate	\$40
B	Hire fee	\$25
	Daily rate	\$60

=

e) My car uses one litre of petrol every 10 km. Petrol costs me \$1.65 per litre. How much money would I save in one week, if I walk 2.5 km to and 2.5 km from work for 6 days?

= \$

f) Lee buys 4 double and 2 single cones for \$16. The next day he buys 2 double and 4 single cones and pays \$14. How much is a double cone?

= \$

g) Lu's home contents insurance direct debit is \$26.30 per month. How much does she save if she pays the up-front annual amount of \$250?

= \$

h) Jo's car insurance direct debit is \$76.40 per month. How much does he save if he pays the up-front annual amount of \$820?

= \$

Skill 14.2 Estimating outcomes.

- Round where appropriate to the nearest whole numbers or multiples of 10.
- Create an equation from the information given.
- Calculate, where necessary, the percentage of the given amount.
  (see skills 6.3, page 61 and 6.4, page 62)

Q. A toothpaste box weighs 8.01 g. Estimate how many would be required to make 1 kg of recyclable waste?

A. $8.01 \approx 8$
$1000 \div 8$
$= \mathbf{125}$

Round 8.01 to 8 g.
1 kg = 1000 g
It would take 125 toothpaste boxes to make 1 kg of recyclable waste.

a) A dinner costs $49.90. You tip 6%. Estimate the size of the tip.

$49.9 \approx 50$ so $\frac{6}{10\cancel{0}} \times \frac{5\cancel{0}}{1} =$ Simplify: ÷ 10

$= 30 \div 10$ = $

b) You weigh 44.8 kg. If you gain 3% of your body weight, estimate your weight gain.

= kg

c) Advertising costs contribute 10% of the $25 050 development. Estimate the cost of advertising.

= $

d) Your backyard is 124.6 m^2 of which 12% is playground. Estimate the size of your playground.

= m^2

e) Concert tickets were $149.95 until you found the internet discount of 12%. Estimate the savings if you buy online.

= $

f) There are an estimated 8 000 000 species of insects in the world of which 24% are beetles. Estimate the number of beetle species.

=

g) Approximately two thirds of Julie's income of $48 249 is spent on bills. Estimate the amount spent on bills.

= $

h) A best-selling musician has sold 138.5 million albums. Estimate the number of albums that will need to be sold to reach 145 million.

=

i) Approximately 5% of Australia's 19 000 000 citizens applied for a passport in the financial year 2001/2002, after Sept 11th, 2001. Estimate the number of Australian passports issued in the fianancial year 2001/2002.

=

j) Dad donates half a round of golf. You pay the remaining $19.85 for the round. Estimate the cost of a full round of golf.

= $

Skill 14.3 Calculating percentages including GST and lay-bys.

- Write a number sentence from the information given.
- Calculate the percentage of the given amount. (see skills 6.3, page 61 and 6.4, page 62)

Q. The laptop cost \$1430 including GST. If the GST is 10%, how much is the cost excluding GST?	**A.** *GST = 10%* *cost including GST = 100% + 10% = 110%* $\Rightarrow$ *\$1430 = 110%* $\Rightarrow$ *GST = \$1430 ÷ 11 = \$130* $\Rightarrow$ *cost excluding GST = \$1430 − \$130* = ***\$1300***

a) A TV was repaired for \$175, then a 10% GST was added to the price. What was the total cost of the TV repairs?

$$\frac{1\cancel{0}}{10\cancel{0}} \times \frac{175}{1} = 17.5$$ Simplify: ÷ 10

175 + 17.5 = \$

b) Archie leaves an extra 5% of the restaurant bill as a tip. The bill was \$150. How much was the tip?

= \$

c) Jean pays a 20% deposit to put a trampoline on lay-by. If the trampoline costs \$1200, how much does he have left to pay?

= \$

d) Fairy puts up 20% as a lay-by deposit on a shuttle board table. The table costs \$380. She will then make 4 equal payments of the balance. What will the last payment be?

= \$

e) The plumbing repair cost \$308 including GST. If the GST is 10%, how much GST was included in the cost?

= \$

f) The cost of a car service was \$671 including GST. If the GST is 10%, how much GST was included in the service?

= \$

g) The house painting costs \$13 200 including GST. If the GST is 10%, how much is the cost excluding GST?

= \$

h) Before adding the GST of 10%, the phone costs \$690. Find the total cost of the phone including GST.

= \$

i) Lena buys a bag online for \$85. If shipping and handling are an additional 40% of the price, how much will she pay altogether?

= \$

j) Jai put a \$300 lay-by deposit on a \$1500 computer as the store demanded. What percentage of the sale price does the store expect on lay-by?

=

Skill 14.4 Calculating percentages including commissions, profit and loss.

Mauve 1 1 2 **2** 3 3 4 4
Lime 1 **1** 2 2 3 3 4 4

- Write a number sentence from the information given.
- Calculate the percentage of the given amount. (see skills 6.3, page 61 and 6.4, page 62)

Commission = % × Selling price

Q. Jai sells a property for $118 000 and earns 3% commission. How much is Jai's commission?

A. $\frac{3}{1\cancel{0}\cancel{0}} \times \frac{1180\cancel{0}\cancel{0}}{1} =$ Simplify: ÷ 100

$= 3 \times 1180$

$= \mathbf{\$3540}$

a) Breanna pays $18 000 for a car and sells it for 15% less. Calculate the loss.

$\frac{15}{1\cancel{0}\cancel{0}} \times \frac{18\,0\cancel{0}\cancel{0}}{1} =$

$= 15 \times 180$ = $

b) Kim pays $14 000 for a diamond ring and sells it for 5% more. Calculate the profit.

= = $

c) A surfboard costing $700 is sold at a loss of 12%. Calculate the selling price.

= $

d) An antique chest costing $1200 is sold at a profit of 15%. Calculate the selling price.

= $

e) David sells a house for $450 000. If his commission is 3%, how much is David's commission?

= $

f) Kate sells a car for $84 000. If her commission is 2%, how much is Kate's commission?

= $

g) Shane bought a second-hand car for $9500. He then sold it for 40% less. What is the selling price of the car?

= $

h) Ian bought a house for $340 000 and renovated it. He then sold it, making a profit of 30%. What was the selling price of the house?

= $

i) A pair of skates is marked up 20%. If the sale price is $150, what profit is made?

= $

j) A coffee table is marked up 25%. If the sale price is $350, what profit is made?

= $

Skill 14.5 Calculating wages.

Mauve 11 22 **3**3 44
Lime 11 **2**2 33 44

- Write a number sentence from what you are given.
- Consider the dollar amount and the time it takes to earn that.

NB: In Australia employers pay 9% of their employees' base income into a superannuation fund.

Q. Goldie earns \$238 for 17 hours work. What is her hourly rate?

A. *\$238 ÷ 17 h =*

= ***\$14/h***

$$\begin{array}{r|l} & 14 \\ \hline 17 \,) & 238 \\ - & 17\downarrow \\ \hline & 68 \\ - & 68 \end{array}$$

a) Sean is an apprentice, and he earns \$7.20 per hour for a 40 hour week. His pay this fortnight is \$595. By how much was Sean overpaid?

40 × 2 = 80

\$7.20 × 80 = \$576/fortnight

\$595 – \$576 = \$

b) Joey earns \$192 for 16 hours work. What is his hourly rate?

= \$

c) Tamara works from 9 pm to 2 am at a rate of \$13.50/h after tax. From midnight however, the pay rate doubles. What is this shift worth to Tamara?

= \$

d) Today Prue and Trudy together earn \$600 for standard hair cuts. They share 15 customers. If Prue cuts 9 heads of hair, how much does Trudy earn?

= \$

e) If Gary's tax for the year is \$3500, and his pay each fortnight is \$750, how much is his yearly wage before tax? [Hint: There are 26 fortnights in a year.]

= \$

f) Kay is paid \$15/hour for a 20 hour week. Her pay this fortnight is \$485. By how much is Kay underpaid?

= \$

g) Your employer's superannuation contribution is 9% of your base income. If your base income is \$30 000, what is your superannuation guarantee?

= \$

h) How much does his employer pay quarterly (13 weeks) into John's superannuation if John's fortnightly wage, before tax, is \$2000? [Superannuation guarantee = 9%]

= \$

Skill 14.6 Calculating net and gross income and tax payable on income.

Mauve 11 22 33 44
Lime 11 22 33 44

- Write a number sentence from what you are given.
- Consider the dollar amount and the time it takes to earn that.

Net wage = Gross wage − Total deductions

Q. Ali's gross wage is $2200 per fortnight. He pays 20% of the gross wage in tax and contributes 9% of the gross wage to his superannuation fund. Calculate Ali's net wage per fortnight.
[net wage = gross wage − total deductions]

A. *total deductions = tax + superannuation*

$$tax = \frac{20}{1\cancel{00}} \times \frac{22\cancel{00}}{1} = \$440$$ Simplify: ÷ 100

$$superannuation = \frac{9}{1\cancel{00}} \times \frac{22\cancel{00}}{1} = \$198$$

total deductions = $440 + $198 = $638

net wage = $2200 − $638 = ***$1562***

a) Sam's gross wage is $2700 per fortnight. She pays 20% of the gross wage in tax and contributes 9% of the gross wage to her superannuation fund. Calculate Sam's net wage per fortnight.
[net wage = gross wage − total deductions]

=

b) Hal's pay cheque is $3000 per fortnight. Hal's tax for the year is $17 000. How much is his yearly gross income?
[Assume 26 fortnights in a year.]

=

c) Part of Carrie's fortnightly payslip is shown. Calculate Carrie's total deductions and net wage for the fortnight.
[net wage = gross wage − total deductions]

Gross wage	$943.15
Income tax	$317.22
Superannuation	$80.65
Union fees	$19.00

Total deductions =

Net wage =

d) Donna is a foreign resident on a holiday working visa. Her taxable income is $20 000. What is the amount of tax payable on her income?

Taxable income	Tax on this income*
0-$80 000	32.5¢ for each $1
$80 001-$180 000	$26 000 plus 37¢ for each $1 over $80 000
$180 001 and over	$63 000 plus 45¢ for each $1 over $180 000

*Income tax rates for the 2013-2014 year

=

e) Xi's taxable income is $77 000. What is the amount of tax payable on his income?

Taxable income	Tax on this income
0-$18 200	nil
$18 201-$37 000	19¢ for each $1 over $18 200
$37 001-$80 000	$3572 plus 32.5¢ for each $1 over $37 000
$80 001-$180 000	$17 547 plus 37¢ for each $1 over $80 000
$180 001 and over	$54 547 plus 45¢ for each $1 over $180 000

*Income tax rates for the 2013-2014 year

=

f) Di's taxable income is $35 200. What is the amount of tax payable on her income?

Taxable income	Tax on this income
0-$18 200	nil
$18 201-$37 000	19¢ for each $1 over $18 200
$37 001-$80 000	$3572 plus 32.5¢ for each $1 over $37 000
$80 001-$180 000	$17 547 plus 37¢ for each $1 over $80 000
$180 001 and over	$54 547 plus 45¢ for each $1 over $180 000

*Income tax rates for the 2013-2014 year

=

Skill 14.7 Calculating simple interest.

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 **3** 3 4 4

- Write an equation from the word problem.
- To find the total investment, after interest, add the interest to the principal.

Simple Interest = Principal × Rate × Time OR $SI = PRT$

Q. Darcy invests \$1000 at a simple interest rate of 12% per year. What did the investment equal at the end of 2 years?

A. $SI = PRT$

$= 10\cancel{00} \times \frac{12}{1\cancel{00}} \times 2$ Simplify: ÷ 100

$= 10 \times 12 \times 2 = 240$

$1000 + 240$ investment = principal + interest

$= \boldsymbol{\$1240}$

a) How much interest would Sean pay on his credit card after 2 years if he owed \$1500 at an interest rate of 8% per year?

$SI = PRT = 15\cancel{00} \times \frac{8}{1\cancel{00}} \times 2 =$ Simplify: ÷ 100

$= 15 \times 8 \times 2$ =

b) Simple Interest = Principal × Rate × Time
Paula invests \$100 for 1 year. If the interest rate is 14% per year, how much interest would Paula get?

$SI =$

= =

c) Simple Interest = Principal × Rate × Time
A bank account of \$1000 earns 11% simple interest. How much interest is earned after 1 year?

$SI =$

= =

d) How much interest is paid on a loan of \$500 at a simple interest rate of 10% after 2 years?

$SI =$

= =

e) Pedro invested \$1500 at 5% simple interest for 2 years. How much interest did he earn?

$SI =$

= =

f) How much interest would Carey pay on his credit card after 3 years if he owed \$1200 at an interest rate of 12% per year?

$SI =$

= =

g) Guy borrowed \$200 for 3 years at a simple interest rate of 7% per year. How much does Guy owe at the end of 3 years?

$SI =$

=

$Total =$ =

h) Marcie invests \$750 for 4 years at a simple interest rate of 8% per year. How much does Marcie get back?

$SI =$

=

$Total =$ =

Skill 14.8 Calculating discount prices and depreciation.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 **3** 4 4

- Write a number sentence from the information given.

Q. A 30% increase followed by a 10% decrease on the same item is greater than (>), less than (<) or equal to a 20% increase of the original value?

A.

$$\frac{3\cancel{0}}{10\cancel{0}} \times \frac{4\cancel{0}}{1} = 12$$

$$40 + 12 = 52$$

+30%

$$\frac{1\cancel{0}}{10\cancel{0}} \times \frac{52}{1} = 5.2$$

$$52 - 5.2 = 46.8$$

−10%

$$\frac{2\cancel{0}}{10\cancel{0}} \times \frac{4\cancel{0}}{1} = 8$$

$$40 + 8 = 48$$

+20% ⇒

$\$6.80 < \$8.00$

Assume an amount e.g. 40

a) A 40% increase followed by a 30% decrease on the same item is >, < or = a 10% increase of the original value?

= ☐

b) A 60% increase followed by a 20% decrease on the same item is >, < or = a 40% increase of the original value?

= ☐

c) A book was discounted by 40% to $15. How much was the book before the discount?

= ☐

d) Ollie's iPhone is now worth $210 or 30% of its original cost. How much did Ollie originally pay for the iPhone?

= ☐

e) Last year Sandra invested $5000 in shares. In the past 12 months they lost 25% of their value. What is the value of her investment?

= ☐

f) You get 15% off your car insurance (cost = $350) and house insurance (cost = $450) if you combine the two payments. What would the joint payment be?

= ☐

g) A toy was discounted by 25% to $60. How much was the toy before the discount?

= ☐

h) A pen was discounted by 30% to $35. How much was the pen before the discount?

= ☐

Skill 14.9 Calculating compound interest.

yearly interest = principal × rate

- Write an equation for the word problem.
- Calculate the amount of interest each year on that year's principal.
  Hint: Deal with each year separately because in each year there will be a new balance or principal which includes all previous interest.

Q. Luke invests $2000 at a compound interest rate of 15% per year. After 3 years, how much does Luke have?

A.

Year 1

$$2000 + \frac{15}{10\cancel{0}} \times \frac{200\cancel{0}}{1} =$$
$$= 2000 + 300 = 2300$$

Year 2

$$2300 + \frac{15}{10\cancel{0}} \times \frac{230\cancel{0}}{1} =$$
$$= 2300 + 345 = 2645$$

Year 3

$$2645 + \frac{15}{100} \times \frac{2645}{1} =$$
$$= 2645 + 396.75 =$$
$$= \mathbf{\$3041.75}$$

Luke starts with $2000

After year 1: Principal is $2000 plus 15% of $2000 = $2300

The next year's interest is paid on the new balance of $2300.

After 2 years: Principal is $2300 plus 15% of $2300 = $2645

After 3 years: Principal is $2645 plus 15% of $2645 = $3041.75

a) What is the total amount of interest repaid on a loan of $800 after 2 years at a compound interest rate of 6%?

Year 1

$$\frac{6}{100} \times \frac{800}{1} = 48$$

Interest after year 1

$$800 + 48 = 848$$

New principal after year 1

Year 2

$$\frac{6}{100} \times \frac{848}{1} = 50.88$$

Interest after year 2

$$Yr\ 1 + Yr\ 2 = 48 + 50.88 =$$

Total Interest after 2 years

b) Jim borrowed $10 000 for 2 years at a compound interest rate of 6%. How much did Jim pay back?

Year 1

Year 2

c) April invests $5000 at a compound interest rate of 20% per year. What is the total amount of interest April gets after 3 years?

Year 1

Year 2

Year 3

d) What is the total amount of interest repaid on a loan of $12 000 after 3 years at a compound interest rate of 5%?

Year 1

Year 2

Year 3

Skill 14.10 Calculating compound growth and depreciation.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 **4**

- Write an equation for the word problem.
- Calculate the amount of growth or depreciation each year on that year's value.

Hint: Deal with each year separately because in each year there will be a new value which includes all previous growth or depreciation.

Q. A computer depreciates in value at an annual rate of 20%. If it costs $4000 when new, calculate its value after 3 years.

A.

Year 1

$$4000 - \frac{20}{10\cancel{0}} \times \frac{400\cancel{0}}{1} =$$
$$= 4000 - 800 = 3200$$

Year 2

$$3200 - \frac{20}{10\cancel{0}} \times \frac{320\cancel{0}}{1} =$$
$$= 3200 - 640 = 2560$$

Year 3

$$2560 - \frac{20}{100} \times \frac{2560}{1} =$$
$$= 2560 - 512 =$$
$$= \mathbf{\$2048}$$

New computer costs $4000.

After year 1: Value is $4000 minus 20% of $4000 = $3200

The next year's depreciation starts from the new value of $3200.

After 2 years: Value is $3200 minus 20% of $3200 = $2560

After 3 years: Value is $2560 minus 20% of $2560 = $2048

a) A car depreciates in value at an annual rate of 10%. If it costs $45 000 when new, calculate its value after 2 years.

Year 1

Year 2

b) The population of a town compounded annually at a rate of 10% per year. Initially 1000 people, what was the population after 2 years?

Year 1

Year 2

c) The population of a town compounded annually at a rate of 10% per year. The population was initially 20 000. What was the population after 3 years?

Year 1

Year 2

Year 3

d) A printer depreciates in value at an annual rate of 20%. If it costs $400 when new, calculate its value after 3 years.

Year 1

Year 2

Year 3

15. [Number Patterns]

Skill 15.1 Completing number patterns in table format by adding, subtracting or multiplying by the same number.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Look at consecutive terms in the second row of the table.
- Find the number and operation used to get from one term to the next.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next term of the pattern.

Q. What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	–1	3	–27	81	–243	?

A. $-1, 3, -27, 81, -243, ?$

$\times(-3)$

Rule: *Multiply each term by –3.*

$-1 \times (-3) = 3$

$3 \times (-3) = -27$

$-27 \times (-3) = 81$

$81 \times (-3) = -243$

$-243 \times (-3) = 729$

$-1, 3, -27, 81, -243,$ **729**

Note that the value of each term in the pattern is a multiple of 3 in increasing order. The signs are changing.

a) What is the value of the missing term in the pattern?

position	1	2	3	4	5
term	3	7	11	15	?

3 + 4 = 7, 7 + 4 = 11, 11 + 4 = 15, 15 + 4 = 19

b) What is the value of the missing term in the pattern?

position	1	2	3	4	5
term	1	6	11	16	?

c) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	17	12	7	2	–3	?

d) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	21	15	9	3	–3	?

e) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	1	–5	25	–125	625	?

f) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	2	–4	8	–16	32	?

g) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	–10	20	–40	80	–160	?

h) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	2	–10	50	–250	1250	?

Skill 15.2 Completing number patterns by using changing values in the rule.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Hints: Every number pattern is created by a rule involving numbers and operations. Counting numbers, even numbers and odd numbers have patterns themselves that can become part of the rule (see below).

Q. Complete the pattern:

3 , 5 , 9 , 15 , 23 , ___ , ___

A. *3 , 5 , 9 , 15 , 23 , ___ , ___*

+2 +4 +6 +8

Rule: *Add 2, then 4, then 6, then 8, etc. The pattern is formed by adding consecutive even numbers.*

$23 + 10 = 33$

$33 + 12 = 45$

3 , 5 , 9 , 15 , 23 , ***33*** *,* ***45***

a) Complete the pattern:

11 , 13 , 16 , 20 , ___ , ___

+2 +3 +4 +5 +6

20 + 5 = 25   25 + 6 = 31

b) Complete the pattern:

2 , 3 , 5 , 8 , 12 , ___ , ___

c) Complete the pattern:

3 , 5 , 9 , 15 , ___ , ___

d) Complete the pattern:

1 , 3 , 7 , 13 , ___ , ___

e) Complete the pattern:

0 , 3 , 9 , 18 , 30 , ___ , ___

f) Complete the pattern:

49 , 48 , 45 , 40 , 33 , ___ , ___

g) Complete the pattern:

1 , 4 , 8 , 13 , 19 , ___ , ___

h) Complete the pattern:

5 , 6 , 9 , 14 , 21 , ___ , ___

i) Complete the pattern:

1 , 4 , 10 , 19 , ___ , ___

j) Complete the pattern:

30 , 28 , 24 , 18 , 10 , ___ , ___

Skill 15.3 Completing number patterns by adding or subtracting the same positive number to integers.

Mauve 1 1 **2** 2 3 3 4 4
Lime 1 **1** 2 2 3 3 4 4

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
  Hint: Every number pattern is created by adding or subtracting the same positive integer.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:

−25 , −17 , −9 , −1 , 7 , ___ , ___

Note that the value of each term in the pattern is a increasing. Then find by how much.

A. *−25 , −17 , −9 , −1 , 7 , ___ , ___*

+8 +8 +8 +8

***Rule:** Add 8 to each term.*

7 + 8 = 15

15 + 8 = 23

*−25 , −17 , −9 , −1 , 7 , **15** , **23***

a) Complete the pattern:

35 , 20 , 5 , −10 , ___ , ___

−15 −15 −15 −15 −15

−10 − 15 = −25   −25 − 15 = −40

b) Complete the pattern:

8 , 5 , 2 , −1 , −4 , ___ , ___

c) Complete the pattern:

−20 , −14 , −8 , −2 , 4 , ___ , ___

d) Complete the pattern:

−16 , −11 , −6 , −1 , 4 , ___ , ___

e) Complete the pattern:

9 , 5 , 1 , −3 , −7 , ___ , ___

f) Complete the pattern:

10 , 7 , 4 , 1 , −2 , ___ , ___

g) Complete the pattern:

−35 , −28 , −21 , −14 , −7 , ___ , ___

h) Complete the pattern:

−19 , −15 , −11 , −7 , −3 , ___ , ___

i) Complete the pattern:

16 , 10 , 4 , −2 , −8 , ___ , ___

j) Complete the pattern:

12 , 7 , 2 , −3 , −8 , ___ , ___

k) Complete the pattern:

46 , 34 , 22 , 10 , −2 , ___ , ___

l) Complete the pattern:

−20 , −11 , −2 , 7 , 16 , ___ , ___

Skill 15.4 Completing number patterns by multiplying by the same integer.

Mauve 1 1 **2** 2 3 3 4 4
Lime 1 1 **2** 2 3 3 4 4

- Look at consecutive terms of the pattern.
- Find the number and operation (in this case multiplication) used to get from one term to the next.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:

$-5 , 15 , -45 , 135 ,$ $\boxed{\_\_\_ , \_\_\_}$

Note that the value of each term in the pattern is increasing, but the signs are changing.

A. *$-5 , 15 , -45 , 135 , \_\_\_ , \_\_\_$*

$\times(-3)\ \times(-3)\ \times(-3)$

Rule: *Multiply each term by −3.*

$135 \times (-3) = -405$

$-405 \times (-3) = 1215$

$-5 , 15 , -45 , 135 , \underline{\mathbf{-405}} , \underline{\mathbf{1215}}$

a) Complete the pattern:

$1 , 3 , 9 , 27 , 81 ,$ $\boxed{\_\_\_ , \_\_\_}$

$\times 3\quad \times 3\quad \times 3\quad \times 3\quad \times 3$

$81 \times 3 = 243 \qquad 243 \times 3 = 729$

b) Complete the pattern:

$3 , 6 , 12 , 24 , 48 ,$ $\boxed{\_\_\_ , \_\_\_}$

$48 \times 2 =$

c) Complete the pattern:

$2 , 6 , 18 , 54 , 162 ,$ $\boxed{\_\_\_ , \_\_\_}$

d) Complete the pattern:

$1 , 5 , 25 , 125 ,$ $\boxed{\_\_\_ , \_\_\_}$

e) Complete the pattern:

$\frac{1}{36} , \frac{1}{6} , 1 , 6 ,$ $\boxed{\_\_\_ , \_\_\_}$

$\times 6\quad \times 6\quad \times 6$

f) Complete the pattern:

$\frac{1}{25} , \frac{1}{5} , 1 , 5 ,$ $\boxed{\_\_\_ , \_\_\_}$

g) Complete the pattern:

$\frac{5}{2} , 5 , 10 , 20 , 40 ,$ $\boxed{\_\_\_ , \_\_\_}$

h) Complete the pattern:

$\frac{1}{49} , \frac{1}{7} , 1 , 7 , 49 ,$ $\boxed{\_\_\_ , \_\_\_}$

i) Complete the pattern:

$2 , -4 , 8 , -16 , 32 ,$ $\boxed{\_\_\_ , \_\_\_}$

j) Complete the pattern:

$1 , -4 , 16 , -64 ,$ $\boxed{\_\_\_ , \_\_\_}$

k) Complete the pattern:

$-3 , 9 , -27 , 81 ,$ $\boxed{\_\_\_ , \_\_\_}$

l) Complete the pattern:

$4 , -20 , 100 , -500 ,$ $\boxed{\_\_\_ , \_\_\_}$

Skill 15.5 Completing number patterns by dividing by the same integer.

- Look at consecutive terms of the pattern.
- Find the number and operation (in this case division) used to get from one term to the next.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:

$-1215 , 405 , -135 , 45 ,$ ____ , ____

NB: The value of each term in the pattern is decreasing, but the signs are changing.

A. *−1215 , 405 , −135 , 45 ,* ____ , ____

$\div(-3) \quad \div(-3) \quad \div(-3)$

Rule: *Divide each term by −3.*

$45 \div (-3) = -9$

$-9 \div (-3) = 3$

−1215 , 405 , −135 , 45 , ***−9*** , ***3***

a) Complete the pattern:

$288 , -144 , 72 , -36 , 18 ,$ ____ , ____

$\div(-2) \quad \div(-2) \quad \div(-2) \quad \div(-2) \quad \div(-2) \quad \div(-2)$

$18 \div (-2) = -9 \qquad -9 \div (-2) = 4.5$

b) Complete the pattern:

$1458 , 486 , 162 , 54 , 18 ,$ ____ , ____

c) Complete the pattern:

$3125 , 625 , 125 , 25 ,$ ____ , ____

d) Complete the pattern:

$1600 , 800 , 400 , 200 , 100 ,$ ____ , ____

e) Complete the pattern:

$-200\,000 , 20\,000 , -2000 , 200 ,$ ____ , ____

f) Complete the pattern:

$310\,000 , -31\,000 , 3100 , -310 ,$ ____ , ____

g) Complete the pattern:

$-6250 , 1250 , -250 , 50 ,$ ____ , ____

h) Complete the pattern:

$-64 , 32 , -16 , 8 ,$ ____ , ____

i) Complete the pattern:

$112 , 56 , 28 , 14 , 7 ,$ ____ , ____

j) Complete the pattern:

$54 , 18 , 6 , 2 ,$ ____ , ____

k) Complete the pattern:

$375 , 75 , 15 , 3 ,$ ____ , ____

l) Complete the pattern:

$7203 , 1029 , 147 , 21 , 3 ,$ ____ , ____

Skill 15.6 Finding a random term in a number pattern.

- Draw a table and list the given terms and the position each term occupies in the pattern.
- Look for a relationship between consecutive terms and/or between the term and its position in the pattern.
- Based on this relationship, find the requested term in the pattern.

Q. Find the 15th term in the pattern
5 , 7 , 9 , 11 , 13 , ...

A.

position	1st	2nd	3rd	4th	5th	.....	15th
term	5	7	9	11	13		?
relationship	**1**×2+3	**2**×2+3	**3**×2+3	**4**×2+3	**5**×2+3		**15**×2+3

×2+3

Relationship:
each term = twice its position plus 3
The 15th term of the pattern is:
$15 \times 2 + 3 = \mathbf{33}$

a) Find the 30th term in the pattern
2 , 4 , 6 , 8 , 10 , ...

position	1st	2nd	3rd	4th	5th	.....	30th
term	2	4	6	8	10		?
relationship	**1** × 2	**2** × 2	**3** × 2	**4** × 2	**5** × 2		**30** × 2

× 2

Relationship: term = ……………………………………

30th term = …………………………… = ☐

b) Find the 18th term in the pattern
5 , 10 , 15 , 20 , 25 , ...

position	1st	2nd	3rd	4th	5th	.....	18th
term	5	10					?
relationship							

Relationship: term = ……………………………………

18th term = …………………………… = ☐

c) Find the 20th term in the pattern
8 , 13 , 18 , 23 , 28 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term							?
relationship							

Relationship: term = ……………………………………

20th term = …………………………… = ☐

d) Find the 25th term in the pattern
4 , 6 , 8 , 10 , 12 , ...

position	1st	2nd	3rd	4th	5th	.....	25th
term							?
relationship							

Relationship: term = ……………………………………

25th term = …………………………… = ☐

e) Find the 20th term in the pattern
1 , 4 , 7 , 10 , 13 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term							?
relationship							

Relationship: term = ……………………………………

20th term = …………………………… = ☐

f) Find the 8th term in the pattern
1 , 2 , 4 , 8 , 16 , ...

position	1st	2nd	3rd	4th	5th	.....	8th
term							?
relationship							

Relationship: term = ……………………………………

8th term = …………………………… = ☐

Skill 15.7 Finding a particular term of a sequence given its general rule.

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 3 **3** 4 4

- Substitute the value of *n* in the formula for the general rule of the sequence.
- Calculate the value of the required term.

Q. If the general rule of a sequence is $\frac{n}{4} - 9$ find the 60th term ($n = 60$).

A. *the nth term of the sequence =*

$= \frac{n}{4} - 9$ substitute n = 60

the 60th term of the sequence =

$= \frac{60}{4} - 9$

$= 15 - 9$

$= \mathbf{6}$

a) If the general rule of a sequence is $5n - 4$ find the 30th term ($n = 30$).

30th term $= 5 \times 30 - 4$

$= 150 - 4$ ……… = ☐

b) If the general rule of a sequence is $4n - 7$ find the 15th term ($n = 15$).

15th term = ………

= ……… = ☐

c) If the general rule of a sequence is $8 - 5n$ find the 10th term ($n = 10$).

10th term = ………

= ……… = ☐

d) If the general rule of a sequence is $25 - n$ find the 40th term ($n = 40$).

40th term = ………

= ……… = ☐

e) If the general rule of a sequence is $15n$ find the 30th term ($n = 30$).

30th term = ………

= ……… = ☐

f) If the general rule of a sequence is $-40n$ find the 25th term ($n = 25$).

25th term = ………

= ……… = ☐

g) If the general rule of a sequence is $-\frac{2n}{7}$ find the 35th term.

35th term = ………

= ……… = ☐

h) If the general rule of a sequence is $\frac{n}{3} + 1$ find the 21st term.

21st term = ………

= ……… = ☐

i) If the general rule of a sequence is $-6(n - 3)$ find the 23rd term.

23rd term = ………

= ……… = ☐

j) If the general rule of a sequence is $3(n - 6)$ find the 24th term.

24th term = ………

= ……… = ☐

Skill 15.8 Finding the general rule of a pattern given a table of values for the pattern (1).

Mauve 11 22 33 44
Lime 11 22 33 44

To decide which general formula is true for the values shown in the table

- Substitute the values for *n* (first row in the table) in the general formula.
- Check if the results match the term values given in the second row of the table.

To express a general term of a sequence

EITHER

- Look for a relationship between consecutive terms and/or between the term and its position in the sequence.

  Hints: This relationship is an expression of n involving one or more operations, i.e. n – 6, 2n, 4n + 2

OR

- Find the difference between consecutive terms of the sequence (common difference).
- Write the term “common difference × *n*” in the expression.
- Check the result by substituting a random value for *n* into the formula.
- Adjust the expression by adding, subtracting, multiplying or dividing by a constant.
- Check the result by substituting all values for *n* = 1, 2, 3, 4, 5 into the final formula.

Q. Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	1	5	9	13	17	....	

$+4 \quad +4 \quad +4 \quad +4$

A. *common difference = 4*

⇒ *term in position n contains 4n*

If term in position n = 4n and n = 1

⇒ *term 1 = 4 × 1 = 4 ≠ 1* Adjust by subtracting 3

If term in position n = 4n – 3

n = 1 ⇒ term 1 = 4 × 1 – 3 = 1 (true)

n = 2 ⇒ term 2 = 4 × 2 – 3 = 5 (true)

n = 3 ⇒ term 3 = 4 × 3 – 3 = 9 (true)

n = 4 ⇒ term 4 = 4 × 4 – 3 = 13 (true)

n = 5 ⇒ term 5 = 4 × 5 – 3 = 17 (true)

The term in position n is ***4n – 3***

a)

position	1	2	3	4	5	....	*n*
term	7	9	11	13	15	....	?

The rule for the term in position *n* is:

A) 5*n* + 2 B) 2*n* – 5
C) 5*n* + 5 D) 2*n* + 5

A n = 1 ⇒ term 1 = 5 × 1 + 2 = 7 ⇒ true

n = 2 ⇒ term 2 = 5 × 2 + 2 = 12 ⇒ false

B n = 1 ⇒ term 1 = 2 × 1 – 5 = –3 ⇒ false

C n = 1 ⇒ term 1 = 5 × 1 + 5 = 10 ⇒ false

D n = 1 ⇒ term 1 = 2 × 1 + 5 = 7 ⇒ true

n = 2 ⇒ term 2 = 2 × 2 + 5 = 9 ⇒ true

n = 3 ⇒ term 3 = 2 × 3 + 5 = 11 ⇒ true

n = 4 ⇒ term 4 = 2 × 4 + 5 = 13 ⇒ true

n = 5 ⇒ term 5 = 2 × 5 + 5 = 15

D

b)

position	1	2	3	4	5	....	*n*
term	7	10	13	16	19	....	?

The rule for the term in position *n* is:

A) 3*n* – 4 B) 3*n* + 4
C) 4*n* + 3 D) 4*n* – 3

A

Skill 15.8 Finding the general rule of a pattern given a table of values for the pattern (2).

c) Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	8	10	12	14	16	....	$2n + 6$

+ 2 + 2 + 2 + 2

common difference = 2

term $n = 2n \Rightarrow$ *term* $1 = 2 \times 1 = 2$ (false)

adjust *term* $n = 2n + 6 \Rightarrow$ *term* $1 = 8$ (true)

d) Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	5	4	3	2	1	....	

– 1 – 1 – 1 – 1

common difference = –1

term $n = -n \Rightarrow$ *term* $1 = -1$ (false)

adjust *term* $n = 6 - n \Rightarrow$ *term* $1 = 5$ (true)

e) Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	2	5	8	11	14	....	

common difference =

term n

adjust *term n*

f) Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	5	9	13	17	21	....	

common difference =

term n

adjust *term n*

g) Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	5	10	15	20	25	....	

common difference =

term n

adjust *term n*

h) Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	7	10	13	16	19	....	

common difference =

term n

adjust *term n*

i) Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	2	1	0	–1	–2	....	

common difference =

term n

adjust *term n*

j) Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	–3	–6	–9	–12	–15	....	

common difference =

term n

adjust *term n*

Skill 15.9 Completing number patterns involving decimals and fractions.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 **4**

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
  Hint: Every number pattern is created by adding, subtracting, multiplying or dividing by rational numbers (whole numbers, fractions or decimals).
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:

$$\frac{5}{18}, \frac{1}{2}, \frac{13}{18}, \frac{17}{18}, 1\frac{1}{6}, \boxed{\_\_\_ , \_\_\_}$$

A. $\frac{5}{18}, \frac{1}{2}, \frac{13}{18}, \frac{17}{18}, 1\frac{1}{6},$

Look at the 3rd and 4th terms: their difference is $\frac{4}{18}$

Rule: *Add* $\frac{4}{18}$ *to each term.*

$$1\frac{1}{6} + \frac{4}{18} = \frac{7}{6} + \frac{4}{18} = \frac{21}{18} + \frac{4}{18} = \frac{25}{18} = 1\frac{7}{18}$$

$$\frac{25}{18} + \frac{4}{18} = \frac{29}{18} = 1\frac{11}{18}$$

$$\frac{5}{18}, \frac{1}{2}, \frac{13}{18}, \frac{17}{18}, 1\frac{1}{6}, \underline{\boldsymbol{1\frac{7}{18}}}, \underline{\boldsymbol{1\frac{11}{18}}}$$

a) Complete the pattern:

$$1, 2, 3.5, 5.5, 8, \boxed{\_\_\_ , \_\_\_}$$

+1 +1.5 +2 +2.5 +3 +3.5

8 + 3 = 11 *11 + 3.5 = 14.5*

b) Complete the pattern:

$$0.8, 2, 3.4, 5, 6.8, \boxed{\_\_\_ , \_\_\_}$$

+1.2

c) Complete the pattern:

$$1.5, 3.5, 6, 9, 12.5, \boxed{\_\_\_ , \_\_\_}$$

d) Complete the pattern:

$$4, 5.5, 7.5, 10, 13, \boxed{\_\_\_ , \_\_\_}$$

e) Complete the pattern:

$$1.75, 3.5, 7, 14, \boxed{\_\_\_ , \_\_\_}$$

f) Complete the pattern:

$$1\frac{1}{4}, 2\frac{1}{2}, 5, 10, \boxed{\_\_\_ , \_\_\_}$$

g) Complete the pattern:

$$36, 18, 9, \frac{9}{2}, \frac{9}{4}, \boxed{\_\_\_ , \_\_\_}$$

h) Complete the pattern:

$$32, 8, 2, \frac{1}{2}, \frac{1}{8}, \boxed{\_\_\_ , \_\_\_}$$

i) Complete the pattern:

$$3\frac{1}{4}, 4, 4\frac{3}{4}, 5\frac{1}{2}, 6\frac{1}{4}, \boxed{\_\_\_ , \_\_\_}$$

j) Complete the pattern:

$$\frac{8}{15}, \frac{4}{5}, 1\frac{1}{15}, 1\frac{1}{3}, 1\frac{3}{5}, \boxed{\_\_\_ , \_\_\_}$$

16. [Expressions]

Skill 16.1 Writing expressions to represent word problems.

Mauve 11 22 33 44
Lime 11 22 33 44

- Write the expression using the variables and/or the numbers mentioned in the word problem.
- Decide about the operation or operations needed in the expression.
  Example: $a + b$ (sum of a and b), $4n$ (product of 4 and n), $m - 20$ (20 less than m)
  Hint: "Sum, altogether, in total, more than" ⇒ addition ⇒ +
  "Difference, less than, change" ⇒ subtraction ⇒ –
  "Product, times, lots of" ⇒ multiplication ⇒ ×
  "A fraction (half, third, quarter) of" ⇒ division ⇒ ÷

Q. Lisa earns a weekly wage of w dollars. How much money did she earn in a fortnight, if she received a $300 bonus?

A. *w dollars a week*
2 weeks in a fortnight } ⇒ *2 times w*
$300 bonus ⇒ +

The × sign can be left out

$\Rightarrow \mathbf{2 \times w + 300}$ ***or*** $\mathbf{2w + 300}$

a) Write as an expression:
The sum of d and 20.

sum ⇒ + ⇒ $\boxed{d + 20}$

b) Write as an expression:
The number seven times y.

⇒ $\boxed{}$

c) Write as an expression:
The number 15 less than p.

⇒ $\boxed{}$

d) Write as an expression:
Nine lots of s.

⇒ $\boxed{}$

e) Write as an expression:
The product of –8 and t.

⇒ $\boxed{}$

f) Write as an expression:
The sum of $2u$ and $3v$.

⇒ $\boxed{}$

g) Lily had d dollars and spent a third of her money. How much money did she spend?

a third of ⇒ ÷ ⇒ $\boxed{\frac{d}{3}}$
or $d \div 3$

h) Out of the t tickets for sale, a quarter remained unsold. How many tickets remained unsold?

⇒ $\boxed{}$

i) You pay $50 dollars at the petrol station. How much change do you get if the petrol was p dollars?

⇒ $\boxed{}$

j) There are a local and b imported products at the supermarket. How many products are there altogether?

⇒ $\boxed{}$

k) Write as an expression:
Twice the product of p and q.

⇒ $\boxed{}$

l) Write as an expression:
The number 6 less than the product of a and b.

⇒ $\boxed{}$

Skill 16.2 Simplifying expressions.

Mauve 1 1 **2** 2 3 3 4 4
Lime 1 **1** 2 2 3 3 4 4

- Leave out the multiplication sign between variables (letters) or between variables and numbers.
  Example: $1 \times a = 1a = a$
- Write the number first, followed by the variables.
  Example: $m \times 3 = 3m$
- Write the variables in alphabetical order.
  Example: $c \times a \times b = abc$
- Replace the division sign with a fraction line.
  Example: $m \div n = \frac{m}{n}$
- Use the sign rules. (see skill 9.1, page 93)

Q. Simplify $p \times 3 \times m$

A. $p \times 3 \times m =$ The × signs can be left out
$= \mathbf{3mp}$
number first
alphabetical order

a) Simplify $j \times 5$ $\boxed{5j}$
number first
The × sign can be left out

b) Simplify $y \times 7$

c) Simplify $n \times m$

d) Simplify $h \times g$

e) Simplify $6 \times z \times y$

f) Simplify $4 \times u \times r$

g) Simplify $3 \times x \div 2$ $\boxed{\frac{3x}{2}}$
÷ becomes fraction line
The × sign can be left out

h) Simplify $6 \times z \div 5$

i) Simplify $4 \times b \times b$
$b \times b = b^2$

j) Simplify $3 \times a \times -a$
$+ \times - = -$

k) Simplify $w \times z \times w$

l) Simplify $c \times d \times -c$

m) Simplify $s \times r^2 \times 2$

n) Simplify $j \times k^2 \times -1$

o) Simplify $r \times 5 \times s \div t$

p) Simplify $2 \times a \times b \div c$

q) Simplify $u \times 10 \times v \div -w$

r) Simplify $g \times 6 \times h \div -i$

Skill 16.3 Finding like terms.

- Look at the combination of variables in all terms.

EITHER

- Find the **like terms**, terms that have the same combination of variables.

Example: $4c$ and $-c$
$-2x^2$ and $5x^2$ — like terms
$-ab$, $5ba$ and $3ba$

OR

- Find the **unlike terms**, terms that do not have the same combination of variables.

Example: $2\cancel{k}$ and $2\cancel{k^2}$
$5\cancel{uv}$ and $\cancel{vw}$ — unlike terms
$3\cancel{xy}$, $\cancel{x}$ and $\cancel{y}$

Hint: The order of the variables in a term does not matter.
$gh = hg$, $mn^2 = n^2m$

Q. Choose the like terms:
$a^2b, -ab, 4ba$

A. $4ba = 4ab$
$-ab$ and $4ab$ have the same combination of variables (ab)
$\Rightarrow$ ***−ab*** *and* ***4ba*** *are like terms.*

a) Choose the like terms:
$8a, 3, 5a$ — like terms
$\boxed{8a, 5a}$

b) Choose the like terms:
$-2, -2m, 3m$

c) Choose the like terms:
$m^2, 3\cancel{m^2}, 3\cancel{m}$ — unlike terms

d) Choose the like terms:
$t^2, 2t, -t^2$

e) Choose the like terms:
$3cd, dc, 3c$
$\boxed{3cd, dc}$

f) Choose the like terms:
$-bc, 5c, 5cb$

g) Choose the like terms:
$3t^2, -2t, 4, 3t$

h) Choose the like terms:
$-6w, 8, w^2, w$

i) Choose the like terms:
$3s, 2.3s, s^2, 2.3$

j) Choose the like terms:
$-0.2y, -0.2y^2, 2y, 2.2$

k) Choose the like terms:
$v^2, -2v, u^2, -2v^2$

l) Choose the like terms:
$4k, 4k^2, l^2, -k^2$

m) Choose the like terms:
$z^2, 8z, -8z^2, z^3$

n) Choose the like terms:
$g, g^2, -4g^2, g^3$

o) Choose the like terms:
$-5w, -5w^4, -5, w^4$

p) Choose the like terms:
$a^2b, 2ab, 2ba^2, -ab^2$

q) Choose the like terms:
$-xy, x^2y, 2yx^2, 2xy^2$

r) Choose the like terms:
$3t^2u^2, 3tu, -tu^2, 3u^2t$

Skill 16.4 Simplifying expressions by adding and subtracting like terms.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Group like terms. (see skill 16.3, page 169)
- Read the sign in front of each term.
- Add and/or subtract only the like terms.
- Add and/or subtract the coefficients first, then copy the variable.
  Example: $3g + 5g = (3 + 5)g = 8g$
  coefficients
- Write coefficient 1 in front of any variable.
  Example: $a = 1a$, $-b = -1b$, $c^2 = 1c^2$
  Hint: Unlike terms cannot be added or subtracted.

Q. Simplify $3x^2 - 6x + x^2 + 7x$

A. $\underline{3x^2} \underline{\underline{- 6x}} \underline{+ x^2} \underline{\underline{+ 7x}} =$ group like terms

$x^2 = 1x^2$

$= 3x^2 + 1x^2 - 6x + 7x$

$3 + 1 = 4$ $-6 + 7 = 1$

$= 4x^2 + 1x$

$= \mathbf{4x^2 + x}$

a) Simplify $2m + m$

$= 2m + 1m$ .... $= \boxed{3m}$

b) Simplify $5cd + dc$

$= 5cd + 1cd$ .... $=$ ☐

c) Simplify $4j - 3j + 2j$

$=$ .... $=$ ☐

d) Simplify $7xy - 5xy + xy$

$=$ .... $=$ ☐

e) Simplify $\underline{5a} + 3b \underline{- 2a}$ group like terms

$= 5a - 2a + 3b$ .... $=$ ☐

f) Simplify $t^2 + 3t + 2t^2$

$=$ .... $=$ ☐

g) Simplify $\underline{6ad} \underline{\underline{+ 2d}} \underline{- 5da} \underline{\underline{+ 3d}}$ group like terms

$= 6ad - 5ad + 2d + 3d$ .... $=$ ☐

h) Simplify $3m + 5n - 4m - n$

$=$ .... $=$ ☐

i) Simplify $4p^2 - p^2 - 3p + 2p^2$

$=$ .... $=$ ☐

j) Simplify $3y^2 - 2yz - y^2 + 3zy$

$=$ .... $=$ ☐

k) Simplify $2r^2 + s^2 + r^2 - 4s^2$

$=$ .... $=$ ☐

l) Simplify $-3x - x^2 + x + 4x^2$

$=$ .... $=$ ☐

m) Simplify $3d - d^2e - 2ed^2 - 4d$

$=$ .... $=$ ☐

n) Simplify $3ab^2 - 2ab^2 - 4a^2b + a^2b$

$=$ .... $=$ ☐

Skill 16.5 Simplifying expressions by multiplying terms.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Read the sign in front of each term.
- Multiply the coefficients.
- Multiply the variables (join the letters).
- Use the sign rules. (see skill 9.1, page 93)
  Example: $2u \times -3v = (2 \times -3) \times (u \times v) = -6 \times uv = -6uv$
  coefficients $+ \times - = -$
- Write coefficient 1 in front of any variable.
  Example: $a = 1a$, $-b = -1b$, $c^2 = 1c^2$
  Hint: Any terms can be multiplied.

Q. Simplify $-3cd \times 4c \times -d$

A. $-3cd \times 4c \times -d =$

× the coefficients   × the letters

$= (-3 \times 4 \times -1) \times (cd \times c \times d) =$

$- \times - = +$

$= (-12 \times -1) \times (c^2d \times d) =$

$= 12 \times c^2d^2$

$= \mathbf{12c^2d^2}$

a) Simplify $4 \times 3v$

× the coefficients

$= (4 \times 3) \times v$ .... $= \boxed{12v}$

b) Simplify $3xy \times 5$

= .... $= \boxed{\quad}$

c) Simplify $2m \times 7n$

= .... $= \boxed{\quad}$

d) Simplify $-8j \times 5k$

= .... $= \boxed{\quad}$

e) Simplify $-4d \times -5e$

$- \times - = +$

$= (-4 \times -5) \times (d \times e)$ .... $= \boxed{20de}$

f) Simplify $3b \times 6b$

$b \times b = b^2$

= .... $= \boxed{\quad}$

g) Simplify $2v \times -12w$

= .... $= \boxed{\quad}$

h) Simplify $-4ab \times 7b$

= .... $= \boxed{\quad}$

i) Simplify $-10xz \times 3z$

= .... $= \boxed{\quad}$

j) Simplify $-4gh \times 5g$

= .... $= \boxed{\quad}$

k) Simplify $2s \times -5t \times 3s$

$= (2 \times -5 \times 3) \times (s \times t \times s)$ = ...

= .... $= \boxed{\quad}$

l) Simplify $-4p \times 2q \times 3p$

= .... = ...

= .... $= \boxed{\quad}$

m) Simplify $3jk \times -5k \times -j$

= .... = ...

= .... $= \boxed{\quad}$

n) Simplify $-bc \times -5c \times 5c$

= .... = ...

= .... $= \boxed{\quad}$

Skill 16.6 Simplifying expressions by dividing terms.

- Read the sign in front of each term.
- Write the division as a fraction.
- Simplify by dividing the coefficients.
- Simplify by dividing the variables.
- Use the sign rules. (see skill 9.1, page 93)
- Write coefficient 1 in front of any variable.
  Example: $a = 1a$, $-b = -1b$, $c^2 = 1c^2$

Q. Simplify $-30x^2y \div 3y$

A. $-30x^2y \div 3y =$

$$= -\frac{\overset{10}{\cancel{30}}x^2y}{\underset{1}{\cancel{3}}y}$$

Simplify: ÷ 3

$$= -\frac{10x^2\overset{1}{\cancel{y}}}{\underset{1}{\cancel{y}}}$$

Simplify: ÷ y

$$= \mathbf{-10x^2}$$

a) Simplify $12y \div 3$

$= \frac{\overset{4}{\cancel{12}}y}{\underset{1}{\cancel{3}}}$ $= \boxed{4y}$

b) Simplify $24pq \div 4$

$= \frac{\overset{6}{\cancel{24}}pq}{\underset{1}{\cancel{4}}}$ $=$

c) Simplify $14a \div 2a$ a ÷ a = 1

$=$ $=$

d) Simplify $-35mn \div -5n$ − ÷ − = +

$=$ $=$

e) Simplify $-15z^2 \div 3z$ − ÷ + = −

$=$ $z^2 \div z = z$ $=$

f) Simplify $-12xy \div 2y$

$=$ $=$

g) Simplify $18x \div 15x$

$=$ $=$

h) Simplify $20cd \div cd$

$=$ $=$

i) Simplify $-24t^2 \div 8t$

$=$ $=$

j) Simplify $11ab \div -11b$

$=$ $=$

k) Simplify $-25v^2w \div 5w$

$= -\frac{\overset{5}{\cancel{25}}v^2\overset{1}{\cancel{w}}}{\underset{1}{\cancel{5}}\underset{1}{\cancel{w}}}$ Simplify: ÷ 5 then ÷ w $= \boxed{-5v^2}$

l) Simplify $-45ab^2 \div 9b$

$=$ $=$

m) Simplify $20xy \div 4x \times xz$

$= \frac{\overset{5}{\cancel{20}}\overset{1}{\cancel{x}}y}{\underset{1}{\cancel{4}}\underset{1}{\cancel{x}}} \times xz = 5y \times xz$ $=$

n) Simplify $27gh \div 9g \times gi$

$=$ $=$

17. [Substitution]

Skill 17.1 Substituting value 0 into simple expressions.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Replace the variable (letter) with 0.
- Solve the mathematical sentence to find the value of the expression.

number + 0 = 0 + number = number
number − 0 = number
number × 0 = 0 × number = 0
0 ÷ number = 0

Q. If $x = 0$, find the value of $10x$

A. $10x =$ 10x = 10 × x
$= 10 \times x$ Substitute $x = 0$
$= 10 \times 0$ Multiply 10 by 0
$= \mathbf{0}$

a) If $y = 0$, find the value of $12 + y$

$12 + 0$ = $\boxed{12}$

b) If $a = 0$, find the value of $a + 45$

= $\boxed{}$

c) If $m = 0$, find the value of $100 + m$

= $\boxed{}$

d) If $f = 0$, find the value of $f + 6$

= $\boxed{}$

e) If $b = 0$, find the value of $8 - b$

$8 - 0$ = $\boxed{8}$

f) If $v = 0$, find the value of $17 - v$

= $\boxed{}$

g) If $d = 0$, find the value of $d - 40$

= $\boxed{}$

h) If $z = 0$, find the value of $z - 200$

= $\boxed{}$

i) If $t = 0$, find the value of $8t$

$8t = 8 \times t = 8 \times 0$ = $\boxed{}$

j) If $j = 0$, find the value of $25j$

= $\boxed{}$

k) If $g = 0$, find the value of $12g$

= $\boxed{}$

l) If $p = 0$, find the value of $81p$

= $\boxed{}$

m) If $h = 0$, find the value of $\frac{h}{5}$

= $\boxed{}$

n) If $n = 0$, find the value of $\frac{n}{10}$

= $\boxed{}$

o) If $u = 0$, find the value of $\frac{u}{7}$

= $\boxed{}$

p) If $q = 0$, find the value of $\frac{q}{24}$

= $\boxed{}$

Skill 17.2 Substituting one value into expressions involving +, –, × and ÷

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Replace the variable (letter) with the given value.
- Solve the mathematical sentence to find the value of the expression.
- Use the order of operations rules: Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (–) in order from left to right.

Q. If $x = 4$, find the value of $18 - 3x$

A. $18 - 3x =$ — $3x = 3 \times x$

$= 18 - 3 \times x$ Substitute $x = 4$

$= 18 - 3 \times 4$ Multiply 3 by 4

$= 18 - 12$ Subtract 12 from 18

$= \mathbf{6}$

a) If $c = 5$, find the value of $36 + c$

$36 + c = 36 + 5$ = $\boxed{41}$

b) If $k = 7$, find the value of $k + 56$

.... = ☐

c) If $y = 12$, find the value of $88 + y$

.... = ☐

d) If $r = 50$, find the value of $r + 150$

.... = ☐

e) If $z = 20$, find the value of $25 - z$

.... = $\boxed{5}$

f) If $a = 40$, find the value of $a - 28$

.... = ☐

g) If $e = 35$, find the value of $e - 30$

.... = ☐

h) If $x = 8$, find the value of $7x$

$7x = 7 \times x = 7 \times 8$ = ☐

i) If $b = 12$, find the value of $12b$

.... = ☐

j) If $y = 22$, find the value of $5y$

.... = ☐

k) If $j = 3$, find the value of $\frac{48}{j}$

.... = ☐

l) If $p = 4$, find the value of $\frac{56}{p}$

.... = ☐

m) If $u = 5$, find the value of $4u - 19$

....

= .... = ☐

n) If $f = 6$, find the value of $25 - 3f$

....

= .... = ☐

o) If $x = 8$, find the value of $2x + 6$

....

= .... = ☐

p) If $z = 3$, find the value of $15 + 6z$

....

= .... = ☐

Skill 17.3 Substituting two values into expressions involving +, −, × and ÷

Mauve 1 1 **2** 2 3 3 4 4
Lime **1** 1 2 2 **3** 3 4 4

- Replace the two variables with the given values.
- Solve the mathematical sentence to find the value of the expression.
- Use the order of operations rules: Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (−) in order from left to right.

Q. If $x = 4$ and $y = 8$,
find the value of $\frac{4x - y}{2}$

A. $\frac{4x - y}{2} =$ 4x = 4 × x

$= \frac{4 \times x - y}{2}$ Substitute $x = 4$ and $y = 8$

$= \frac{4 \times 4 - 8}{2}$ Multiply 4 by 4

$= \frac{16 - 8}{2}$ Subtract 8 from 16

$= 8 \div 2$ Divide 8 by 2

$= \mathbf{4}$

a) If $c = 5$ and $d = 3$,
find the value of $3c - 5d$

$3 \times c - 5 \times d$

$= 3 \times 5 - 5 \times 3 = 15 - 15 = \boxed{0}$

b) If $v = 6$ and $w = 7$,
find the value of $4v - 3w$

c) If $a = 7$ and $b = 1$,
find the value of $6a + 5b$

d) If $m = 5$ and $n = 3$,
find the value of $24 - mn$

e) If $p = 6$ and $q = 8$,
find the value of $\frac{pq}{4}$

f) If $x = 6$ and $y = 1$,
find the value of $\frac{2xy}{3}$

g) If $y = 1$ and $z = 9$,
find the value of $\frac{z}{3} - y$

h) If $d = 12$ and $e = 2$,
find the value of $\frac{d}{4} - e$

i) If $m = 2$ and $l = 6$,
find the value of $\frac{m + 3l}{4}$

j) If $j = 5$ and $k = 4$,
find the value of $\frac{2j - k}{3}$

Skill 17.4 Substituting into rules.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Replace the variables *x* with the given value.
- Solve the mathematical sentence to find the value of *y*.
- Use the order of operations rules: Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (−) in order from left to right.

Q. If $y = 4x^2 - 3$, find y when $x = 2$

A. $y = 4x^2 - 3$ — $4x^2 = 4 \times x^2$

$= 4 \times x^2 - 3$ Substitute $x = 2$

$= 4 \times 2^2 - 3$ Evaluate 2^2

$= 4 \times 4 - 3$ Multiply 4 by 4

$= 16 - 3$ Subtract 3 from 16

$= \mathbf{13}$

a) If $y = x - 9$, find y when $x = 12$

$y = 12 - 9$ = $\boxed{3}$

b) If $y = 25 - x$, find y when $x = 7$

$y =$ =

c) If $y = 4x + 8$, find y when $x = 2$

$y = 4 \times 2 + 8 = 8 + 8$ =

d) If $y = 3x - 9$, find y when $x = 9$

$y =$ =

e) If $y = 5x - 6$, find y when $x = 3$

$y =$ =

f) If $y = 2x + 7$, find y when $x = 12$

$y =$ =

g) If $y = \frac{18}{x} - 7$, find y when $x = 2$

$y = 18 \div 2 - 7 = 9 - 7$ =

h) If $y = \frac{24}{x} - 10$, find y when $x = 6$

$y =$ =

i) If $y = x^2 + 18$, find y when $x = 3$

$y =$ =

j) If $y = x^2 - 7$, find y when $x = 4$

$y =$ =

k) If $y = 3x^2 + 2$, find y when $x = 5$

$y =$ =

l) If $y = 5x^2 - 18$, find y when $x = 2$

$y =$ =

m) If $y = \frac{3x}{4}$, find y when $x = 8$

$y =$ =

n) If $y = \frac{6x}{5}$, find y when $x = 10$

$y =$ =

Skill 17.5 Substituting into formulae.

Mauve 1 1 2 2 **3** 3 4 4
Lime 1 1 **2** 2 3 3 4 **4**

- Replace the variables with the given values.
- Solve the mathematical sentence to find the requested value in the formula.
- Use the order of operations rules: Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (–) in order from left to right.

Q. Use $V = \pi r^2 h$ to find the volume of a cylinder when $r = 10$, $h = 5$ and $\pi \approx 3.14$

A. $V = \pi r^2 h$ — $\pi r^2 h = \pi \times r^2 \times h$

$= \pi \times r^2 \times h$	Substitute $r = 10, h = 5$ and $\pi \approx 3.14$ and evaluate 10^2
$\approx 3.14 \times 10^2 \times 5$	
$= 3.14 \times 100 \times 5$	Multiply 3.14 by 100
$= 314 \times 5$	Multiply the result by 5
$= \mathbf{1570}$	

a) Use $P = 4l$ to find the perimeter P of a square when $l = 4.5$

$P = 4 \times l = 4 \times 4.5$ = 18

b) Use $M = 0.6K$ to find the number of miles M when $K = 2000$

.... =

c) Use $A = lw$ to find the area A of a rectangle when $l = 12$ and $w = 8$

$A = l \times w =$ .... =

d) Use $C = \pi d$ to find the circumference C of a circle when $d = 15$ and $\pi \approx 3.14$

.... =

e) Use $A = \dfrac{d_1 d_2}{2}$ to find the area A of a rhombus when $d_1 = 15$ and $d_2 = 6$

.... =

f) Use $M = \dfrac{1}{2}(x + y)$ to find the average M of $x = 20$ and $y = 16$

.... =

g) Use $v = \dfrac{d}{t}$ to find the speed v when $d = 400$ and $t = 5$

.... =

h) Use $A = \dfrac{l^2\sqrt{3}}{4}$ to find the area A of an equilateral triangle when $l = 4$ and $\sqrt{3} \approx 1.73$

.... =

i) Use $V = lwh$ to find the volume V of a prism when $l = 5$, $w = 3$ and $h = 10$

.... =

j) Use $TSA = 6l^2$ to find the total surface area TSA of a cube when $l = 20$

.... =

k) Use $TSA = 4\pi r^2$ to find the total surface area TSA of a sphere when $r = 10$ and $\pi \approx 3.14$

....

= .... =

l) Use $a^2 = c^2 - b^2$ to find the value of $a > 0$ when $c = 15$ and $b = 9$

....

= .... $a =$

Skill 17.6 Substituting into rules, expressions and formulae with brackets.

Mauve 1 1 2 2 3 **3** 4 4
Lime 1 1 **2 2 3** 3 4 **4**

- Replace the variables with the given values.
- Solve the mathematical sentence to find the value of the expression.
- Use the order of operations rules: Simplify within the brackets.
  Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (−) in order from left to right.

Q. If $y = (x + 3)(x - 4)$, find y when $x = 6$

A. $y = (x + 3)(x - 4)$ $( ) = ( ) \times ( )$
$= (x + 3) \times (x - 4)$ Substitute $x = 6$
$= (6 + 3) \times (6 - 4)$ Evaluate each bracket.
$= 9 \times 2$ Multiply the results.
$= \mathbf{18}$

a) If $y = 4(x + 3)$, find y when $x = 0$

$y = 4 \times (0 + 3) = 4 \times 3 = \boxed{12}$

b) If $y = 5(x - 2)$, find y when $x = 6$

$y = 5 \times (6 - 2) =$

c) If $y = -3(x - 6)$, find y when $x = 10$

$y = \qquad =$

d) If $y = -4(x + 8)$, find y when $x = 0$

$y = \qquad =$

e) If $y = x(x - 7)$, find y when $x = 9$

$y = \qquad =$

f) If $y = x(x + 2)$, find y when $x = 0$

$y = \qquad =$

g) If $y = (x + 1)(x - 3)$, find y when $x = -2$

$y = \qquad =$

h) If $y = (x - 1)(x + 5)$, find y when $x = 11$

$y = \qquad =$

i) If $c = 5$ and $d = 15$,
find the value of $c(d - 10)$

$c \times (d - 10) =$

$= 5 \times (15 - 10) = 5 \times 5 =$

j) If $x = 2$ and $y = 4$,
find the value of $y(x + 16)$

$y \times (x + 16) =$

$= \qquad =$

k) If $j = 2$ and $k = 1$,
find the value of $3j(2k - j)$

$= \qquad =$

l) If $a = 5$ and $b = 0$,
find the value of $4a(a - 3b)$

$= \qquad =$

m) Use $S = (n - 2) \times 180°$ to find the sum S of all interior angles when $n = 6$ (hexagon).

$=$

n) Use $TSA = \pi r(r + s)$ to find the total surface area TSA of a cone when $r = 2$, $s = 3$ and $\pi \approx 3.14$

$=$

Skill 17.7 Substituting negative values into rules and expressions.

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 **3** 3 4 4

- Replace the variables with the given values.
- Solve the mathematical sentence to find the value of the expression.
- Use the order of operations rules: Simplify within the brackets.
  Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (−) in order from left to right.
- Use the sign rules. (see skill 9.1, page 93)

Q. If $x = -2$ and $y = 3$,
find the value of $-3y - x$

A. $-3y - x =$ 3y = 3 × y
$= -3 \times y - x$ Substitute $x = -2$ and $y = 3$
$= -3 \times 3 - -2$ Multiply −3 by 3
− − = +
$= -9 + 2$ Add −9 and 2
$= \mathbf{-7}$

a) If $y = -x + 5$, find y when $x = -3$

$y = - -3 + 5 = 3 + 5$ $= \boxed{8}$
− − = +

b) If $y = -3 + x$, find y when $x = -6$

$y = -3 + - 6$ $= \boxed{}$
+ − = −

c) If $y = 8x$, find y when $x = -4$

$y =$ $= \boxed{}$

d) If $y = -3x$, find y when $x = -2$

$y =$ $= \boxed{}$

e) If $y = \frac{15}{x}$, find y when $x = -5$

$y =$ $= \boxed{}$

f) If $y = \frac{12}{x}$, find y when $x = -6$

$y =$ $= \boxed{}$

g) If $y = 2x - 5$, find y when $x = -3$

$y = 2 \times -3 - 5 = -6 - 5$ $= \boxed{}$

h) If $y = 3x - 4$, find y when $x = -1$

$y =$ $= \boxed{}$

i) If $m = -5$ and $n = 0$,
find the value of $2m - 3n$

$2 \times m - 3 \times n =$

$= 2 \times -5 - 3 \times 0 = -10 - 0$ $= \boxed{}$
+ × − = −

j) If $a = 6$ and $b = -2$,
find the value of $2b - 5a$

$=$ $= \boxed{}$

k) If $p = 2$ and $q = -10$,
find the value of $p(3p + q)$

$=$ $= \boxed{}$

l) If $y = 1$ and $z = -4$,
find the value of $8 - 3z + 2y$

$=$ $= \boxed{}$

Skill 17.8 Substituting into more complex rules and expressions.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 **3** 4 4

- Replace the variables with the given values.
- Solve the mathematical sentence to find the value of the expression.
- Use the order of operations rules: Simplify within the brackets.
  Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (–) in order from left to right.

Q. If $a = 2$, $b = -5$ and $c = 3$, find the value of $\frac{1}{2a}(3b - c)$

A. $\frac{1}{2a}(3b - c) =$ 3b = 3 × b

$= \frac{1}{2 \times a} \times (3 \times b - c)$ Substitute $a = 2, b = -5$ and $c = 3$

$= \frac{1}{2 \times 2} \times (3 \times -5 - 3)$ Evaluate the bracket

Multiply 2 by 2

$= \frac{1}{4} \times (-15 - 3)$

$= \frac{1}{4} \times -18$ Multiply the results

$=$

a) If $y = x^3 + 2$, find y when $x = 3$

$y = 3^3 + 2 = 27 + 2 \qquad = \boxed{29}$

b) If $y = x^3 - 100$, find y when $x = 5$

$y = \qquad = \boxed{}$

c) If $x = 5$ and $y = 2$, find the value of $\frac{x}{3} + \frac{y}{5}$

$\frac{5}{3} + \frac{2}{5} = \frac{25 + 6}{15} = \frac{31}{15} \qquad = \boxed{2\frac{1}{15}}$

d) If $a = 7$ and $b = 3$, find the value of $\frac{a}{5} - \frac{b}{7}$

$= \boxed{}$

e) If $y = \frac{3x - 5}{x}$, find y when $x = 5$

$y = \qquad = \boxed{}$

f) If $y = x^2(x + 2)$, find y when $x = -3$

$y = \qquad = \boxed{}$

g) If $a = 8$ and $b = -10$, find the value of $\frac{a}{4}(b - 12)$

$= \boxed{}$

h) If $x = -3$, $y = 3$ and $z = 6$, find the value of $\frac{9}{y}(yz + x)$

$= \boxed{}$

i) If $x = -4$, find the value of $\frac{x^2 - 3x}{2}$

$\frac{(-4)^2 - 3 \times -4}{2} = \frac{16 + 12}{2} = \frac{28}{2} \qquad = \boxed{}$

j) If $a = -4$ and $b = -10$, find the value of $a^2 + \frac{2b}{5}$

$= \boxed{}$

Skill 17.9 Substituting into quadratic rules.

- Replace the variable *x* with the given value.
- Solve the mathematical sentence to find the value of *y*.
- Use the order of operations rules: Multiply (×) and/or divide (÷) in order from left to right.
  Add (+) and/or subtract (–) in order from left to right.

Q. If $y = 3x^2 + x - 6$, find y when $x = -2$

A.

$y = 3x^2 + x - 6$

$= 3 \times x^2 + x - 6$ Substitute $x = -2$

$= 3 \times (-2)^2 + -2 - 6$ Evaluate $(-2)^2$

$= 3 \times 4 - 2 - 6$ Multiply 3 by 4

$= 12 - 2 - 6$ Subtract 2 and 6 from 12

$= \mathbf{4}$

a) If $y = x^2 + 2x$, find y when $x = 4$

$y = 4^2 + 2 \times 4 = 16 + 8$ = 24

b) If $y = x^2 + 3x$, find y when $x = 0$

$y =$ .... = ☐

c) If $y = x^2 - 3x + 2$, find y when $x = 1$

$y =$ .... = ☐

d) If $y = x^2 - 4x + 3$, find y when $x = 3$

$y =$ .... = ☐

e) If $y = x^2 + 6x - 5$, find y when $x = 2$

$y =$ .... = ☐

f) If $y = x^2 - 4x - 10$, find y when $x = 5$

$y =$ .... = ☐

g) If $y = 2x^2 - 3x + 1$, find y when $x = 1$

$y =$ .... = ☐

h) If $y = 3x^2 - 11x + 6$, find y when $x = 3$

$y =$ .... = ☐

i) If $y = 4x^2 + x - 7$, find y when $x = 2$

$y =$ .... = ☐

j) If $y = 5x^2 - 2x - 1$, find y when $x = 0$

$y =$ .... = ☐

k) If $y = 3x^2 - x + 4$, find y when $x = 3$

$y =$ .... = ☐

l) If $y = 2x^2 + 6x$, find y when $x = -2$

$y =$ .... = ☐

m) If $y = x^2 - 5x + 6$, find y when $x = -1$

$y =$ .... = ☐

n) If $y = x^2 - 16$, find y when $x = -4$

$y =$ .... = ☐

o) If $y = x^2 - 3x - 4$, find y when $x = -2$

$y =$ .... = ☐

p) If $y = x^2 + 2x - 9$, find y when $x = -3$

$y =$ .... = ☐

18. [Expansion]

Skill 18.1 Expanding brackets in expressions like $2(a + 1)$

Mauve **1 1** 2 2 3 3 4 4
Lime **1** 1 2 2 3 3 4 4

- Multiply the number outside the brackets by every term inside the brackets.
- Keep the sign from inside the brackets.

  Hint: Once you multiply across the brackets the multiplication sign can be left out.
  $2(a) = 2 \times a = 2a$

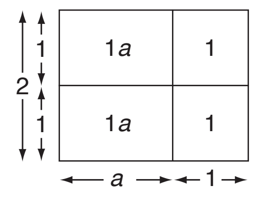
Expand the brackets ① ②

The × sign can be left out

$$2\,(a + 1) = 2 \times a + 2 \times 1$$

Keep the sign

$$= \underset{①}{2a} + \underset{②}{2}$$



Q. Expand $5(2 - b)$

A. $5(2 - b)$
$= 5 \times 2 - 5 \times b$
$= \mathbf{10 - 5b}$

a) Expand $3(4b - 5)$ Expand the brackets
Keep the sign
$= 3 \times 4b - 3 \times 5$ = $\boxed{12b - 15}$

b) Expand $2(z + 4)$
$= 2 \times z + 2 \times 4$ = ☐

c) Expand $3(5 + w)$
= .... = ☐

d) Expand $7(n - 2)$
= .... = ☐

e) Expand $9(4 - u)$
= .... = ☐

f) Expand $5(e - 8)$
= .... = ☐

g) Expand $8(1 + 2a)$
= .... = ☐

h) Expand $4(2g - 6)$
= .... = ☐

i) Expand $2(2k - 3)$
= .... = ☐

j) Expand $9(2h + 3)$
= .... = ☐

k) Expand $6(7 - 2c)$
= .... = ☐

l) Expand $8(4x - 5y + 3)$
= .... = ☐

m) Expand $3(4 - 6w + 4x)$
= .... = ☐

n) Expand $2(5 - 7d + 4e)$
= .... = ☐

Skill 18.2 Expanding brackets in expressions like $a(a + 1)$

- Multiply the variable outside the brackets by every term inside the brackets.
- Keep the sign from inside the brackets.

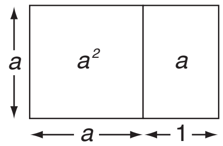
  Hint: Once you multiply across the brackets the multiplication sign can be left out.
  $a(a) = a \times a = a^2$

Expand the brackets

The × sign can be left out

$$a(a \overset{}{+} 1) = a \times a + a \times 1$$
$$= \underset{①}{a^2} + \underset{②}{a}$$

Keep the sign



Q. Expand $k(k - 6)$

A. $k(k - 6)$
$= k \times k - k \times 6$
$= \boldsymbol{k^2 - 6k}$

Expand the brackets

a) Expand $a(2 - 2a)$

$= a \times 2 - a \times 2a$ .... $= \boxed{2a - 2a^2}$

Keep the sign

b) Expand $e(e + 4)$

$= e \times e + e \times 4$ .... =

c) Expand $r(9 + r)$

= .... =

d) Expand $s(5 - s)$

= .... =

e) Expand $d(d + 3)$

= .... =

f) Expand $e(e - 7)$

= .... =

g) Expand $a(1 + 2a)$

= .... =

h) Expand $d(5d + 6)$

= .... =

i) Expand $p(4 + 2p)$

= .... =

j) Expand $z(6 - 6z)$

= .... =

k) Expand $c(2c - 3)$

= .... =

l) Expand $w(4 - 5w)$

= .... =

m) Expand $x(3x - 2y + 7)$

= .... =

n) Expand $t(u - 5 + 9t)$

= .... =

o) Expand $s(7t - 4s - 8)$

= .... =

p) Expand $e(f + 4 - 9e)$

= .... =

Skill 18.3 Expanding brackets in expressions like $2a(b+1)$

- Multiply the term outside the brackets by every term inside the brackets.
- Keep the sign from inside the brackets.

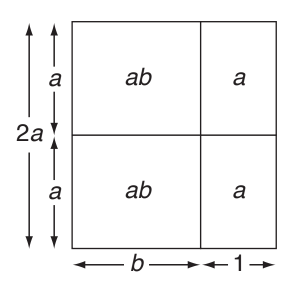
Hint: Once you multiply across the brackets the multiplication sign can be left out.
$2a(b) = 2 \times a \times b = 2ab$

Expand the brackets

The × sign can be left out

Keep the sign

$$\overset{\text{①}\ \ \text{②}}{2a\,(b + 1)} = 2a \times b + 2a \times 1$$
$$= \underset{①}{2ab} + \underset{②}{2a}$$



Q. Expand $2x(x - 7)$

A. $2x(x - 7)$
$= 2x \times x - 2x \times 7$
$= \mathbf{2x^2 - 14x}$

Expand the brackets

a) Expand $2d(3d + 6)$
Keep the sign
$= 2d \times 3d + 2d \times 6$ = $\boxed{6d^2 + 12d}$

b) Expand $3a(a - 5)$
$= 3a \times a - 3a \times 5$ = ☐

c) Expand $5s(2 - 4s)$
= .... = ☐

d) Expand $3y(4y - 3)$
= .... = ☐

e) Expand $3k(5 + 2k)$
= .... = ☐

f) Expand $5g(2g - 4)$
= .... = ☐

g) Expand $4d(2d + 3)$
= .... = ☐

h) Expand $3a(7 + 2a)$
= .... = ☐

i) Expand $9c(4 + 2c)$
= .... = ☐

j) Expand $6h(5h - 2)$
= .... = ☐

k) Expand $3e(7e + 8)$
= .... = ☐

l) Expand $4z(8 - 2z)$
= .... = ☐

m) Expand $2q(6 - 2r)$
= .... = ☐

n) Expand $4i(6j + 4)$
= .... = ☐

o) Expand $7p(4p + q)$
= .... = ☐

p) Expand $5n(m - 5n)$
= .... = ☐

Skill 18.4 Expanding brackets in expressions like $-2a(b+1)$

Mauve 1 1 2 2 **3** 3 4 4
Lime 1 1 **2** 2 3 3 4 4

- Multiply the negative term outside the brackets by every term inside the brackets.
- Use the sign rules. (see skill 9.1, page 93)

  Hint: Once you multiply across the brackets the multiplication sign can be left out.
  $-a(b) = -a \times b = -ab$

Expand the brackets

The × sign can be left out

Use the sign rules

$$-2a\,(\overset{①}{b} + \overset{②}{1}) = -2a \times b + -2a \times 1$$
$$= \underset{①}{-2ab} - \underset{②}{2a}$$

Q. Expand $-2(x-4)$

A. $-2(x-4)$
$= -2 \times x - -2 \times 4$
$= \mathbf{-2x + 8}$

$- - = +$

Expand the brackets

a) Expand $-5m(m+4)$

$= -5m \times m + -5m \times 4 = \boxed{-5m^2 - 20m}$

$+ - = +$

b) Expand $-4(f+3)$

$= -4 \times f + -4 \times 3 =$

c) Expand $-(b+9)$

$=$ .... $=$

d) Expand $-3(r+6)$

$=$ .... $=$

e) Expand $-8a(a-2)$

$=$ .... $=$

f) Expand $-2w(3+4w)$

$=$ .... $=$

g) Expand $-7q(q+3)$

$=$ .... $=$

h) Expand $-6b(4-5b)$

$=$ .... $=$

i) Expand $-2cd(2-3d)$

$=$ .... $=$

j) Expand $-tu(5t+2u)$

$=$ .... $=$

k) Expand $-5jk(8-4j)$

$=$ .... $=$

l) Expand $-gh(7g-3h)$

$=$ .... $=$

m) Expand $-4i(6hi+2h)$

$=$ .... $=$

n) Expand $-9y(yz+2z)$

$=$ .... $=$

o) Expand $-2s(8st+3t)$

$=$ .... $=$

p) Expand $-3m(6mn-4n)$

$=$ .... $=$

Skill 18.5 Expanding and evaluating expressions.

Mauve 1 1 2 2 3 **3** **4** 4
Lime 1 1 2 **2** 3 3 4 4

- Multiply the term outside the brackets by every term inside the brackets.
- Group like terms. (see skills 16.3, page 169 and 16.4, page 170)
- Use the sign rules. (see skill 9.1, page 93)

Q. Expand and evaluate
$2(ef - 5) + 4(ef + 3)$

A. ① ② ① ②
$2(ef - 5) + 4(ef + 3)$ Expand the brackets
$= \underline{\underline{2ef}} - \underline{10} + \underline{\underline{4ef}} + \underline{12}$ Group like terms
$= 2ef + 4ef - 10 + 12$
$= \mathbf{6ef + 2}$

a) Expand and evaluate
$2(8c + 4) - 7c$ Expand the brackets
$= \underline{16c} + 8 \underline{- 7c}$ Group like terms
$= 16c - 7c + 8$ = $\boxed{9c + 8}$

b) Expand and evaluate
$3(2x + 1) + 4x$
$= 6x + 3 + 4x$
= .... = ☐

c) Expand and evaluate
$2(x + 1) - 4x$
= ....
= .... = ☐

d) Expand and evaluate
$4s + s(2s - 5)$
= ....
= .... = ☐

e) Expand and evaluate
$3p(q - 6) + 4p$
= ....
= .... = ☐

f) Expand and evaluate
$5z(y + 3) - 8z$
= ....
= .... = ☐

g) Expand and evaluate
$5(hi - 3) - 8(hi + 3)$
= ....
= .... = ☐

h) Expand and evaluate
$n(n - 5) + 3(2n + 7)$
= ....
= .... = ☐

i) Expand and evaluate
$6(de + 5) - 3(de - 2)$
= ....
= .... = ☐

j) Expand and evaluate
$w(w + 4) - 2(4w - 7)$
= ....
= .... = ☐

k) Expand and evaluate
$2b(b - 5) - 8(b - 5)$
= ....
= .... = ☐

l) Expand and evaluate
$a(bc + 4) - 3(2a + 5)$
= ....
= .... = ☐

Skill 18.6 Expanding and evaluating more complex expressions.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 **3** 3 4 4

- Multiply the term outside the brackets by every term inside the brackets.
- Group like terms. (see skills 16.3, page 169 and 16.4, page 170)
- Use the sign rules. (see skill 9.1, page 93)

Q. Expand and evaluate
$-2(t^2 - u) + 5t(t - 3)$

A. ① ② ① ②
$-2(t^2 - u) + 5t(t - 3)$ Expand the brackets
$= \underline{-2t^2} + 2u \underline{+ 5t^2} - 15t$ Group like terms
$= -2t^2 + 5t^2 + 2u - 15t$
$= \mathbf{3t^2 + 2u - 15t}$

a) Expand and evaluate
$-4a(a - 2) + 7(a^2 - b)$ Expand the brackets

$= \underline{-4a^2} + 8a \underline{+ 7a^2} - 7b$

$= -4a^2 + 7a^2 + 8a - 7b = \boxed{3a^2 + 8a - 7b}$

Group like terms

b) Expand and evaluate
$x(2x + 3) - 3(x + 7)$

$= 2x^2 + 3x - 3x - 21$

= .... = $\boxed{}$

c) Expand and evaluate
$3(2t - 4) + t(t - 2)$

= ....

= .... = $\boxed{}$

d) Expand and evaluate
$-2s(5s^2 + 3s) + (s - s^2)$

= ....

= .... = $\boxed{}$

e) Expand and evaluate
$tu(t - 1) + 8u(t^2 - t)$

= ....

= .... = $\boxed{}$

f) Expand and evaluate
$3e(f - e) + 8e(f^2 - e)$

= ....

= .... = $\boxed{}$

g) Expand and evaluate
$-6kl(k - 2) - 2l(2k^2 - 2k)$

= ....

= .... = $\boxed{}$

h) Expand and evaluate
$-5m(m - 1) + 6(m^2 - 1)$

= ....

= .... = $\boxed{}$

i) Expand and evaluate
$2pq(p - 6) - 3q(p^2 - 3p)$

= ....

= .... = $\boxed{}$

j) Expand and evaluate
$3(r^2 - 4) - 2r(r - 5)$

= ....

= .... = $\boxed{}$

k) Expand and evaluate
$-8y(xy - 1) + 4xy(x + 2y)$

= ....

= .... = $\boxed{}$

l) Expand and evaluate
$-3(q^2 + q) + 4q(q + 1)$

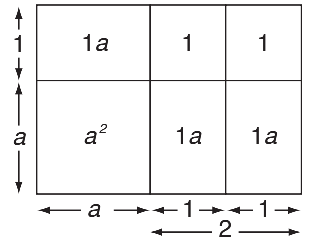
= ....

= .... = $\boxed{}$

Skill 18.7 Expanding brackets in expressions like $(a+1)(a+2)$

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Multiply each term inside the first set of brackets by each term inside the second set of brackets.
- Simplify the products.
- Group like terms. (see skills 16.3, page 169 and 16.4, page 170)
- Use the sign rules. (see skill 9.1, page 93)



Expand the brackets

$$(a+1)(a+2) = a\times a + a\times 2 + 1\times a + 1\times 2$$

$$= a^2 + 2a + a + 2$$

① ② ③ ④

Simplify the products

$$= a^2 + 3a + 2$$

Group like terms

Q. Expand and evaluate
$(w-3)(w-2)$

A. $(w-3)(w-2)$

Expand the brackets

$$= w\times w + w\times -2 + -3\times w + -3\times -2$$

$+ - = -$

Simplify the products

$$= w^2 - 2w - 3w + 6$$

Group like terms

$$= \mathbf{w^2 - 5w + 6}$$

a) Expand and evaluate
$(h-5)(h+2)$

$+ - = -$

$= h\times h + h\times 2 + -5\times h + -5\times 2$

$= h^2 + 2h - 5h - 10$ = $\boxed{h^2 - 3h - 10}$

b) Expand and evaluate
$(x+3)(x+1)$

$= x\times x + x\times 1 + 3\times x + 3\times 1$

= …… = $\boxed{}$

c) Expand and evaluate
$(w+4)(w-3)$

= ……

= …… = $\boxed{}$

d) Expand and evaluate
$(u+4)(5-u)$

= ……

= …… = $\boxed{}$

e) Expand
$(f-2)(g+8)$

= ……

= …… = $\boxed{}$

f) Expand
$(j-5)(k-3)$

= ……

= …… = $\boxed{}$

g) Expand and evaluate
$(2h-4)(h+5)$

= ……

= …… = $\boxed{}$

h) Expand and evaluate
$(r+6)(3r-7)$

= ……

= …… = $\boxed{}$

i) Expand and evaluate
$(3v+4)(v-9)$

= ……

= …… = $\boxed{}$

j) Expand and evaluate
$(y-2)(5y-6)$

= ……

= …… = $\boxed{}$

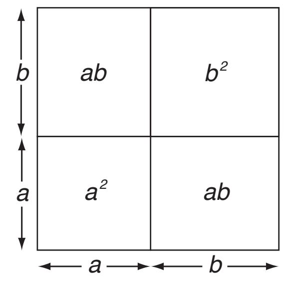
Skill 18.8 Expanding brackets in binomial squares like $(a+b)^2$

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Multiply each term inside the first set of brackets by each term inside the second set of brackets.
- Simplify the products.
- Group like terms. (see skills 16.3, page 169 and 16.4, page 170)

OR

- Substitute values into the binomial square formula $(a+b)^2 = a^2 + 2ab + b^2$ (see skill 17.9, page 181)



Expand the brackets

$$(a+b)^2 = (a+b)(a+b) = a \times a + a \times b + b \times a + b \times b$$

$$= \underset{①}{a^2} + \underset{②}{ab} + \underset{③}{ba} + \underset{④}{b^2}$$

Simplify the products

$$= a^2 + 2ab + b^2$$

Group like terms

Q. Expand and evaluate $(n+9)^2$

A. $(n+9)^2$
$= (n+9)(n+9)$
$= n \times n + n \times 9 + 9 \times n + 9 \times 9$
$= n^2 + 9n + 9n + 81$
$= \boldsymbol{n^2 + 18n + 81}$

OR

$(n+9)^2$
Using $a^2 + 2ab + b^2$
where $a = n$ *and* $b = 9$
$= n^2 + 2 \times n \times 9 + 9^2$
$= \boldsymbol{n^2 + 18n + 81}$

a) Expand and evaluate $(s+4)^2$

$a^2 + 2ab + b^2$ *where* $a = s$ *and* $b = 4$

$= s^2 + 2 \times s \times 4 + 4^2 \quad = \boxed{s^2 + 8s + 16}$

b) Expand and evaluate $(y+1)^2$

=

= =

c) Expand and evaluate $(h+2)^2$

=

= =

d) Expand and evaluate $(t+6)^2$

=

= =

e) Expand and evaluate $(p+7)^2$

=

= =

f) Expand and evaluate $(m+5)^2$

=

= =

g) Expand and evaluate $(a+3)^2$

=

= =

h) Expand and evaluate $(c+10)^2 - 75$

=

= =

i) Expand and evaluate $(r+8)^2 + 4$

=

= =

j) Expand and evaluate $(g+3)^2 - 3g$

=

= =

Skill 18.9 Expanding brackets in binomial squares like $(a-b)^2$

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 **4**

- Multiply each term inside the first set of brackets by each term inside the second set of brackets.
- Simplify the products.
- Group like terms. (see skills 16.3, page 169 and 16.4, page 170)

OR

- Substitute values into the perfect square formula $(a-b)^2 = a^2 - 2ab + b^2$ (see skill 17.9, page 181)

Expand the brackets

$$(a-b)^2 = (a-b)(a-b) = a\times a + a\times -b + -b\times a + -b\times -b$$

$$= a^2 \underbrace{- ab - ba}_{} + b^2$$

Simplify the products

① ② ③ ④

$$= a^2 - 2ab + b^2$$

Group like terms

Q. Expand and evaluate $(n-3)^2$

A. $(n-3)^2$

$= (n-3)(n-3)$

$= n\times n + n\times -3 + -3\times n + -3\times -3$

$= n^2 - 3n - 3n + 9$

$= \mathbf{n^2 - 6n + 9}$

OR

$(n-3)^2$

Using $a^2 - 2ab + b^2$

where $a = n$ *and* $b = 3$

$= n^2 - 2\times n\times 3 + 3^2$

$= \mathbf{n^2 - 6n + 9}$

a) Expand and evaluate $(s-4)^2$

$a^2 - 2ab + b^2$ *where* $a = s$ *and* $b = 4$

$= s^2 - 2\times s\times 4 + 4^2 = \boxed{s^2 - 8s + 16}$

b) Expand and evaluate $(k-1)^2$

=

= =

c) Expand and evaluate $(m-2)^2$

=

= =

d) Expand and evaluate $(q-5)^2$

=

= =

e) Expand and evaluate $(j-7)^2$

=

= =

f) Expand and evaluate $(e-9)^2$

=

= =

g) Expand and evaluate $(x-8)^2$

=

= =

h) Expand and evaluate $(x-10)^2 + 15x$

=

= =

i) Expand and evaluate $(z-6)^2 + 8$

=

= =

j) Expand and evaluate $(b-4)^2 - 3b$

=

= =

19. [Factorisation]

Skill 19.1 Factorising by finding the HCF of the coefficients.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Write the HCF in front of the brackets.
- Divide each term by the HCF to find the remaining factors.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factorise $15a - 24$

A. $15a$ and 24

$3 \times 5 = 15$ and $3 \times 8 = 24$

HCF of 15 and 24 is 3

$15a \div 3 = 5a$ and $24 \div 3 = 8$

Remaining factors are $5a$ and 8

$15a - 24 = \mathbf{3(5a - 8)}$

Write the HCF before the ( )

Keep the sign

a) Factorise $4k - 16 = \boxed{4(k-4)}$

$4 \times k = 4k$ and $4 \times 4 = 16 \Rightarrow$ HCF is 4

b) Factorise $4x + 8 =$

c) Factorise $6s + 18 =$

d) Factorise $3u - 15 =$

e) Factorise $9m - 24 =$

f) Factorise $14n + 21 =$

g) Factorise $2y + 10z =$

h) Factorise $4a - 12b =$

i) Factorise $6d + 14e =$

j) Factorise $16uv - 40 =$

k) Factorise $12k - 8l =$

l) Factorise $4g + 4h - 6 =$

m) Factorise $3m - 6n + 9 =$

n) Factorise $10v - 5w + 15 =$

o) Factorise $5h^2 - 10i + 25j =$

p) Factorise $6r^2 - 27s + 9t =$

Skill 19.2 Factorising by finding the HCF of coefficients and variables.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variables in each term.
- Write the HCF and any other CF in front of the brackets.
- Divide each term by any common factors to find the remaining factors.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factorise $18kl - 24k$

A. *18kl and 24k*

$6 \times 3 = 18$ *and* $6 \times 4 = 24$

HCF of 18 and 24 is 6

k is common to both terms CF is k

$18kl \div 6k = 3l$ *and* $24k \div 6k = 4$

Remaining factors are $\textcircled{3l}$ *and* $\textcircled{4}$

Write all CF's before the ( )

$18kl - 24k = \mathbf{6k(3l - 4)}$

Keep the sign

a) Factorise $ab + 5b$

ab = ba

b is common to both terms = $b(a + 5)$

b) Factorise $de + d$

.... =

c) Factorise $7e + ef$

.... =

d) Factorise $3st + 4s$

.... =

e) Factorise $8ab - 4b$

.... =

f) Factorise $15g + 20gh$

.... =

g) Factorise $wx - xy$

.... =

h) Factorise $2jk + 2kl$

.... =

i) Factorise $uv - 3vw$

.... =

j) Factorise $8ab + 4bc$

.... =

k) Factorise $12qr + 8rs$

.... =

l) Factorise $15de - 6ef$

.... =

m) Factorise $10cd - 8d$

.... =

n) Factorise $15m - 10mn$

.... =

o) Factorise $21qr + 14pq$

.... =

p) Factorise $6tu + 18uv$

.... =

q) Factorise $6xy + 9yz$

.... =

r) Factorise $10gh - 25gi$

.... =

Skill 19.3 Factorising to simplify expressions involving large numbers.

- Find the number repeating in both products.
- Write this number in front of the brackets.

  Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factorise and evaluate
$45 \times 7 + 45 \times 3$

A. $45 \times 7 + 45 \times 3$ 45 is repeating
$= 45 \times (7 + 3)$
$= 45 \times 10$
$= \mathbf{450}$

a) Factorise and evaluate
$99 \times 99 - 98 \times 99$

$= 99 \times (99 - 98)$

$= 99 \times 1 = \boxed{99}$

b) Factorise and evaluate
$15 \times 14 + 15 \times 6$

$= 15 \times (14 + 6)$

= .... = ☐

c) Factorise and evaluate
$987 \times 2 + 987 \times 8$

= ....

= .... = ☐

d) Factorise and evaluate
$40 \times 8 + 40 \times 12$

= ....

= .... = ☐

e) Factorise and evaluate
$23 \times 37 + 23 \times 63$

= ....

= .... = ☐

f) Factorise and evaluate
$25 \times 26 + 25 \times 24$

= ....

= .... = ☐

g) Factorise and evaluate
$999 \times 9 - 999 \times 8$

= ....

= .... = ☐

h) Factorise and evaluate
$87 \times 19 - 87 \times 9$

= ....

= .... = ☐

i) Factorise and evaluate
$-4 \times 14 - 4 \times 6$

Both terms are negative so CF is negative

$- + = -$

$= -4 \times (14 + 6)$

= .... = ☐

j) Factorise and evaluate
$-9 \times 33 - 9 \times 67$

= ....

= .... = ☐

Skill 19.4 Factorising involving squared terms.

Mauve 11 22 33 44
Lime 11 22 33 44

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variables in each term.
- Write the HCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factorise $2wx - 12w^2x$

A. $2wx$ and $12w^2x$

$2 \times 1 = 2$ and $2 \times 6 = 12$

HCF of 2 and 12 is 2

wx is common to both terms CF is wx

Remaining factors are 1 and $6w$

$2wx - 12w^2x = \mathbf{2wx(1 - 6w)}$

Write all CF's before the ( )

Keep the sign

a) Factorise $2j^2k + 5j$

$CF = j$ = $j(2jk + 5)$

Write all CF's outside the ( )

b) Factorise $e^2 + 7e$

=

c) Factorise $h + 4h^2$

=

d) Factorise $m^2 - 9m$

=

e) Factorise $3c - 12c^2$

=

f) Factorise $4f^2 + 6f$

=

g) Factorise $fg^2 + f$

=

h) Factorise $10b - 16ab^2$

=

i) Factorise $p^2q - 3p$

=

j) Factorise $12i - 18hi^2$

=

k) Factorise $14bc + 2b^2c$

=

l) Factorise $5r^2s - r^2t$

=

m) Factorise $vw + 7v^2 - 3vwx$

=

n) Factorise $8j^2 - 24jk + 12jl$

=

o) Factorise $f^3g^2 + fg^2$

=

p) Factorise $p^3q^2 + p^2q + pq$

=

Skill 19.5 Factorising negative terms.

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variables in each term.
- Write the HCF and any other CF in front of the brackets.
  Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factorise $-10r^2 - 5r$

A. $-10r^2$ and $5r$

$5 \times 2 = 10$ and $5 \times 1 = 5$

HCF of 10 and 5 is 5

r is common to both terms CF is r

$-$ is common to both terms CF is $-$

Remaining factors are r and 1

Write all CF before the ( )

$-10r^2 - 5r = \mathbf{-5r(r + 1)}$

"–" is common to both terms

a) Factorise $-7a - 21 = \boxed{-7(a + 3)}$

$7 \times 1 = 7$ and $7 \times 3 = 21 \Rightarrow$ HCF is 7

b) Factorise $-4k - 12 =$ ☐

$4 \times 1 = 4$ and $4 \times 3 = 12 \Rightarrow$ HCF is ……

c) Factorise $-6g - 15 =$ ☐

……………………

d) Factorise $-6e - 14 =$ ☐

……………………

e) Factorise $-2h^2 - 6h =$ ☐

……………………

f) Factorise $-8z^2 - 28z =$ ☐

……………………

g) Factorise $-12i^3 - 9ij =$ ☐

……………………

h) Factorise $-t^3 - 5t^2u =$ ☐

……………………

i) Factorise $-6bc^2 + 3c^2 =$ ☐

……………………

j) Factorise $-5x^2y^2 - xy^3 =$ ☐

……………………

k) Factorise
$-2x^3 - 4xy =$ ☐

……………………

l) Factorise
$-4m^3 - 12mn^2 + 18m =$ ☐

……………………

m) Factorise
$-2k^3 + 6k^3l + 8k =$ ☐

……………………

n) Factorise
$-2hi^3 + 3h^2i - 5h^2 =$ ☐

……………………

Skill 19.6 Factorising by finding binomial factors.

Mauve 11 22 33 44
Lime 11 22 33 44

- Find any common factors (CF).

  Hint: It might help to think of common factors that are expressions like (d + 2) as a blob ■.
- Write the CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factorise $2(r-1)-r(r-1)$

A. $2(r-1)-r(r-1)$ Consider (r − 1) = ■

$= 2■ - r■$ Keep the sign

$= ■(2-r)$

$= \mathbf{(r-1)(2-r)}$

a) Factorise

$d(d+2)+8(d+2)$ Consider (d + 2) = ■

$= d■+8■$ Keep the sign

$= ■(d+8)$ $= \boxed{(d+2)(d+8)}$

b) Factorise

$2(h-3)+h(h-3)$

$= 2■+h■$

= ……………… = ☐

c) Factorise

$5(x+4)+x(x+4)$

= ………………

= ……………… = ☐

d) Factorise

$b(b-7)+6(b-7)$

= ………………

= ……………… = ☐

e) Factorise

$a(a+2)-9(a+2)$

= ………………

= ……………… = ☐

f) Factorise

$z(z-5)-(z-5)$

= ………………

= ……………… = ☐

g) Factorise

$j(j+4)+j+4$

= ………………

= ……………… = ☐

h) Factorise

$m(n-2)+4(n-2)$

= ………………

= ……………… = ☐

i) Factorise

$3x(2x-5)-4(2x-5)$

= ………………

= ……………… = ☐

j) Factorise

$d(c+5)-(c+5)$

= ………………

= ……………… = ☐

k) Factorise

$q(s-3)+t(s-3)$

= ………………

= ……………… = ☐

l) Factorise

$6v(2w-1)+4(2w-1)$

= ………………

= ……………… = ☐

Skill 19.7 Factorising four terms by grouping 2 and 2.

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 **3** 3 **4** 4

- Begin factorising by grouping the 4 terms in 2 groups of 2.
- Take out the CF from the first group of 2 and write it in front of the brackets.
- Take out the CF from the second group of 2 and write it in front of the brackets.
- Keep the signs.
- Factorise again by finding the common binomial factor. (see skill 19.6, page 198)
  Hint: It might help to think of binomial factors that are expressions like (d + 2) as a blob ■.
- Take out the binomial factor or blob and write it in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the result by expanding the brackets.

Q. Factorise $m^2 + 3m + 5m + 15$

A. $m^2 + 3m + 5m + 15$

Keep the sign

$= m(m + 3) + 5(m + 3)$

$= m(■) + 5(■)$

Consider (m + 3) = ■

$= ■(m + 5)$

$= \mathbf{(m + 3)(m + 5)}$

a) Factorise
$c^2 + 8c + 3c + 24$

Group 2 and 2
Factorise each group

Factorise again
Consider (c + 8) = ■

$= c(c + 8) + 3(c + 8)$

$= ■\ (c + 3)$ $=$ $\boxed{(c + 8)(c + 3)}$

b) Factorise
$a^2 + 3a + 2a + 6$

$= a(a + 3) + 2(a + 3)$

= .... = $\boxed{\quad}$

c) Factorise
$s^2 + 6s + 5s + 30$

= ....

= .... = $\boxed{\quad}$

d) Factorise
$h^2 + 5h + 4h + 20$

= ....

= .... = $\boxed{\quad}$

e) Factorise
$v^2 + 7v + 3v + 21$

= ....

= .... = $\boxed{\quad}$

f) Factorise
$4n + n^2 + 16 + 4n$

= ....

= .... = $\boxed{\quad}$

g) Factorise
$6t + t^2 - 42 - 7t$

= ....

= .... = $\boxed{\quad}$

h) Factorise
$4b + 4 - b^2 - b$

= ....

= .... = $\boxed{\quad}$

i) Factorise
$5p - 10 + p^2 - 2p$

= ....

= .... = $\boxed{\quad}$

j) Factorise
$q^2 + 5q - 4q - 20$

= ....

= .... = $\boxed{\quad}$

Skill 19.8 Factorising using the difference of perfect squares.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find any common factors (CF) of the terms.
- Write any CF in front of the brackets.
- Use the difference of perfect squares formula.
- Check the result by expanding the brackets.

Expand the brackets

$$(a + b)(a - b) = a \times a + a \times -b + b \times a + b \times -b$$

$$= \underset{①}{a^2} - \underset{②}{ab} + \underset{③}{ba} - \underset{④}{b^2}$$

$$= a^2 - b^2$$

Group like terms

Q. Factorise $5w^2 - 20$

A. $5w^2 - 20$ Take out the CF of 5

$= 5(w^2 - 4)$ $4 = 2^2$

$= 5(w^2 - 2^2)$ Use $a^2 - b^2 = (a + b)(a - b)$ where $a = w$ and $b = 2$

$= \mathbf{5(w + 2)(w - 2)}$

a) Factorise $c^2 - 81$ $b^2 = 81$, $b = 9$

Use $a^2 - b^2 = (a + b)(a - b)$

$= c^2 - 9^2$ = $\boxed{(c + 9)(c - 9)}$

b) Factorise $y^2 - 4$

= .... = ☐

c) Factorise $d^2 - e^2$

= .... = ☐

d) Factorise $36 - h^2$

= .... = ☐

e) Factorise $4j^2 - 9$

= .... = ☐

f) Factorise $2c^2 - 50$

= .... = ☐

g) Factorise $p^2 - 81q^2$

= .... = ☐

h) Factorise $80 - 5y^2$

= .... = ☐

i) Factorise $9a^2 - 36b^2$

= .... = ☐

j) Factorise $75 - 3z^2$

= .... = ☐

k) Factorise $3d^2 - 27$

= .... = ☐

l) Factorise $100 - 4k^2$

= .... = ☐

Skill 19.9 Factorising quadratic trinomials.

Mauve 11 22 33 44
Lime 11 22 33 44

- Write two sets of brackets. Because x^2 can only be produced from $x \times x$, write the factors of the squared pronumeral in the brackets $(x \quad)(x \quad)$.
- Make a list of all pairs of factors, positive and negative, that produce the whole number.
- From this list determine which pair can be added to get the correct number of x terms.
- Write the result in the brackets with their signs.
- Check the result by expanding the brackets.

Q. Factorise
$x^2 - 9x + 8$

A. $x^2 - 9x + 8$

$= (x \quad)(x \quad)$

Write x in the brackets

List the factors of +8

$8 = 1 \times 8 = -1 \times -8 = 2 \times 4 = -2 \times -4$

Only $-1x$ and $-8x$ can make $-9x$

Determine the x terms

$= x^2 - 1x - 8x + 8$

$= \mathbf{(x - 1)(x - 8)}$

CHECK

$= (x - 1)(x - 8)$

Expand the brackets

$= x^2 - 1x - 8x + 8$

$= x^2 - 9x + 8 = \mathbf{(x - 1)(x - 8)}$ ✓

AND/OR consider

$= x^2 - 1x - 8x + 8$

Group 2 and 2

$= x(x - 1) - 8(x - 1)$

Factorise each group

$= x(■) - 8(■)$

$= ■(x - 8)$

Factorise again
Consider $(x - 1) = ■$

$= \mathbf{(x - 1)(x - 8)}$

a) Find the missing factor
$x^2 + 7x + 10$

$10 = 2 \times 5 = -2 \times -5$

$5x + 2x = 7x$ $= (x + 5)(x + 2)$

b) Find the missing factor
$d^2 - 4d + 4$

$4 = 2 \times 2 = -2 \times -2$

$= (d - 2)(\quad)$

c) Find the missing factor
$s^2 + 4s + 3$

$= (s + 3)(\quad)$

d) Find the missing factor
$g^2 + 8g + 15$

$= (g + 5)(\quad)$

e) Factorise
$m^2 + 2m - 24$

Which pair can be added to get +2m?

$-24 = -4 \times 6 = 4 \times -6$

$6m - 4m = 2m$ $=$

f) Factorise
$j^2 + 11j + 24$

$=$

g) Factorise
$y^2 + 5y + 4$

$=$

h) Factorise
$z^2 - 6z + 8$

$=$

i) Factorise
$c^2 - 6c + 5$

$=$

j) Factorise
$p^2 - 6p - 16$

$=$

20. [Equations]

Skill 20.1 Solving one-step equations by using the inverse operations of + and – (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Consider the operation used to construct the expression involving the variable.
- Perform the inverse operation on both sides of the equation.

Operation $+$	Inverse Operation $-$	**Operation** $-$	Inverse Operation $+$
$x + 3 = 6$ $x + 3 - 3 = 6 - 3$ $x = 3$		$x - 3 = 6$ $x - 3 + 3 = 6 + 3$ $x = 9$	

Q. Solve for x: $x - 7 = 4$

A. $x - 7 = 4$ Operation: – 7

$x \cancel{-7} \cancel{+7} = 4 + 7$ Inverse of – 7 is + 7

Simplify: – 7 + 7 = 0

$x = \mathbf{11}$

a) Solve for x: $x + 2 = 5$ Operation: + 2

$x + \cancel{2} - \cancel{2} = 5 - 2$

$x = \boxed{3}$

b) Solve for x: $x + 4 = 9$

$x + \cancel{4} - \cancel{4} = 9 - 4$

$x = \boxed{}$

c) Solve for x: $x + 6 = 9$

$x = \boxed{}$

d) Solve for x: $x + 4 = 2$

$x = \boxed{}$

e) Solve for x: $x + 7 = -3$

$\boxed{}$

f) Solve for x: $5 + x = 2$

$\boxed{}$

g) Solve for x: $x - 3 = 5$

$x \cancel{-3} \cancel{+3} = 5 + 3$

$x = \boxed{8}$

h) Solve for x: $x - 8 = 4$

$x \cancel{-8} \cancel{+8} = 4 + 8$

$\boxed{}$

i) Solve for x: $x - 7 = 9$

$\boxed{}$

j) Solve for x: $8 - x = 4$

$\boxed{}$

k) Solve for x: $x - 5 = -7$

$\boxed{}$

l) Solve for x: $6 - x = 9$

$\boxed{}$

Skill 20.1 Solving one-step equations by using the inverse operations of + and − (2).

m) Solve for x: $21 + x = 32$

n) Solve for x: $x - 4 = 9$

o) Solve for x: $x - 14 = 7$

p) Solve for x: $6 + x = 23$

q) Solve for x: $18 - x = 9$

r) Solve for x: $15 - x = 7$

s) Solve for x: $x + 12 = 21$

t) Solve for x: $x + 9 = 45$

u) Solve for x: $11 - x = 3$

v) Solve for x: $x - 2 = 14$

w) Solve for x: $x - 7 = 7$

x) Solve for x: $x - 9 = 12$

y) Solve for x: $13 - x = 8$

z) Solve for x: $x + 7 = 16$

A) Solve for x: $x + 11 = 19$

B) Solve for x: $x - 8 = 32$

C) Solve for x: $x - 12 = 8$

D) Solve for x: $x + 5 = 42$

Skill 20.2 Solving one-step equations by using the inverse operations of × and ÷ (1).

- Consider the operation used to construct the expression involving the variable.
- Perform the inverse operation on both sides of the equation.

Operation ×	Inverse Operation ÷	Operation ÷	Inverse Operation ×
$3x = 6$ $\frac{3x}{3} = \frac{6}{3}$ $x = 2$		$\frac{x}{3} = 6$ $\frac{x}{3} \times 3 = 6 \times 3$ $x = 18$	

Q. Solve for x: $\frac{x}{3} = 5$

A.

$$\frac{x}{\textcircled{3}} = 5$$

Operation: ÷ 3

$$\frac{x}{\cancel{3}} \times \cancel{3} = 5 \times 3$$

Inverse of ÷ 3 is × 3

$$x = \mathbf{15}$$

a) Solve for x: $\frac{x}{7} = 4$

Operation: ÷ 7

$$\frac{x}{\cancel{7}} \times \cancel{7} = 4 \times 7$$

Inverse of ÷ 7 is × 7

$x = \boxed{28}$

b) Solve for x: $\frac{x}{3} = 3$

$x = \boxed{}$

c) Solve for x: $\frac{x}{2} = 3$

$x = \boxed{}$

d) Solve for x: $4x = 16$

Operation: × 4

$$\frac{{}^{1}\cancel{4}x}{{}_{1}\cancel{4}} = \frac{\cancel{16}^{4}}{\cancel{4}_{1}}$$

Inverse of × 4 is ÷ 4

Simplify: ÷ 4

$x = \boxed{4}$

e) Solve for x: $3x = 12$

$x = \boxed{}$

f) Solve for x: $2x = 14$

$\boxed{}$

g) Solve for x: $\frac{x}{2} = 6$

$x = \boxed{}$

h) Solve for x: $\frac{x}{5} = 2$

$\boxed{}$

i) Solve for x: $\frac{x}{8} = 6$

$\boxed{}$

j) Solve for x: $3x = 27$

$\boxed{}$

k) Solve for x: $4x = 28$

$\boxed{}$

l) Solve for x: $5x = 45$

$\boxed{}$

Skill 20.2 Solving one-step equations by using the inverse operations of × and ÷ (2).

m) Solve for x: $\frac{x}{4} = 10$

$x =$

n) Solve for x: $\frac{x}{6} = 7$

o) Solve for x: $6x = 72$

p) Solve for x: $\frac{x}{8} = 5$

$x =$

q) Solve for x: $\frac{x}{9} = 11$

r) Solve for x: $7x = 140$

s) Solve for x: $\frac{x}{7} = 7$

$x =$

t) Solve for x: $\frac{x}{10} = 12$

u) Solve for x: $\frac{x}{5} = 6$

v) Solve for x: $2x = 34$

w) Solve for x: $5x = 250$

x) Solve for x: $7x = 70$

y) Solve for x: $\frac{x}{9} = 20$

$x =$

z) Solve for x: $\frac{x}{12} = 2$

A) Solve for x: $4x = 32$

B) Solve for x: $3x = 30$

C) Solve for x: $9x = 54$

D) Solve for x: $8x = 48$

Skill 20.3 Solving two-step equations by using the inverse operations of +, –, × and ÷ (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- To isolate the variable (x) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x: $5+\frac{2x}{3}=1$

A.

$$5+\frac{2x}{3}=1$$

$$5-5+\frac{2x}{3}=1-5$$

Inverse of + 5 is – 5

$$\frac{2x}{\cancel{3}}\times\cancel{3}=-4\times 3$$

Inverse of ÷ 3 is × 3

$$\frac{2x}{2}=\frac{-12}{2}$$

Inverse of × 2 is ÷ 2

$$x=\mathbf{-6}$$

a) Solve for x: $4x - 1 = 11$

Inverse of – 1 is + 1

$4x - 1 + 1 = 11 + 1$

$4x = 12$

Inverse of × 4 is ÷ 4

$\frac{{}^{1}\cancel{4}x}{{}_{1}\cancel{4}} = \frac{\cancel{12}^{3}}{\cancel{4}_{1}}$

$x = \boxed{3}$

b) Solve for x: $7 + 3x = 22$

$7 - 7 + 3x = 22 - 7$

$3x =$

$=$

$x = \boxed{}$

c) Solve for x: $2x + 7 = -3$

$=$

$=$

$=$

$x = \boxed{}$

d) Solve for x: $5x - 1 = 24$

$=$

$=$

$=$

$x = \boxed{}$

e) Solve for x: $15 + 10x = 45$

$=$

$=$

$=$

$x = \boxed{}$

f) Solve for x: $3x + 12 = 3$

$=$

$=$

$=$

$x = \boxed{}$

g) Solve for x: $\frac{x}{4}+3=5$

$\frac{x}{4} + 3 - 3 = 5 - 3$

$=$

$=$

$x = \boxed{}$

h) Solve for x: $\frac{x}{5}+3=1$

$=$

$=$

$=$

$x = \boxed{}$

i) Solve for x: $\frac{5x}{2}-3=-1$

$=$

$=$

$=$

$x = \boxed{}$

Skill 20.3 Solving two-step equations by using the inverse operations of + , – , × and ÷ (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

j) Solve for x: $2x - 8 = 14$

=

=

=

x = ☐

k) Solve for x: $12 + 4x = 20$

=

=

=

x = ☐

l) Solve for x: $3x - 6 = 9$

=

=

=

x = ☐

m) Solve for x: $2x + 7 = -3$

=

=

=

x = ☐

n) Solve for x: $6 + 5x = 1$

=

=

=

x = ☐

o) Solve for x: $2x + 3 = 11$

=

=

=

x = ☐

p) Solve for x: $6x - 5 = 0$

=

=

=

x = ☐

q) Solve for x: $5 + 8x = 1$

=

=

=

x = ☐

r) Solve for x: $\frac{3x}{7} + 4 = 1$

=

=

=

x = ☐

s) Solve for x: $\frac{x}{2} - 1 = 3$

=

=

=

x = ☐

t) Solve for x: $\frac{x}{5} + 6 = 1$

=

=

=

x = ☐

u) Solve for x: $3 - \frac{x}{3} = 6$

=

=

=

x = ☐

Skill 20.4 Solving equations by first expanding the brackets (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Expand the brackets.
- To isolate the variable (*x*) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x: $6(2 - x) = -18$

A.

$6(2 - x) = -18$ Expand the ( )

$12 - 6x = -18$

$12 - 12 - 6x = -18 - 12$ Inverse of + 12 is − 12

$-6x = -30$

$\frac{-6x}{-6} = \frac{-30}{-6}$ Inverse of × −6 is ÷ −6

$x = \mathbf{5}$

a) Solve for x: $3(x - 2) = 12$

Expand the ( )

$3x - 6 = 12$

Inverse of − 6 is + 6

$3x - 6 + 6 = 12 + 6$

Inverse of × 3 is ÷ 3

$\frac{{}^{1}\cancel{3}x}{\cancel{3}_{1}} = \frac{\cancel{12}^{4}}{\cancel{3}_{1}}$

$x = \boxed{4}$

b) Solve for x: $3(2 + x) = 21$

=

=

=

$x =$ ☐

c) Solve for x: $2(x - 3) = 14$

=

=

=

$x =$ ☐

d) Solve for x: $5(1 + x) = 20$

=

=

=

$x =$ ☐

e) Solve for x: $7(2 + x) = 35$

=

=

=

$x =$ ☐

f) Solve for x: $4(x - 3) = 4$

=

=

=

$x =$ ☐

g) Solve for x: $4(x - 5) = 8$

=

=

=

$x =$ ☐

h) Solve for x: $2(9 - x) = 8$

=

=

=

$x =$ ☐

i) Solve for x: $3(2x - 3) = 15$

=

=

=

$x =$ ☐

Skill 20.4 Solving equations by first expanding the brackets (2).

Mauve 1 1 2 **2** 3 3 4 4
Lime 1 1 2 2 3 3 4 4

j) Solve for x: $2(x - 6) = 10$

=

=

=

x = ☐

k) Solve for x: $6(3 - x) = 18$

=

=

=

x = ☐

l) Solve for x: $3(x + 8) = 30$

=

=

=

x = ☐

m) Solve for x: $8(2 + x) = 88$

=

=

=

x = ☐

n) Solve for x: $7(x - 1) = 21$

=

=

=

x = ☐

o) Solve for x: $4(5 - x) = 16$

=

=

=

x = ☐

p) Solve for x: $5(x + 7) = 45$

=

=

=

x = ☐

q) Solve for x: $9(3 + x) = 36$

=

=

=

x = ☐

r) Solve for x: $3(x - 5) = 24$

=

=

=

x = ☐

s) Solve for x: $2(8 - x) = 12$

=

=

=

x = ☐

t) Solve for x: $4(x + 3) = 20$

=

=

=

x = ☐

u) Solve for x: $5(7 + x) = 35$

=

=

=

x = ☐

Skill 20.5 Solving equations with variables in more than one place (1).

- If necessary, expand the brackets. (see skill 20.4, page 209)
- Combine all variables on one side of the equation by using inverse operations.
- To isolate the variable (*x*) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x: $x = 3x + 12$

A.

$$x = 3x + 12$$

Combine x's: −3x

$$x - 3x = 3x - 3x + 12$$

$$-2x = 12$$

$$\frac{-2x}{-2} = \frac{12}{-2}$$

Inverse of × −2 is ÷ −2

$$x = \mathbf{-6}$$

a) Solve for x: $6 - 3x = 3x$

$6 - 6 - 3x = 3x - 6$

$-3x - 3x = 3x - 3x - 6$

$\frac{{}^{1}\cancel{-6}x}{{}_{1}\cancel{-6}} = \frac{\cancel{-6}^{1}}{\cancel{-6}_{1}}$

Inverse of × −6 is ÷ −6

$x = \boxed{1}$

b) Solve for x: $6x + 4 = 8x$

=

=

=

$x =$ ☐

c) Solve for x: $25 - 4x = x$

=

=

=

$x =$ ☐

d) Solve for x: $6x - 4 = 5x$

=

=

=

$x =$ ☐

e) Solve for x: $15 - 3x = 2x$

=

=

=

$x =$ ☐

f) Solve for x: $7x - 24 = 4x$

=

=

=

$x =$ ☐

g) Solve for x: $5x = 2x - 6$

=

=

=

$x =$ ☐

h) Solve for x: $3x = 21 - 4x$

=

=

=

$x =$ ☐

i) Solve for x: $8x = 3x - 15$

=

=

=

$x =$ ☐

Skill 20.5 Solving equations with variables in more than one place (2).

Mauve 1 1 2 2 **3** 3 4 4
Lime 1 1 **2** 2 3 3 4 4

j) Solve for x:

$4x + 2(3x - 4) = 22$

Expand the ( )

Combine the variables

$4x + 6x - 8 = 22$

$10x - 8 + 8 = 22 + 8$

$$\frac{{}^{1}\cancel{10}x}{{}_{1}\cancel{10}} = \frac{\cancel{30}^{3}}{\cancel{10}_{1}}$$

$x = \boxed{3}$

k) Solve for x:

$2x + 3(4x - 3) = 19$

$x =$

l) Solve for x:

$2(x - 3) - 3x = -12$

$x =$

m) Solve for x:

$3x + 5(2 - 3x) = 10$

$x =$

n) Solve for x:

$x + 4(3 - 2x) = 5$

$x =$

o) Solve for x:

$5x + 2(x - 8) = 5$

$x =$

p) Solve for x:

$5(x - 4) = 3x$

$x =$

q) Solve for x:

$6(x - 7) = -x$

$x =$

r) Solve for x:

$3(x - 8) = 5x$

$x =$

s) Solve for x:

$2(4x - 10) = 3(x + 5)$

$x =$

t) Solve for x:

$3(2x + 4) = 4(2x - 1)$

$x =$

u) Solve for x:

$5(2x - 6) = 2(3x + 1)$

$x =$

Skill 20.6 Solving equations involving algebraic fractions (1).

- Use inverse operations rules to isolate any algebraic fractions.
- Rewrite all expressions as fractions if necessary.
- Cross multiply. (see skill 10.11, page 109)
- Combine all variables on one side of the equation by using inverse operations. (see skill 20.5, page 211)
- To isolate the variable (*x*) perform the inverse operations, in order, to both sides of the equation.

Q. Solve for x: $\frac{x}{3} = x + 4$

A.

$$\frac{x}{3} = x + 4$$

$$\frac{x}{3} \times \frac{x+4}{1}$$

Cross multiply

$$x = 3(x + 4)$$

$$x = 3x + 12$$

$$x - 3x = 3x - 3x + 12$$

Combine x's: −3x

$$-2x = 12$$

$$\frac{-2x}{-2} = \frac{12}{-2}$$

Inverse of × −2 is ÷ −2

$$x = \mathbf{-6}$$

a) Solve for x: $\frac{x}{4} - 10 = -x$

Isolate the fraction

$$\frac{x}{4} - 10 + 10 = -x + 10$$

$$\frac{x}{4} \times \frac{-x+10}{1}$$

Rewrite expression as fraction

$$x = 4(-x + 10)$$

$$x + 4x = -4x + 4x + 40$$

$$5x = 40$$

$x = \boxed{}$

b) Solve for x: $\frac{18}{x} = 2$

= ……………………

= ……………………

= ……………………

$x = \boxed{}$

c) Solve for x: $\frac{6}{x} = \frac{3}{10}$

= ……………………

= ……………………

= ……………………

= ……………………

$x = \boxed{}$

d) Solve for x: $\frac{10}{x} = 5$

= ……………………

= ……………………

= ……………………

= ……………………

$x = \boxed{}$

e) Solve for x: $\frac{12}{x} = 3$

= ……………………

= ……………………

= ……………………

= ……………………

$x = \boxed{}$

f) Solve for x: $\frac{4}{x} = \frac{2}{7}$

= ……………………

= ……………………

= ……………………

= ……………………

$x = \boxed{}$

Skill 20.6 Solving equations involving algebraic fractions (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

g) Solve for x: $\frac{20-2x}{3}=2$

=

=

=

=

$x =$ ☐

h) Solve for x: $\frac{3x-2}{5}=8$

=

=

=

=

$x =$ ☐

i) Solve for x: $\frac{5x-1}{3}=3$

=

=

=

=

$x =$ ☐

j) Solve for x: $\frac{2x}{5}=x-3$

=

=

=

=

$x =$ ☐

k) Solve for x: $8-x=\frac{2x}{5}$

=

=

=

=

$x =$ ☐

l) Solve for x: $\frac{2x}{3}+10=4x$

=

=

=

=

$x =$ ☐

m) Solve for x: $\frac{x-2}{4}=\frac{x+6}{5}$

=

=

=

=

$x =$ ☐

n) Solve for x: $\frac{x+4}{3}=\frac{10-x}{4}$

=

=

=

=

$x =$ ☐

o) Solve for x: $\frac{x+3}{3}-\frac{x-2}{5}=3$

=

=

=

=

$x =$ ☐

Skill 20.7 Solving inequations (1).

- Manipulate the inequation in the same way as you would an equation.

EXCEPT:

- When both sides are multiplied or divided by a negative number, reverse the inequality signs. < becomes > and ≤ becomes ≥.

Q. Solve the inequation:
$4x - 7 \leq 5$

A.

$$4x - 7 \leq 5$$

$$4x \cancel{-7} \cancel{+7} \leq 5 + 7$$

$$\frac{\cancel{4}x}{\cancel{4}} \leq \frac{\cancel{12}^{3}}{\cancel{4}_{1}}$$

$$x \leq \mathbf{3}$$

a) Solve the inequation:
$20 \geq 5(7 - 2x) - 35$

$$20 \geq \cancel{35} - 10x - \cancel{35}$$

$$\cancel{20} - \cancel{20} + 10x \geq -\cancel{10}x + \cancel{10}x - 20$$

$$10x \geq -20$$

$$\frac{10x}{10} \geq -\frac{20}{10}$$

$$\boxed{x \geq -2}$$

b) Solve the inequation:
$3x - 8 < 7$

$<$

$<$

$<$

$<$

$$\boxed{x <}$$

c) Solve the inequation:
$2x + 6 \leq 10$

$\leq$

$\leq$

$\leq$

$\leq$

d) Solve the inequation:
$2x - 9 \leq 7$

$\leq$

e) Solve the inequation:
$5x - 1 > 12$

$>$

f) Solve the inequation:
$3x + 8 \leq 2$

$\leq$

g) Solve the inequation:
$\frac{x}{4} + 3 \geq 6$

$\geq$

h) Solve the inequation:
$\frac{x}{3} - 2 < 9$

$<$

i) Solve the inequation:
$\frac{x}{6} - 2 \geq 5$

$\geq$

Skill 20.7 Solving inequations (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

j) Solve the inequation:
$12 - x > 2(x + 3)$

$$12 - x > 2x + 6$$

$$\cancel{12} - \cancel{12} - x > 2x + 6 - 12$$

$$- x - 2x > \cancel{2x} - \cancel{2x} - 6$$

$$- 3x > - 6$$

Both sides negative so reverse inequality sign

$$\frac{-\cancel{3}x}{\cancel{-3}} < \frac{-6}{-3} \quad \boxed{x < 2}$$

k) Solve the inequation:
$-5(x + 7) \geq 10$

l) Solve the inequation:
$4 < 2(3 - 2x) - 10$

m) Solve the inequation:
$6(3 - 2x) > -6$

n) Solve the inequation:
$5(3x - 1) - 12 \geq 13$

o) Solve the inequation:
$29 \leq 4(3 - 4x) - 15$

p) Solve the inequation:
$\frac{3(x+4)}{2} > 15$

q) Solve the inequation:
$\frac{4(x+1)}{4} \geq 10$

r) Solve the inequation:
$\frac{4x}{3} - x > -1$

s) Solve the inequation:
$\frac{5(x-2)}{6} > 3$

t) Solve the inequation:
$\frac{3(x+6)}{5} \leq 1$

u) Solve the inequation:
$\frac{4x}{7} - x > 27$

Skill 20.8 Solving quadratic equations (1).

- Make either factor equal zero. Use the zero multiplication property.
  $a \times 0 = 0$ and $0 \times a = 0$
  Hint: A quadratic equation can have 2 solutions, 1 solution or no solution.

Q. Solve for x:
$(x - 8)(x + 9) = 0$

If either $(x - 8) = 0$ or $(x + 9) = 0$ then $(x - 8)(x + 9) = 0$

A. $(x - 8)(x + 9) = 0$
$x - 8 + 8 = 0 + 8$
$x = 8$
OR
$x + 9 - 9 = 0 - 9$
$x = -9$
8, −9

Check:
If $x = 8$
$(8 - 8)(8 + 9) = 0$
$0 \times 17 = 0$ *is true*
If $x = -9$
$(-9 - 8)(-9 + 9) = 0$
$-17 \times 0 = 0$ *is true*

a) Solve for x:
$(x - 6)(x - 5) = 0$

Make $(x - 6) = 0$

Make $(x - 5) = 0$

If $x - 6 = 0$*, then*
$x - 6 + 6 = 0 + 6$
$x = 6$

If $x - 5 = 0$*, then*
$x - 5 + 5 = 0 + 5$
$x = 5$

6, 5

b) Solve for x:
$(x + 7)(x - 2) = 0$

If $x + 7 = 0$*, then*

If $x - 2 = 0$*, then*

c) Solve for x:
$(x - 2)(x + 9) = 0$

d) Solve for x:
$(x + 3)(x + 4) = 0$

e) Solve for x:
$(x - 4)(x + 7) = 0$

f) Solve for x:
$(x + 1)(x + 9) = 0$

g) Solve for x:
$x(x - 8) = 0$

h) Solve for x:
$x(x + 3) = 0$

Skill 20.8 Solving quadratic equations (2).

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 **3** 4 4

i) Solve for x:
$x(x - 4) = 0$

j) Solve for x:
$x(x + 11) = 0$

k) Solve for x:
$(x + 2)(x - 10) = 0$

l) Solve for x:
$(x - 6)(x - 4) = 0$

m) Solve for x:
$(x - 5)(x + 3) = 0$

n) Solve for x:
$(x + 7)(x + 8) = 0$

o) Solve for x:
$(x + 4)(x - 1) = 0$

p) Solve for x:
$(x - 9)(x - 3) = 0$

q) Solve for x:
$(x - 7)(x + 6) = 0$

r) Solve for x:
$(x + 5)(x + 2) = 0$

Skill 20.9 Solving simultaneous equations (1).

Mauve 11 22 33 44
Lime 11 22 33 **4**4

EITHER
- Find the value of one of the variables in relation to the other.
- Substitute this value of the variable into the other equation.
- Solve for one variable.
- Substitute the result into either equation to find the second variable.

OR
- Add or subtract the equations together to eliminate one of the variables.

Q. Solve the simultaneous equations:
$2x + 3y = 3$
$x + 3y = 6$

A. $2x + 3y = 3$ (1)
$x + 3y = 6$ (2)
Eliminate ‘y’ by subtracting (1) – (2)
$x = -3$
$-3 + 3y = 6$
Substitute x = –3 into (2)
$-3 + 3 + 3y = 6 + 3$
$3y = 9$
$$\frac{3y}{3} = \frac{9}{3}$$
$y = 3$
(–3, 3)

a) Solve the simultaneous equations:
$y = 3x - 9$
$x = 4$
Substitute x = 4 into (1)

$y = 3 \times 4 - 9$

$y = 12 - 9$

$y = 3$

(4,3)

b) Solve the simultaneous equations:
$x + y = 5$
$y = x + 1$
Substitute y = x + 1 into (1)

c) Solve the simultaneous equations:
$y = 2x + 1$
$y = 3x - 2$

d) Solve the simultaneous equations:
$4 = 2x + y$
$x - 5 = y$

e) Solve the simultaneous equations:
$x + y = 1$
$x - y = 3$

f) Solve the simultaneous equations:
$x - y = 2$
$3x + y = 14$

Skill 20.9 Solving simultaneous equations (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 **4** 4

g) Solve the simultaneous equations:
$x = -3$
$y = 2x + 1$

h) Solve the simultaneous equations:
$y = -x$
$y = 2x - 6$

i) Solve the simultaneous equations:
$x + y = 5$
$2x - y = 10$

j) Solve the simultaneous equations:
$x - y = 2$
$2x + 3y = 9$

k) Solve the simultaneous equations:
$y = -x + 2$
$y = 2x - 4$

l) Solve the simultaneous equations:
$y = x - 4$
$3y = x - 6$

m) Solve the simultaneous equations:
$x + y = 8$
$4x - y = 7$

n) Solve the simultaneous equations:
$x - y = 4$
$x + 3y = 12$

Skill 20.10 Solving quadratic equations by factorising (1).

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 3 4 **4**

- Factorise the expression. (see skills 19.4, page 196 and 19.7, page 199)
- Make either factor equal zero. Use the zero multiplication property.
  $a \times 0 = 0$ and $0 \times a = 0$

Q. Solve for x:
$x^2 + 7x = 0$

A. $x^2 + 7x = 0$

Factorise

$x(x + 7) = 0$

$x = 0$

If either
x = 0
or (x + 7) = 0
then $x^2 + 7x = 0$

OR

$x + 7 - 7 = 0 - 7$

$x = -7$

0, −7

a) Solve for x:
$x^2 - 16 = 0$

Factorise

If either
(x + 4) = 0
or (x − 4) = 0
then $x^2 - 16 = 0$

(x + 4)(x − 4) = 0 ....

so x = −4 or x = 4 ....

−4, 4

b) Solve for x:
$x^2 - 4 = 0$

....

....

c) Solve for x:
$x^2 - 2x = 0$

....

....

d) Solve for x:
$x^2 - 3x = 0$

....

....

e) Solve for x:
$x^2 + 4x = 0$

....

....

f) Solve for x:
$x^2 + 5x = 0$

....

....

g) Solve for x:
$x^2 - 64 = 0$

....

....

h) Solve for x:
$x^2 - 144 = 0$

....

....

Skill 20.10 Solving quadratic equations by factorising (2).

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 3 4 **4**

i) Solve for x:
$x^2 - 25 = 0$

j) Solve for x:
$x^2 - 6x = 0$

k) Solve for x:
$x^2 - 11x = 0$

l) Solve for x:
$x^2 - 81 = 0$

m) Solve for x:
$x^2 - 100 = 0$

n) Solve for x:
$x^2 + 7x = 0$

o) Solve for x:
$x^2 - 36 = 0$

p) Solve for x:
$x^2 + 9x = 0$

q) Solve for x:
$x^2 - 15x = 0$

r) Solve for x:
$x^2 - 121 = 0$

21. [Coordinate Geometry]

Skill 21.1 Completing a table of values for a linear rule.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Substitute the variable *x* with the given values.
- Calculate the value of *y*.
- Use the order of operations rules: Multiply (×) and/or divide (÷) from left to right.
  Add (+) and/or subtract (–) from left to right.
- Use the sign rules. (see skill 9.1, page 93)

Q. Complete this table of values for the linear rule $y = -4x + 2$

x	$y = -4x + 2$	y	(x , y)
-1	$y = -4 \times (-1) + 2$	6	$(-1 , 6)$
0			
1			

A. $y = -4x + 2$ 4x = 4 × x

$y = -4 \times x + 2$ Substitute $x = 0$

$y = -4 \times 0 + 2$ Multiply -4 by 0

$y = 0 + 2$ Add 0 to 2

$y = 2$

⇒

x	$y = -4x + 2$	y	(x , y)
-1	$y = -4 \times (-1) + 2$	6	$(-1 , 6)$
0	$\boldsymbol{y = -4 \times 0 + 2}$	$\boldsymbol{2}$	$\boldsymbol{(0 , 2)}$
1	$\boldsymbol{y = -4 \times 1 + 2}$	$\boldsymbol{-2}$	$\boldsymbol{(1 , -2)}$

a) Complete this table of values for the linear rule $y = x - 1$

x	$y = x - 1$	y	(x , y)
-2	$y = -2 - 1$	-3	$(-2 , -3)$
-1	$y = -1 - 1$	-2	$(-1 , -2)$
0	$y = 0 - 1$	-1	
1			
2			

b) Complete this table of values for the linear rule $y = 6x$

x	$y = 6x$	y	(x , y)
-2	$y = 6 \times (-2)$	-12	$(-2 , -12)$
-1			
0			
1			
2			

c) Complete this table of values for the linear rule $y = x + 7$

x	$y = x + 7$	y	(x , y)
-7	$y = -7 + 7$	0	$(-7 , 0)$
-2			
0			
2			
7			

d) Complete this table of values for the linear rule $y = x - 4$

x	$y = x - 4$	y	(x , y)
-4	$y = -4 - 4$	-8	$(-4 , -8)$
-2			
0			
2			
4			

e) Complete this table of values for the linear rule $y = -x - 3$

x	$y = -x - 3$	y	(x , y)
-3	$y = -(-3) - 3$	0	$(-3 , 0)$
-1			
0			
1			
3			

f) Complete this table of values for the linear rule $y = -5x + 1$

x	$y = -5x + 1$	y	(x , y)
-2	$y = -5 \times (-2) + 1$	11	$(-2 , 11)$
-1			
0			
1			
2			

Skill 21.2 Graphing lines of equations x = constant and y = constant on a Cartesian plane (e.g. $x = 1$, $y = 2$).

Mauve 11 22 33 44
Lime 11 22 33 44

- Complete the table of values for the equation. (see skill 21.1, page 223)
- Plot each point on the Cartesian plane.
- Draw the line that joins these points.
- Label the line with the rule.

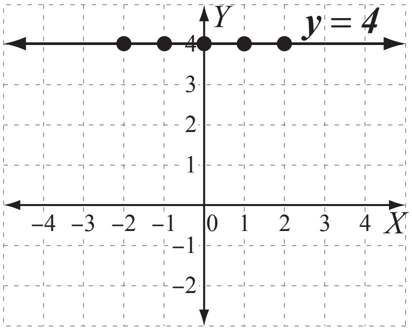
Hints: The lines of equation x = constant are vertical lines.
The lines of equation y = constant are horizontal lines.

Q. Graph the line of equation $y = 4$ by first completing this table of values.
[Label the line with the rule.]

x	−2	−1	0	1	2
y	4				
(x,y)	(−2,4)	(,)	(,)	(,)	(,)

A.

x	−2	−1	0	1	2
y	4	4	4	4	4
(x,y)	(−2,4)	(−1,4)	(0,4)	(1,4)	(2,4)



$y = 4$, no matter the value of x.

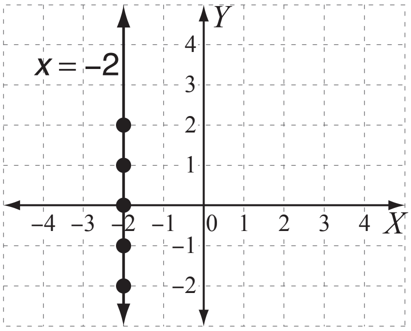
Plot the points.

Join the points with a line.

Label the line with the rule $y = 4$

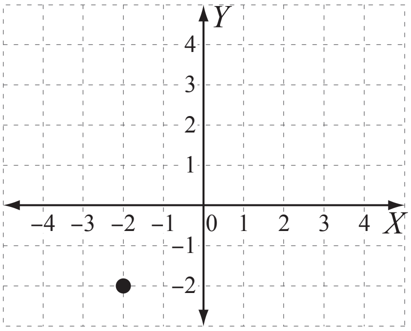
a) Graph the line of equation $x = −2$ by first completing this table of values.
[Label the line with the rule.]

x	−2	−2	−2	−2	−2
y	−2	−1	0	1	2
(x,y)	(−2,−2)	(−2, −1)	(−2, 0)	(−2, 1)	(−2, 2)



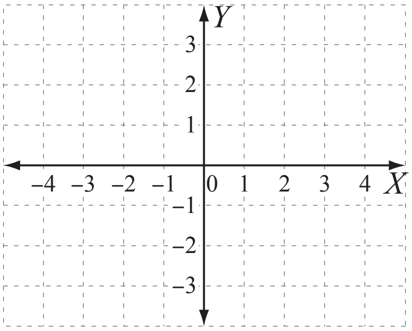
b) Graph the line of equation $y = −2$ by first completing this table of values.
[Label the line with the rule.]

x	−2	−1	0	1	2
y	−2				
(x,y)	(−2,−2)	(,)	(,)	(,)	(,)



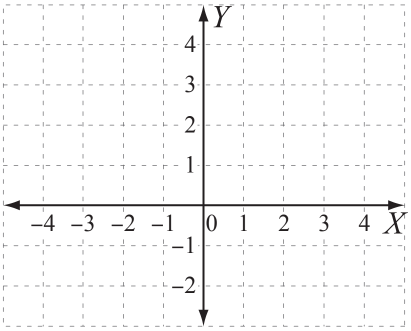
c) Graph the line of equation $y = −3$ by first completing this table of values.
[Label the line with the rule.]

x	−2	−1	0	1	2
y	−3				



d) Graph the line of equation $x = 3$ by first completing this table of values.
[Label the line with the rule.]

x	3				
y	−2	−1	0	1	2



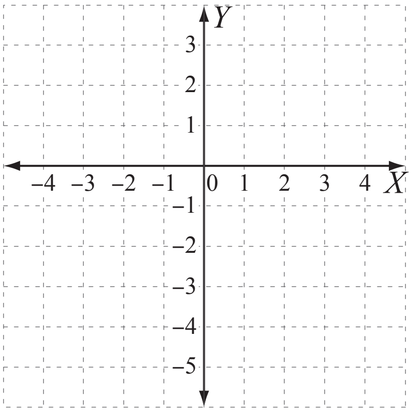
Skill 21.3 Graphing lines of equation $y = mx + c$ on a Cartesian plane (e.g. $y = 3x + 2$) (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Complete the table of values for the equation. (see skill 21.1, page 223)
- Plot each point on the Cartesian plane.
- Draw the line that joins these points.
- Label the line with the rule.

Q. Graph the line of equation $y = -2x - 1$ by first completing this table of values.
[Label the line with the rule.]

x	–2	–1	0	1	2
y	3				
(x,y)	(–2,3)	(,)	(,)	(,)	(,)



A. $y = -2x - 1 = -2 \times x - 1$ — 2x = 2 × x

$x = -1 \Rightarrow y = -2 \times -1 - 1 = 1 \quad \Rightarrow (-1,1)$

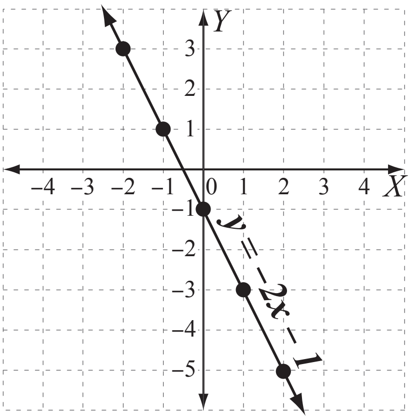
$x = 0 \Rightarrow y = -2 \times 0 - 1 = -1 \quad \Rightarrow (0,-1)$

$x = 1 \Rightarrow y = -2 \times 1 - 1 = -3 \quad \Rightarrow (1,-3)$

$x = 2 \Rightarrow y = -2 \times 2 - 1 = -5 \quad \Rightarrow (2,-5)$

x	–2	–1	0	1	2
y	3	1	–1	–3	–5
(x,y)	(–2,3)	(–1,1)	(0,–1)	(1,–3)	(2,–5)

Complete the table of values.



Plot the points.

Join the points with a line.

Label the line with the rule $y = -2x - 1$

a) Graph the line of equation $y = -x$ by first completing this table of values.
[Label the line with the rule.]

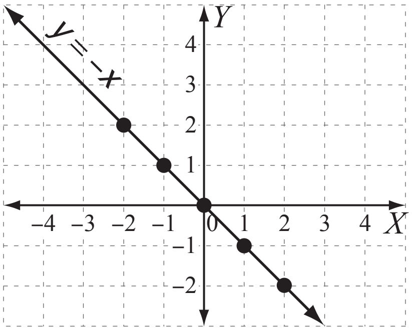
$x = -1 \Rightarrow y = -(-1) = 1 \quad \Rightarrow (-1,1)$

$x = 0 \Rightarrow y = -0 = 0 \quad \Rightarrow (0,0)$

$x = 1 \Rightarrow y = -1 \quad \Rightarrow (1,-1)$

$x = 2 \Rightarrow y = -2 \quad \Rightarrow (2,-2)$

x	–2	–1	0	1	2
y	2	1	0	–1	–2
(x,y)	(–2,2)	(–1, 1)	(0 , 0)	(1 , –1)	(2 ,–2)



b) Graph the line of equation $y = x - 4$ by first completing this table of values.
[Label the line with the rule.]

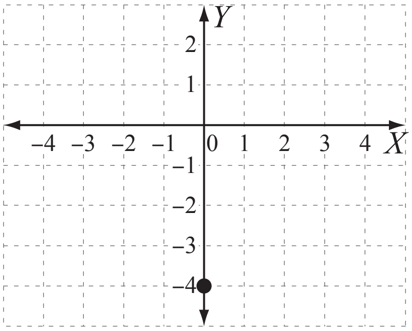
$x = 1 \Rightarrow y = 1 - 4 = -3 \quad \Rightarrow (1,-3)$

$x = 2 \Rightarrow y = \quad \Rightarrow$

$x = 3 \Rightarrow y = \quad \Rightarrow$

$x = 4 \Rightarrow y = \quad \Rightarrow$

x	0	1	2	3	4
y	–4				
(x,y)	(0,–4)	(,)	(,)	(,)	(,)



Skill 21.3 Graphing lines of equation $y = mx + c$ on a Cartesian plane (e.g. $y = 3x + 2$) (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

c) Graph the line of equation $y = 2x$ by first completing this table of values.
[Label the line with the rule.]

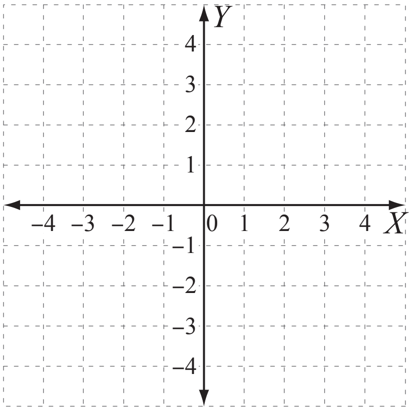
$x = -1 \Rightarrow y = 2 \times -1 = -2 \quad \Rightarrow (-1,-2)$

$x = 0 \quad \Rightarrow y = \qquad \Rightarrow$

$x = 1 \quad \Rightarrow y = \qquad \Rightarrow$

$x = 2 \quad \Rightarrow y = \qquad \Rightarrow$

x	–2	–1	0	1	2
y	–4				



d) Graph the line of equation $y = -x + 5$ by first completing this table of values.
[Label the line with the rule.]

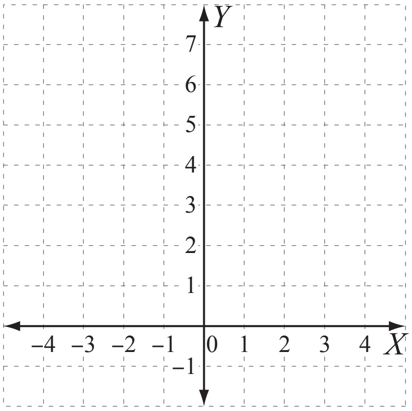
$x = -1 \Rightarrow y = -(-1) + 5 = 6 \quad \Rightarrow (-1,6)$

$x = 0 \quad \Rightarrow y = \qquad \Rightarrow$

$x = 1 \quad \Rightarrow y = \qquad \Rightarrow$

$x = 2 \quad \Rightarrow y = \qquad \Rightarrow$

x	–2	–1	0	1	2
y	7				



e) Graph the line of equation $y = 3x - 2$ by first completing this table of values.
[Label the line with the rule.]

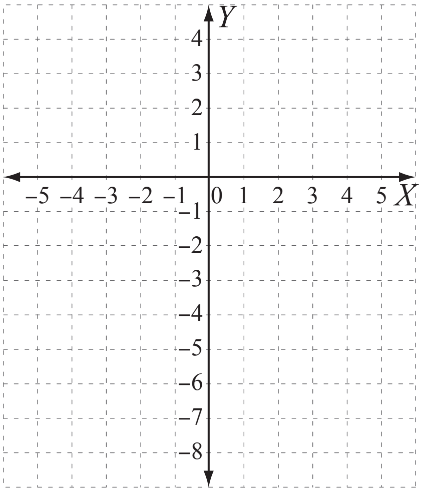
$x = -1 \Rightarrow y = 3 \times -1 - 2 = -5 \quad \Rightarrow (-1,-5)$

$x = 0 \quad \Rightarrow y = \qquad \Rightarrow$

$x = 1 \quad \Rightarrow y = \qquad \Rightarrow$

$x = 2 \quad \Rightarrow y = \qquad \Rightarrow$

x	–2	–1	0	1	2
y	–8				



f) Graph the line of equation $y = -2x - 3$ by first completing this table of values.
[Label the line with the rule.]

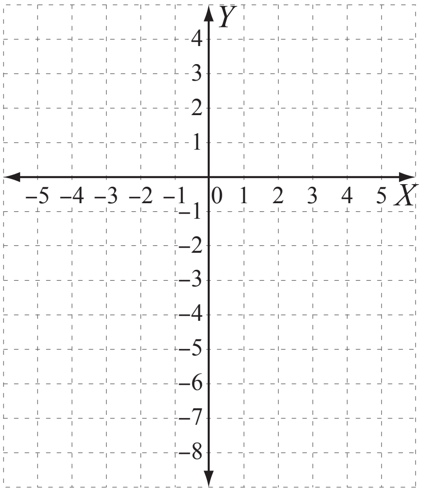
$x = -1 \Rightarrow y = \qquad \Rightarrow$

$x = 0 \quad \Rightarrow y = \qquad \Rightarrow$

$x = 1 \quad \Rightarrow y = \qquad \Rightarrow$

$x = 2 \quad \Rightarrow y = \qquad \Rightarrow$

x	–2	–1	0	1	2
y	1				

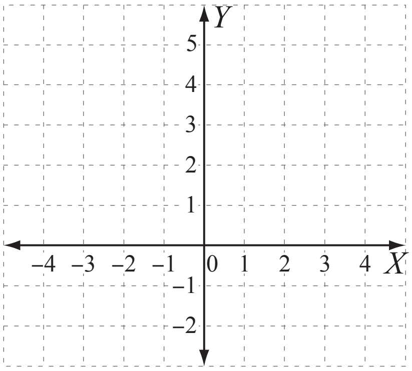


Skill 21.4 Completing the missing coordinate of a point on a given line (1).

- Substitute the given value for *x* and solve the equation for *y*. (see skill 21.1, page 223)
OR
- Substitute the given value for *y* into the rule and solve the equation for *x*. (see skill 20.9, page 219)
- Complete the missing coordinate.
- Plot the points on the graph.

Q. Complete the missing coordinates given that A and B lie on the line defined by the rule $y = 2x + 1$. Plot the points and draw the line.

A(0, $\boxed{}$), B($\boxed{}$,5)



A. $y = 2x + 1 = 2 \times x + 1$ 2x = 2 × x

$x = 0 \Rightarrow y = 2 \times 0 + 1$ Substitute $x = 0$ into the rule.

$y = 1$

$\Rightarrow A(0,\mathbf{1})$

$y = 5 \Rightarrow 5 = 2 \times x + 1$ Substitute $y = 5$ into the rule.

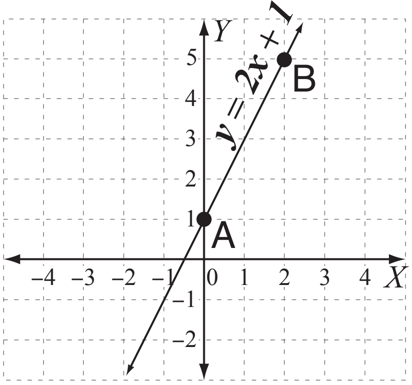
$5 - 1 = 2x + 1 - 1$

$2x = 4$

$2x \div 2 = 4 \div 2$ Solve for *x*.

$x = 2$

$\Rightarrow B(\mathbf{2},5)$



Plot the points A and B. Draw the line by joining A and B.

a) Complete the missing coordinates given that M, N and P lie on the line defined by the rule $y = -x + 3$. Plot the points and draw the line.

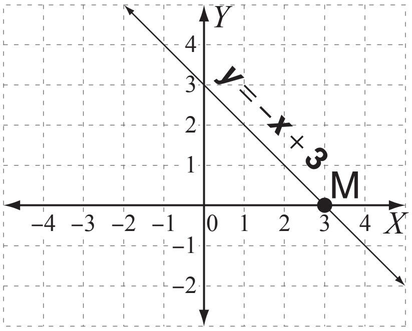
M($\boxed{3}$,0), N(–1, $\boxed{}$), P(1, $\boxed{}$)

$y = 0 \Rightarrow -x + 3 = 0 \Rightarrow -x + 3 - 3 = 0 - 3$

$\Rightarrow -x = -3 \Rightarrow x = 3 \quad \Rightarrow M(3,0)$

$x = -1 \Rightarrow y = \qquad \Rightarrow$

$x = 1 \Rightarrow y = \qquad \Rightarrow$



b) Complete the missing coordinates given that D, E and F lie on the line defined by the rule $y = 3x - 4$. Plot the points and draw the line.

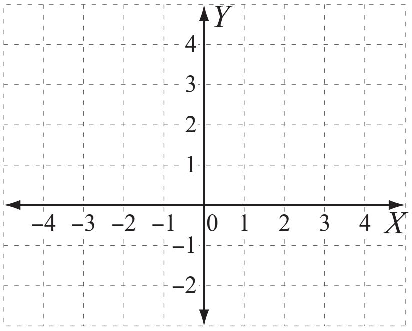
D($\boxed{}$,2), E(1, $\boxed{}$), F($\frac{2}{3}$, $\boxed{}$)

$y = 2 \Rightarrow$

$\Rightarrow \qquad \Rightarrow$

$x = 1 \Rightarrow y = \qquad \Rightarrow$

$x = \frac{2}{3} \Rightarrow y = \qquad \Rightarrow$

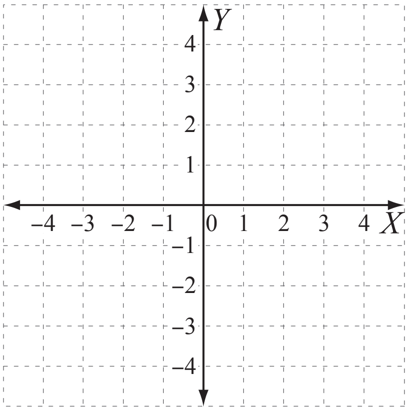


Skill 21.4 Completing the missing coordinate of a point on a given line (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

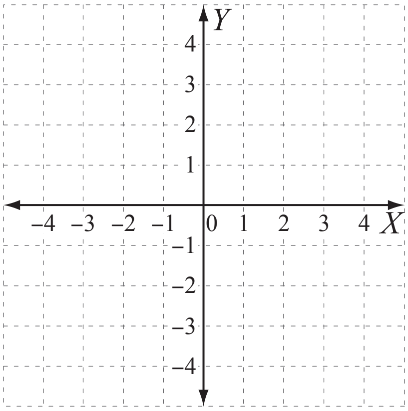
c) Complete the missing coordinates given that B, C and D lie on the line defined by the rule $y = 4x - 5$. Plot the points and draw the line.

B(0, ☐), C(☐,3), D(☐,1)



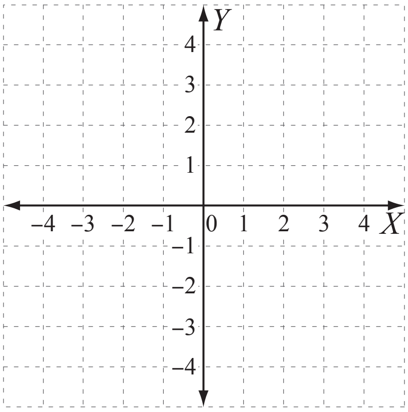
d) Complete the missing coordinates given that S, T and U lie on the line defined by the rule $y = x + 6$. Plot the points and draw the line.

S(☐,2), T(−2, ☐), U(−1, ☐)



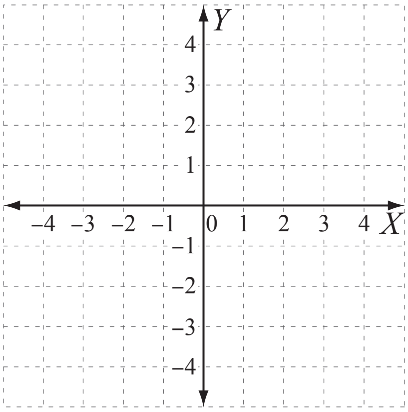
e) Complete the missing coordinates given that G, H and I lie on the line defined by the rule $y = \frac{1}{4}x + 2$. Plot the points and draw the line.

G(0, ☐), H(−4, ☐), I(☐,3)



f) Complete the missing coordinates given that A, B and C lie on the line defined by the rule $y = 2x - \frac{1}{2}$. Plot the points and draw the line.

A(☐, $\frac{3}{2}$), B(2, ☐), C(−1, ☐)



Skill 21.5 Deciding if a point is on a line of a given rule.

- Substitute the values of the coordinates *x* and *y* of the point into the given rule.
- Simplify both sides of the equation.
- Check if the statement is true, which means that the point of coordinates (*x*,*y*) lies on the line defined by the given rule.

Q. Which line does the point (1,–2) lie on?

A) $y = x$
B) $y = 2x - 3$
C) $y = 6x - 8$

A. A) $y = x \Rightarrow -2 = 1$ *(false)*
B) $y = 2x - 3 \Rightarrow -2 = 2 \times 1 - 3$
$\Rightarrow -2 = -1$ *(false)*
C) $y = 6x - 8 \Rightarrow -2 = 6 \times 1 - 8$
$\Rightarrow -2 = -2$ *(true)*
The answer is ***C***.

Substitute $x = 1$ and $y = -2$ into each rule.

a) Which of these points lies on the line defined by the rule $y = 4x + 3$?

A(–3,0)
B(2,2)
C(–1,–1)

A) $x = -3, y = 0 \Rightarrow 0 = 4 \times -3 + 3$
$\Rightarrow 0 = -9$ *(false)*
B) $x = 2, y = 2 \Rightarrow 2 = 4 \times 2 + 3$
$\Rightarrow 2 = 11$ *(false)*
C) $x = -1, y = -1 \Rightarrow -1 = 4 \times -1 + 3$
$\Rightarrow -1 = -1$ *(true)*

C

b) Which of these points lies on the line defined by the rule $y = -2x + 5$?

A(3,–3)
B(–1,7)
C(0,–2)

A) $x = 3, y = -3 \Rightarrow$
$\Rightarrow$
B) $x = -1, y = 7 \Rightarrow$
$\Rightarrow$
C) $x = 0, y = -2 \Rightarrow$
$\Rightarrow$

c) Which line does the point (2,–1) lie on?

A) $y = x + 1$
B) $y = 5 - 3x$
C) $y = 2x$

A) $y = x + 1 \Rightarrow$
$\Rightarrow$
B) $y = 5 - 3x \Rightarrow$
$\Rightarrow$
C) $y = 2x \Rightarrow$
$\Rightarrow$

d) Which line does the point (–1,1) lie on?

A) $y = -3x$
B) $y = 7 - 4x$
C) $y = 5x + 6$

A) $y = -3x \Rightarrow$
$\Rightarrow$
B) $y = 7 - 4x \Rightarrow$
$\Rightarrow$
C) $y = 5x + 6 \Rightarrow$
$\Rightarrow$

Skill 21.6 Finding the *x*-intercept and the *y*-intercept of a linear graph (1).

- To find the *x*-intercept, substitute $y = 0$ into the rule.
- Solve for *x*.
- To find the *y*-intercept, substitute $x = 0$ into the rule.
- Solve for *y*.

Q. Find the *x*-intercept and the *y*-intercept of the line defined by the equation $2x - 5y = 10$.

A. *x-intercept* $\Rightarrow y = 0$

$$\Rightarrow 2x - 0 = 10$$
$$2x \div 2 = 10 \div 2$$
$$x = 5$$

x-intercept is **(5,0)**

y-intercept $\Rightarrow x = 0$

$$\Rightarrow 0 - 5y = 10$$
$$-5y \div -5 = 10 \div -5$$
$$y = -2$$

y-intercept is **(0,–2)**

a) Find the *x*-intercept of the line defined by the equation $y = -3x + 6$

$y = 0 \Rightarrow 0 = -3x + 6$

$0 - 6 = -3x + 6 - 6$

$-3x = -6$

$-3x \div -3 = -6 \div -3$

$x = 2 \Rightarrow$ *x-intercept is* (2,0)

b) Find the *x*-intercept of the line defined by the equation $y = 2x + 8$

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is* ☐

c) Find the *y*-intercept of the line defined by the equation $y = 7x - 3$

$x = 0 \Rightarrow y = 0 - 3$

$\Rightarrow y = -3 \Rightarrow$ *y-intercept is* ☐

d) Find the *y*-intercept of the line defined by the equation $y = -5x + 4$

$x = 0 \Rightarrow$

$\Rightarrow \qquad \Rightarrow$ *y-intercept is* ☐

e) Find the *x*-intercept of the line defined by the equation $3x - 2y = -12$

$\Rightarrow$ *x-intercept is* ☐

f) Find the *y*-intercept of the line defined by the equation $4y - x = 16$

$\Rightarrow$ *y-intercept is* ☐

Skill 21.6 Finding the *x*-intercept and the *y*-intercept of a linear graph (2).

g) Find the x-intercept of the line defined by the equation $y = -2x + 10$

⇒ *x-intercept is* ☐

h) Find the y-intercept of the line defined by the equation $y = 3x - 9$

⇒ *y-intercept is* ☐

i) Find the y-intercept of the line defined by the equation $y = 6x - 8$

⇒ *y-intercept is* ☐

j) Find the x-intercept of the line defined by the equation $y = -5x + 1$

⇒ *x-intercept is* ☐

k) Find the y-intercept of the line defined by the equation $y = -4x + 5$

⇒ *y-intercept is* ☐

l) Find the y-intercept of the line defined by the equation $y = 3x - 2$

⇒ *y-intercept is* ☐

m) Find the x-intercept of the line defined by the equation $4x - y = 8$

⇒ *x-intercept is* ☐

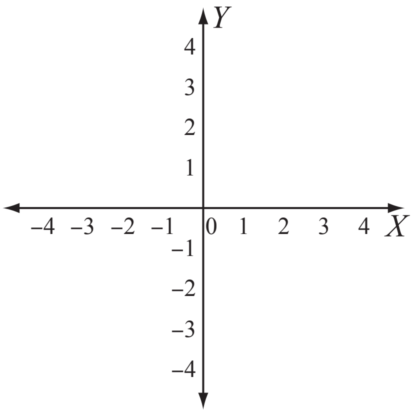
n) Find the x-intercept of the line defined by the equation $2y - x = 7$

⇒ *x-intercept is* ☐

Skill 21.7 Sketching a linear graph by finding the *x*-intercept and the *y*-intercept (1).

- Find the *x*-intercept and the *y*-intercept. (see skill 21.6, page 230)
- Mark each intercept point on the Cartesian plane.
- Draw the line that joins these points.
- Label the line with the rule.

Q. Sketch the line of equation $y = 2x - 2$ by marking the x-intercept and the y-intercept.
[Label the graph with the rule.]



A. *x-intercept* $\Rightarrow y = 0 \Rightarrow \quad 2x - 2 = 0$

$2x - 2 + 2 = 0 + 2$

$2x \div 2 = 2 \div 2$

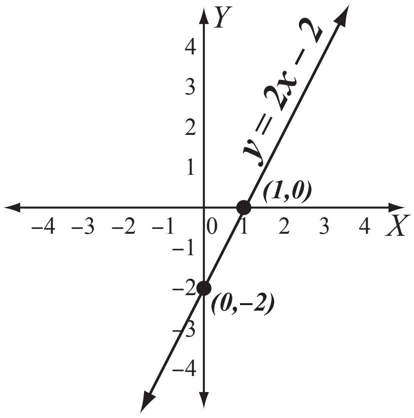
$x = 1$

$\Rightarrow$ *x-intercept is* ***(1,0)***

y-intercept $\Rightarrow x = 0 \Rightarrow y = 2 \times 0 - 2$

$\Rightarrow y = -2$

$\Rightarrow$ *y-intercept is* ***(0,−2)***



Mark the intercept points.

Join the points with a line.

Label the line with the rule $y = 2x - 2$

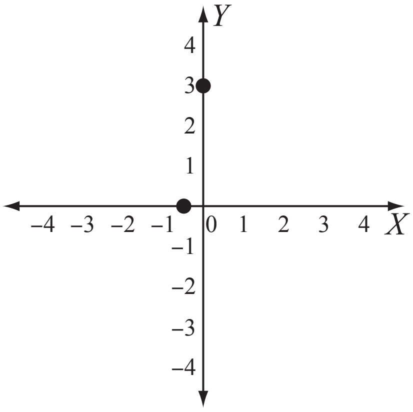
a) Sketch the line of equation $y = 6x + 3$ by marking the x-intercept and the y-intercept.
[Label the graph with the rule.]

$y = 0 \Rightarrow 6x + 3 = 0 \Rightarrow 6x = -3 \Rightarrow x = -\frac{1}{2}$

$\Rightarrow$ *x-intercept is* $(-\frac{1}{2},0)$

$x = 0 \Rightarrow y = 6 \times 0 + 3 \Rightarrow y = 3$

$\Rightarrow$ *y-intercept is* $(0,3)$



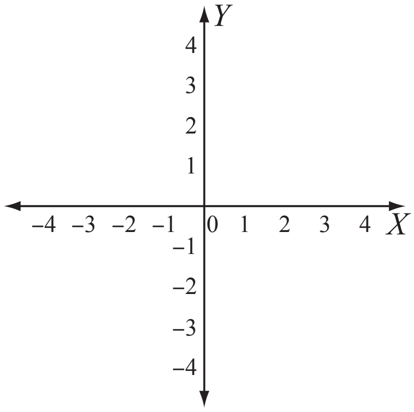
b) Sketch the line of equation $y = -3x + 4$ by marking the x-intercept and the y-intercept.
[Label the graph with the rule.]

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is*

$x = 0 \Rightarrow$

$\Rightarrow$ *y-intercept is*



Skill 21.7 Sketching a linear graph by finding the *x*-intercept and the *y*-intercept (2).

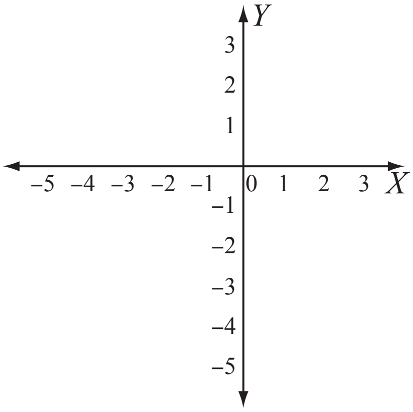
c) Sketch the line of equation $y = -x - 5$ by marking the *x*-intercept and the *y*-intercept.
[Label the graph with the rule.]

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is*

$x = 0 \Rightarrow$

$\Rightarrow$ *y-intercept is*



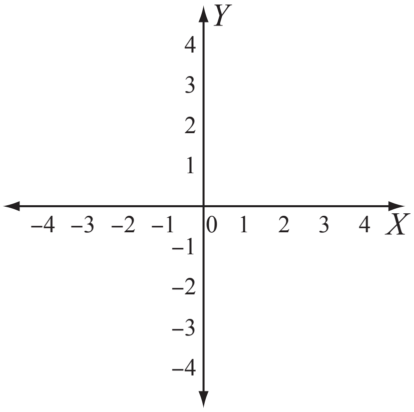
d) Sketch the line of equation $y = 9x - 3$ by marking the *x*-intercept and the *y*-intercept.
[Label the graph with the rule.]

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is*

$x = 0 \Rightarrow$

$\Rightarrow$ *y-intercept is*



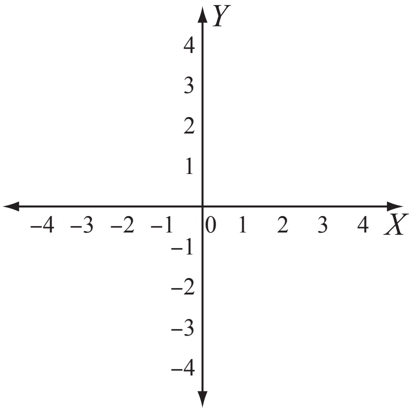
e) Sketch the line of equation $y = -4x - 2$ by marking the *x*-intercept and the *y*-intercept.
[Label the graph with the rule.]

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is*

$x = 0 \Rightarrow$

$\Rightarrow$ *y-intercept is*



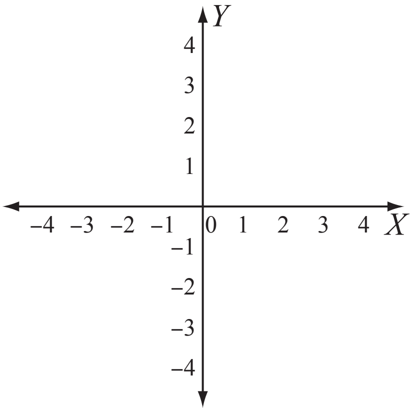
f) Sketch the line of equation $y = -2x + 4$ by marking the *x*-intercept and the *y*-intercept.
[Label the graph with the rule.]

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is*

$x = 0 \Rightarrow$

$\Rightarrow$ *y-intercept is*



Skill 21.8 Finding the gradient of a line by using the rise/run formula (1).

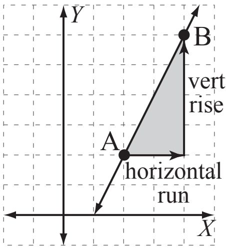
Mauve 11 22 33 44
Lime 11 22 33 44

- Choose two convenient points on the graph and draw a right-angled triangle using the line of the graph as the hypotenuse.
- Measure the vertical rise of the graph (the vertical side of the triangle):
  - positive value if the graph rises from left to right
  - negative value if the graph drops from left to right
- Measure the horizontal run of the graph (the horizontal side of the triangle):
  - always a positive value.

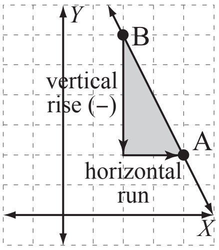
Hints: The gradient gives an indication of how steep a line is.
The gradient is positive if the graph rises from left to right.
The gradient is negative if the graph falls from left to right.

$$\text{Gradient} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

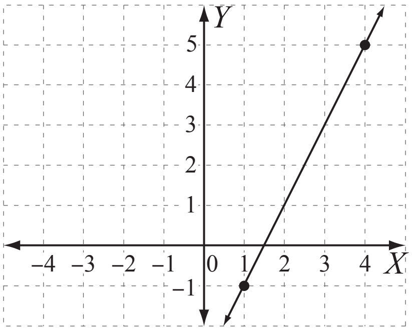
Positive gradient



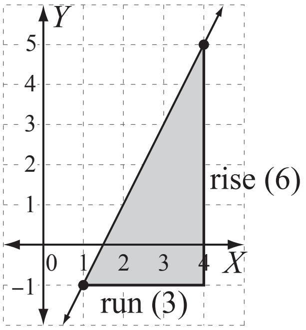
Negative gradient



Q. Find the gradient of the line passing through the points (1,–1) and (4,5).



A.



Draw a right-angled triangle.

Measure the rise and run.

rise = 6

run = 3

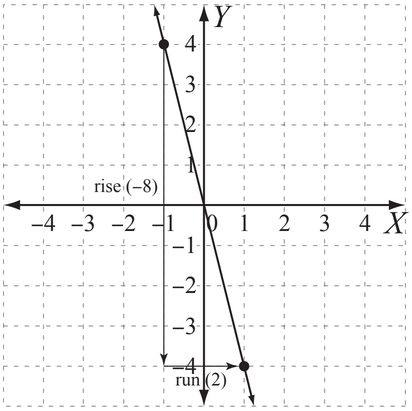
$$gradient = \frac{rise}{run} = \frac{6}{3} = \mathbf{2}$$

a) Find the gradient of the line passing through the points (–1,4) and (1,–4).

rise = –8

run = 2

$$gradient = \frac{rise}{run} = -\frac{8}{2} =$$

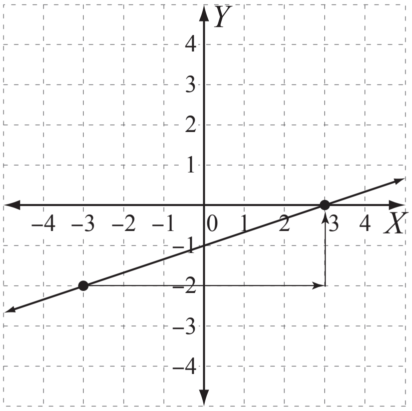


b) Find the gradient of the line passing through the points (–3,–2) and (3,0).

rise = 2

run = 6

$$gradient = \frac{rise}{run} = \quad =$$

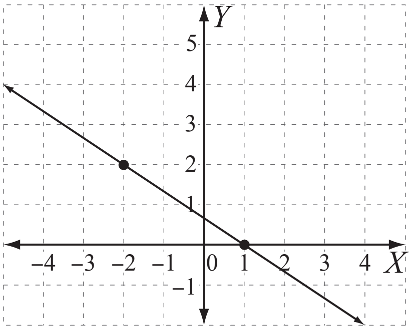


Skill 21.8 Finding the gradient of a line by using the rise/run formula (2).

c) Find the gradient of the line passing through the points (−2,2) and (1,0).

rise = *run* =

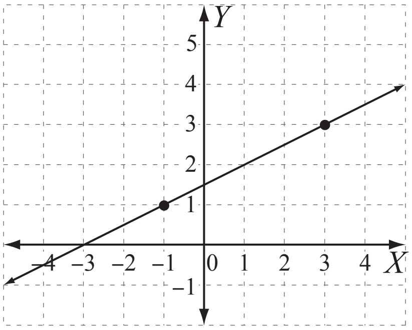
gradient = = =



d) Find the gradient of the line passing through the points (−1,1) and (3,3).

rise = *run* =

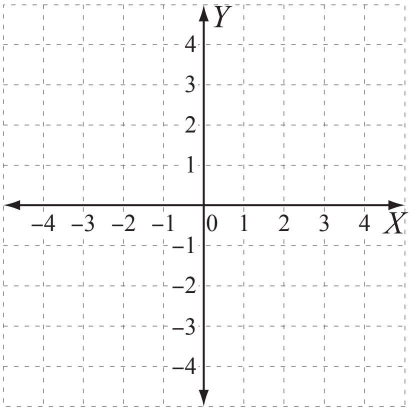
gradient = = =



e) Graph the line passing through the points (−1,1) and (3,2). What is the gradient of the line?

rise = *run* =

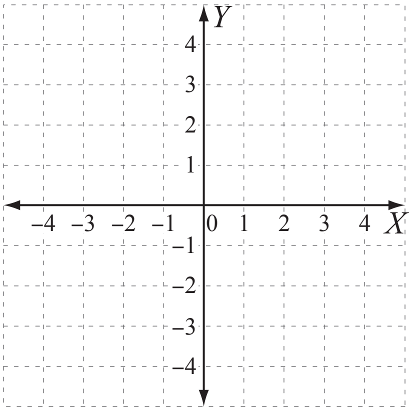
gradient = = =



f) Graph the line passing through the points (−2,4) and (2,−2). What is the gradient of the line?

rise = *run* =

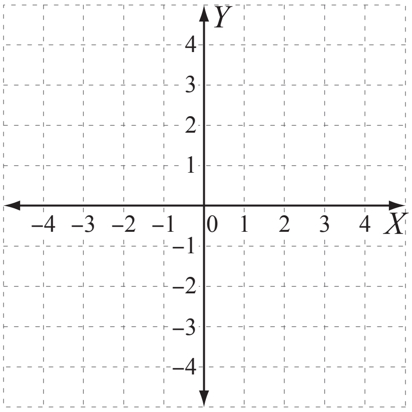
gradient = = =



g) Graph the line passing through the points (−1,−2) and (4,4). What is the gradient of the line?

rise = *run* =

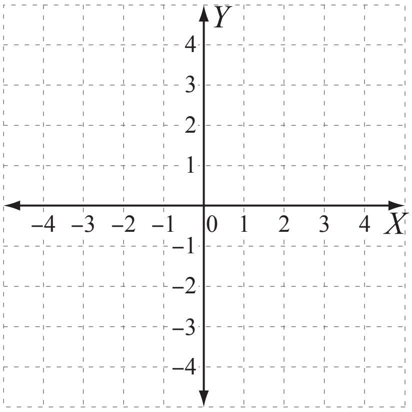
=



h) Graph the line passing through the points (−4,3) and (2,−4). What is the gradient of the line?

rise = *run* =

=



Skill 21.9 Finding the coordinates of the midpoint of an interval.

Mauve 11 22 33 44
Lime 11 22 33 44

- Identify (x_1,y_1) and (x_2,y_2) as the coordinates of the given points.
- Write the formula for the midpoint of an interval.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- Substitute the values of x_1, x_2, y_1 and y_2 into the formula.

Q. Use $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to find the coordinates of the midpoint M of the interval joining the points (6,–4) and (2,8).

A. $(x_1,y_1) = (6,-4) \Rightarrow x_1 = 6$ *and* $y_1 = -4$

$(x_2,y_2) = (2,8) \Rightarrow x_2 = 2$ *and* $y_2 = 8$

$\frac{x_1 + x_2}{2} = \frac{6+2}{2} = \frac{8}{2} = 4$

$\frac{y_1 + y_2}{2} = \frac{-4+8}{2} = \frac{4}{2} = 2$

The coordinates of the midpoint M are **(4,2)**

a) Use $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to find the coordinates of the midpoint M of the interval joining the points (–5,–4) and (1,–3).

$x_1 =$, $y_1 =$, $x_2 =$, $y_2 =$

$\frac{x_1 + x_2}{2} = \qquad =$

$\frac{y_1 + y_2}{2} = \qquad = \qquad \Rightarrow$

b) Use $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to find the coordinates of the midpoint M of the interval joining the points (3,7) and (–2,3).

$x_1 =$, $y_1 =$, $x_2 =$, $y_2 =$

$= \qquad =$

$= \qquad = \qquad \Rightarrow$

c) Use $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to find the coordinates of the midpoint M of the interval joining the points (5,0) and (7,9).

$x_1 =$, $y_1 =$, $x_2 =$, $y_2 =$

$= \qquad =$

$= \qquad = \qquad \Rightarrow$

d) Use $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to find the coordinates of the midpoint M of the interval joining the points (1,–3) and (–4,6).

$x_1 =$, $y_1 =$, $x_2 =$, $y_2 =$

$= \qquad =$

$= \qquad = \qquad \Rightarrow$

e) Use $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to find the coordinates of the midpoint M of the interval joining the points (–2,–5) and (–1,0).

$x_1 =$, $y_1 =$, $x_2 =$, $y_2 =$

$= \qquad =$

$= \qquad = \qquad \Rightarrow$

f) Use $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to find the coordinates of the midpoint M of the interval joining the points (0,6) and (–2,–4).

$x_1 =$, $y_1 =$, $x_2 =$, $y_2 =$

$= \qquad =$

$= \qquad = \qquad \Rightarrow$

Skill 21.10 Rewriting a linear equation in the gradient-intercept form.

Mauve 1 1 2 2 **3** 3 4 4
Lime 1 1 2 **2** 3 3 4 4

- Use the inverse operations of addition, subtraction, multiplication and/or division to rearrange the terms in the rule:
  - *y* on the left hand side of the equal sign without coefficient
  - *x* on the right hand side of the equal sign.

Hint: The general form of a linear function (rule) is y = mx + c
The coefficient of x is the gradient of the graph (m).
The number that is not attached to either x or y is the y-intercept (c).

Q. Write the equation $-x + 2y = -3$ in the gradient-intercept form $y = mx + c$, where m represents the gradient and c the y-intercept.

A.

$$-x + 2y = -3$$
$$-x + x + 2y = -3 + x$$
$$2y = x - 3$$
$$\frac{2y}{2} = \frac{x-3}{2}$$
$$y = \frac{1}{2}x - \frac{3}{2}$$

a) Write the equation $5 - y = 4x$ in the gradient-intercept form $y = mx + c$, where m represents the gradient and c the y-intercept.

$5 - y - 5 = 4x - 5 \Rightarrow -y = 4x - 5$

$- -y = -(4x - 5)$

$y = -4x - (-5)$ $\boxed{y = -4x + 5}$

b) Write the equation $x - 5y = 5$ in the gradient-intercept form $y = mx + c$, where m represents the gradient and c the y-intercept.

$\boxed{y =}$

c) Write the equation $3x + 2y = 1$ in the gradient-intercept form $y = mx + c$, where m represents the gradient and c the y-intercept.

$\boxed{y =}$

d) Write the equation $4x - 2y = 3$ in the gradient-intercept form $y = mx + c$, where m represents the gradient and c the y-intercept.

$\boxed{y =}$

e) Write the equation $-2x - y = 6$ in the gradient-intercept form $y = mx + c$, where m represents the gradient and c the y-intercept.

$\boxed{y =}$

f) Write the equation $3x + 4y = 12$ in the gradient-intercept form $y = mx + c$, where m represents the gradient and c the y-intercept.

$\boxed{y =}$

Skill 21.11 Finding the gradient, the *x*-intercept and the *y*-intercept of an equation written in the gradient-intercept form $y = mx + c$ (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Write the equation given in the table in the gradient-intercept form $y = mx + c$ (see skill 21.10, page 237)
- Identify the gradient (m) of the linear function as the coefficient of x.
- Identify the y-intercept of the linear function as the constant (c).
- Find the x-intercept of the function. (see skill 21.6, page 230)
- Fill in the table.

Q. Complete the following table:

equation	gradient (m)	x-intercept	y-intercept (c)
$y = -2x - 10$			

A. $y = -2x - 10$

$y = -2x + -10$

$y = mx + c \quad \Rightarrow \; m = \mathbf{-2}$ (gradient)

$\Rightarrow \; c = -10$ (y-intercept)

y-intercept is **(0,–10)**

x-intercept $\Rightarrow \; y = 0$

$\Rightarrow -2x - 10 = 0$

$-2x - 10 + 10 = 0 + 10$

$-2x = 10$

$$\frac{-\cancel{2}x}{-\cancel{2}} = \frac{\cancel{10}^{5}}{-\cancel{2}_{1}}$$

$x = -5$

x-intercept is **(–5,0)**

equation	gradient (m)	x-intercept	y-intercept (c)
$y = -2x - 10$	**–2**	**(–5,0)**	**(0,–10)**

a) Complete the following table:

equation	gradient (m)	x-intercept	y-intercept (c)
$y = 2x - 6$	2	(3,0)	(0,–6)

$y = 2x - 6$

$y = mx + c \Rightarrow m = 2$

$\Rightarrow c = -6 \Rightarrow y\text{-intercept is } (0,-6)$

$y = 0 \Rightarrow \quad 2x - 6 = 0$

$2x - 6 + 6 = 0 + 6$

$2x = 6$

$2x \div 2 = 6 \div 2$

$x = 3 \Rightarrow x\text{-intercept is } (3,0)$

b) Complete the following table:

equation	gradient (m)	x-intercept	y-intercept (c)
$y = -x + 5$			

$y = -x + 5$

$y = mx + c \Rightarrow m =$

$\Rightarrow c = \quad \Rightarrow y\text{-intercept is}$

$y = 0 \Rightarrow$

$\Rightarrow x\text{-intercept is}$

Skill 21.11 Finding the gradient, the *x*-intercept and the *y*-intercept of an equation written in the gradient-intercept form $y = mx + c$ (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

c) Complete the following table:

equation	gradient (m)	x-intercept	y-intercept (c)
$y = \frac{1}{3}x - 2$			

$y = \frac{1}{3}x - 2$

$y = mx + c \Rightarrow m =$

$\Rightarrow c = \quad \Rightarrow$ *y-intercept is*

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is*

d) Complete the following table:

equation	gradient (m)	x-intercept	y-intercept (c)
$y = \frac{2}{5}x + 4$			

$y = \frac{2}{5}x + 4$

$y = mx + c \Rightarrow m =$

$\Rightarrow c = \quad \Rightarrow$ *y-intercept is*

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is*

e) Complete the following table:

equation	gradient (m)	x-intercept	y-intercept (c)
$y = 5x + 3$			

$y = 5x + 3$

$y = mx + c \Rightarrow m =$

$\Rightarrow c = \quad \Rightarrow$ *y-intercept is*

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is*

f) Complete the following table:

equation	gradient (m)	x-intercept	y-intercept (c)
$y = -2x + 1$			

$y = -2x + 1$

$y = mx + c \Rightarrow m =$

$\Rightarrow c = \quad \Rightarrow$ *y-intercept is*

$y = 0 \Rightarrow$

$\Rightarrow$ *x-intercept is*

Skill 21.12 Finding the gradient of a line when two points are given.

Mauve 11 22 33 44
Lime 11 22 33 44

- Identify (x_1,y_1) and (x_2,y_2) as the coordinates of the given points.
- Write the formula for the gradient of a linear graph.
- Substitute the values of x_1, x_2, y_1 and y_2 into the formula.
- Simplify and evaluate the value of m.

$$\text{Gradient } m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Q. Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line passing through the points (6,–2) and (–2,4).

A. $(x_1,y_1) = (6,-2) \Rightarrow x_1 = 6$ and $y_1 = -2$

$(x_2,y_2) = (-2,4) \Rightarrow x_2 = -2$ and $y_2 = 4$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

– – = +

$$= \frac{4 - (-2)}{-2 - 6}$$

Simplify: ÷2

$$= \frac{6}{-8} = -\frac{\mathbf{3}}{\mathbf{4}}$$

a) Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line passing through the points (–1,3) and (2,0).

$x_1 = -1,\ y_1 = 3,\ x_2 = 2,\ y_2 = 0$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - 3}{2 - (-1)} = \dfrac{-3}{3} = \boxed{}$

b) Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line passing through the points (2,1) and (–2,–7).

$x_1 = \quad ,\ y_1 = \quad ,\ x_2 = \quad ,\ y_2 =$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \qquad = \qquad = \boxed{}$

c) Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line passing through the points (1,–1) and (3,3).

$= \boxed{}$

d) Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line passing through the points (3,–1) and (–1,–2).

$= \boxed{}$

e) Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line passing through the points (0,3) and (–6,0).

$= \boxed{}$

f) Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line passing through the points (–4,1) and (2,–1).

$= \boxed{}$

g) Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line passing through the points (3,–1) and (1,2).

$= \boxed{}$

h) Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line passing through the points (–4,0) and (–1,5).

$= \boxed{}$

Skill 21.13 Writing the equation of a line when two points are given (1).

- Identify (x_1,y_1) and (x_2,y_2) as the coordinates of the given points.
- Find the gradient of the line joining the two points. (see skill 21.12, page 240)
- Write the equation $y - y_1 = m(x - x_1)$ of a straight line.
- Substitute the values of y_1, m and x_1 into the equation.
- Simplify to rearrange the equation:
  - y on its own on the left hand side of the equal sign without coefficient
  - x and the remaining number on the right hand side of the equal sign

$$\boxed{y - y_1 = m(x - x_1)}$$

Q. Use $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ to write the equation of the line passing through the points (–3,2) and (4,–1).

A. $(x_1,y_1) = (-3,2) \Rightarrow x_1 = -3 \text{ and } y_1 = 2$

$(x_2,y_2) = (4,-1) \Rightarrow x_2 = 4 \text{ and } y_2 = -1$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1 - 2}{4 - (-3)}$$

$$= \frac{-3}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{7} \times (x - -3)$$

$- - = +$

$$y - 2 = -\frac{3}{7} \times (x + 3)$$

$+ - = -$

$$y - 2 = -\frac{3x}{7} + -\frac{9}{7}$$

$$y - 2 + 2 = -\frac{3x}{7} - \frac{9}{7} + 2$$

$-\frac{9}{7} + 2 = -\frac{9}{7} + \frac{14}{7} = \frac{5}{7}$

$$y = -\frac{\mathbf{3}}{\mathbf{7}}\boldsymbol{x} + \frac{\mathbf{5}}{\mathbf{7}}$$

a) Use $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ to write the equation of the line passing through the points (7,0) and (–1,8).

$x_1 = 7, y_1 = 0, x_2 = -1, y_2 = 8$

$m = \frac{8 - 0}{-1 - 7} = \frac{8}{-8} = -1$

$y - y_1 = m(x - x_1)$

$y - 0 = -1 \times (x - 7)$

$y = -1 \times x - -1 \times 7$

$- - = +$

$\boxed{y = -x + 7}$

b) Use $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ to write the equation of the line passing through the points (–4,5) and (–3,7).

$x_1 = \quad , y_1 = \quad , x_2 = \quad , y_2 =$

$m = \quad = \quad =$

$y - y_1 = m(x - x_1)$

$\boxed{y =}$

Skill 21.13 Writing the equation of a line when two points are given (2).

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 **3** 3 4 4

c) Use $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ to write the equation of the line passing through the points (1,5) and (3,11).

$x_1 = \quad , y_1 = \quad , x_2 = \quad , y_2 =$

$m = \quad = \quad =$

$y - y_1 = m(x - x_1)$

$y =$

d) Use $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ to write the equation of the line passing through the points (1,–3) and (–4,–2).

$x_1 = \quad , y_1 = \quad , x_2 = \quad , y_2 =$

$m = \quad = \quad =$

$y - y_1 = m(x - x_1)$

$y =$

e) Use $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ to write the equation of the line passing through the points (4,–3) and (–4,5).

$x_1 = \quad , y_1 = \quad , x_2 = \quad , y_2 =$

$m = \quad = \quad =$

$y =$

f) Use $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ to write the equation of the line passing through the points (–6,2) and (–2,–2).

$x_1 = \quad , y_1 = \quad , x_2 = \quad , y_2 =$

$m = \quad = \quad =$

$y =$

g) Use $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ to write the equation of the line passing through the points (3,0) and (–1,5).

$x_1 =$

$y =$

h) Use $y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ to write the equation of the line passing through the points (2,–4) and (0,–1).

$x_1 =$

$y =$

Skill 21.14 Completing a table of values for a non-linear rule.

- Substitute the variable *x* with the given values.
- Solve the equation for *y*.
- Use the order of operations rules. (see skill 16.1, page 131)
- Use the sign rules. (see skill 9.1, page 93)
- Complete the table of values for the non-linear function.

Q. Complete the table of values for the non-linear rule $y = x^2 - 3$ (parabola).

x	−2	−1	0	1	2
y	1				

A. $y = x^2 - 3 = x \times x - 3$

$x^2 = x \times x$

$x = -1 \Rightarrow y = -1 \times -1 - 3 = 1 - 3 \qquad \Rightarrow y = -2$

$x = 0 \Rightarrow y = 0 \times 0 - 3 = 0 - 3 \qquad \Rightarrow y = -3$

$x = 1 \Rightarrow y = 1 \times 1 - 3 = 1 - 3 \qquad \Rightarrow y = -2$

$x = 2 \Rightarrow y = 2 \times 2 - 3 = 4 - 3 \qquad \Rightarrow y = 1$

x	−2	−1	0	1	2
y	1	**−2**	**−3**	**−2**	**1**

Complete the table of values.

a) Complete the table of values for the non-linear rule $y = 2x^2$ (parabola).

$x = -1 \Rightarrow y = 2 \times -1 \times -1 = -2 \times -1 \qquad \Rightarrow y = 2$

$x = 0 \Rightarrow y = 2 \times 0 \times 0 = 2 \times 0 \qquad \Rightarrow y = 0$

$x = 1 \Rightarrow y = 2 \times 1 \times 1 = 2 \times 1 \qquad \Rightarrow y = 2$

$x = 2 \Rightarrow y = 2 \times 2 \times 2 = 4 \times 2 \qquad \Rightarrow y = 8$

x	−2	−1	0	1	2
y	8				
(x,y)	(−2,8)	(,)	(,)	(,)	(,)

b) Complete the table of values for the non-linear rule $y = x^2 + 2$ (parabola).

$x = -1 \Rightarrow y = -1 \times -1 + 2 = 1 + 2 \qquad \Rightarrow y = 3$

$x = 0 \Rightarrow y = \qquad \Rightarrow$

$x = 1 \Rightarrow y = \qquad \Rightarrow$

$x = 2 \Rightarrow y = \qquad \Rightarrow$

x	−2	−1	0	1	2
y	6				
(x,y)	(−2,6)	(,)	(,)	(,)	(,)

c) Complete the table of values for the non-linear rule $y = -\frac{1}{x}$ (hyperbola).

$x = -2 \Rightarrow y = \qquad \Rightarrow$

$x = -1 \Rightarrow y = \qquad \Rightarrow$

$x = 1 \Rightarrow y = \qquad \Rightarrow$

$x = 2 \Rightarrow y = \qquad \Rightarrow$

x	−2	−1	0	1	2
y			☒		

d) Complete the table of values for the non-linear rule $y = 4^x$ (exponential function).

$x = -1 \Rightarrow y = 4^{-1} \qquad \Rightarrow y = \frac{1}{4}$

$x = 0 \Rightarrow y = \qquad \Rightarrow$

$x = 1 \Rightarrow y = \qquad \Rightarrow$

$x = 2 \Rightarrow y = \qquad \Rightarrow$

x	−2	−1	0	1	2
y	$\frac{1}{16}$				

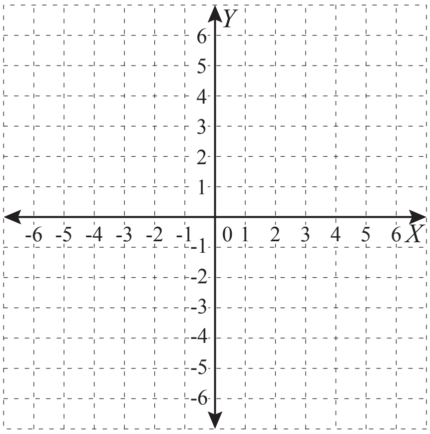
Skill 21.15 Sketching non-linear rules by completing a table of values.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Complete the table of values for the equation. (see skill 21.14, page 243)
- Plot each point on the Cartesian plane.
- Draw the curved line that joins these points.
- Label the line with the rule.

Q. Sketch the non-linear rule $y = x^2 - 3$ (parabola) by first completing this table of values.

x	−3	−2	−1	0	1	2	3
y	4						



A. $y = x^2 - 3$

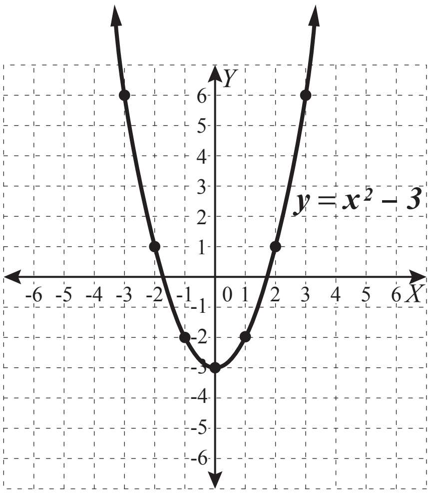
$x = -2 \Rightarrow y = (-2)^2 - 3 = 1 \quad \Rightarrow (-2,1)$

$x = -1 \Rightarrow y = (-1)^2 - 3 = -2 \quad \Rightarrow (-1,-2)$

$x = 0 \Rightarrow y = 0 - 3 = -3 \quad \Rightarrow (0,-3)$

$x = 1 \Rightarrow y = 1^2 - 3 = -2 \quad \Rightarrow (1,-2)$

x	−3	−2	−1	0	1	2	3
y	6	***1***	***−2***	***−3***	***−2***	***1***	***6***



Complete the table of values.

Plot the points.

Join the points with a curved line.

Label the line with the rule.

a) Sketch the non-linear rule $y = 2x^2$ (parabola) by first completing this table of values.

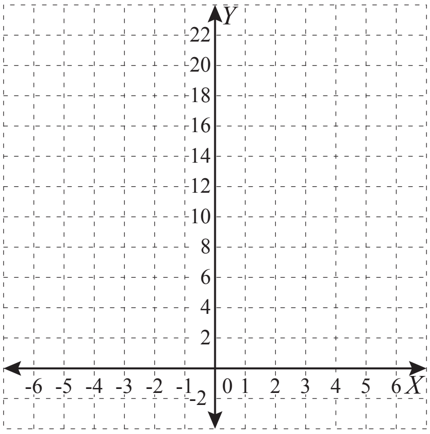
$x = -3 \Rightarrow y = 2(-3)^2 = 18 \quad \Rightarrow (-3,18)$

$x = -2 \Rightarrow y = 2(-2)^2 = 8 \quad \Rightarrow (-2,8)$

$x = -1 \Rightarrow y = \quad \Rightarrow$

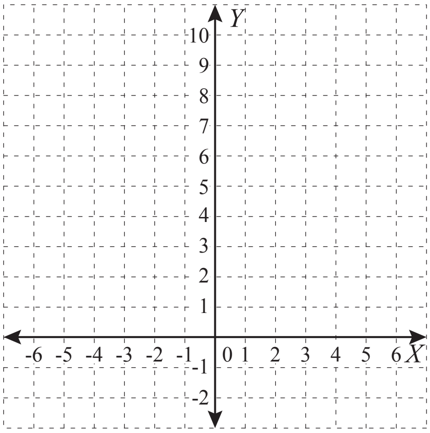
$x = 0 \Rightarrow y = \quad \Rightarrow$

x	−3	−2	−1	0	1	2	3
y							



b) Sketch the non-linear rule $y = x^2 - 2$ (parabola) by first completing this table of values.

x	−3	−2	−1	0	1	2	3
y	7						



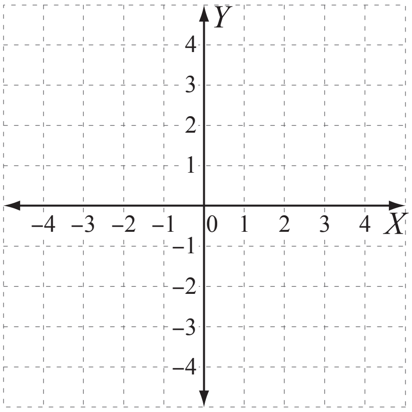
Skill 21.16 Solving simultaneous equations by graphing their lines on a Cartesian plane (1).

- Graph the equations by finding the *x*-intercept and the *y*-intercept on the same Cartesian plane. (see skill 21.11, page 238)

OR

- Graph the equations by finding two convenient points that belong to each equation.
- Mark the solution of the simultaneous equations as the intersection point of the two lines.

Q. Solve the simultaneous equations $3x + 4y = -10$ and $5x - 2y = 18$ by graphing their lines on the Cartesian plane.



A.

Equation 1

x-intercept $\Rightarrow y = 0 \Rightarrow \quad 3x = -10$

$$3x \div 3 = -10 \div 3$$
$$x = -\frac{10}{3}$$
$$\Rightarrow x\text{-intercept is } (-\frac{10}{3}, 0)$$

y-intercept $\Rightarrow x = 0 \Rightarrow \quad 4y = -10$

$$4y \div 4 = -10 \div 4$$
$$y = -\frac{5}{2}$$
$$\Rightarrow y\text{-intercept is } (0, -\frac{5}{2})$$

Equation 2

x-intercept $\Rightarrow y = 0 \Rightarrow \quad 5x = 18$

$$5x \div 5 = 18 \div 5$$
$$x = \frac{18}{5}$$
$$\Rightarrow x\text{-intercept is } (\frac{18}{5}, 0)$$

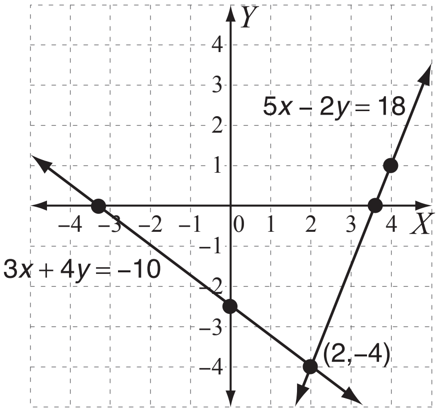
y-intercept $\Rightarrow x = 0 \Rightarrow \quad -2y = 18$

$$-2y \div -2 = 18 \div -2$$
$$y = -9$$
$$\Rightarrow y\text{-intercept is } (0, -9)$$

−9 is too big, so choose a different point:

$x = 4 \quad \Rightarrow \quad 20 - 2y = 18$

$$-2y \div -2 = -2 \div -2$$
$$y = 1$$
$$\Rightarrow \text{point } (4, 1)$$



Plot all the points.

Sketch both graphs by joining the respective pairs of points.

Mark the intersection.

The lines intersect at the point (2,−4)

Solution is ***(2,−4)***

Skill 21.16 Solving simultaneous equations by graphing their lines on a Cartesian plane (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 **4** 4

a) Solve the simultaneous equations $y = x - 1$ and $y = 2x - 3$ by graphing their lines on the Cartesian plane.

Equation 1

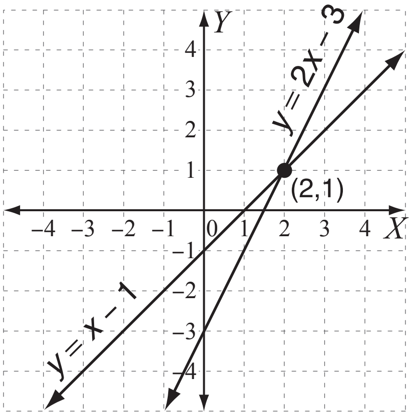
$y = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1 \quad \Rightarrow (1,0)$

$x = 0 \Rightarrow y = 0 - 1 = -1 \quad \Rightarrow (0,-1)$

Equation 2

$y = 0 \Rightarrow 2x - 3 = 0 \Rightarrow x = \frac{3}{2} \quad \Rightarrow (\frac{3}{2},0)$

$x = 0 \Rightarrow y = 0 - 3 = -3 \quad \Rightarrow (0,-3)$



b) Solve the simultaneous equations $y = 3x + 1$ and $x + 1 = 0$ by graphing their lines on the Cartesian plane.

Equation 1

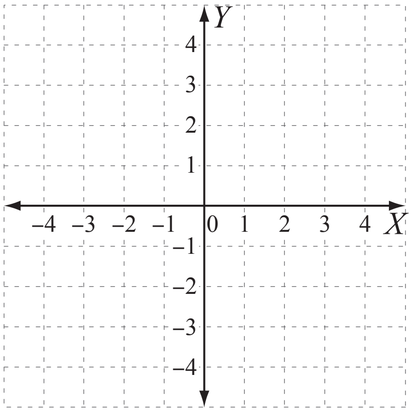
$y = 0 \Rightarrow$

$x = 0 \Rightarrow$

Equation 2

$\Rightarrow$

$\Rightarrow$



c) Solve the simultaneous equations $x + y = 4$ and $2x + y = 6$ by graphing their lines on the Cartesian plane.

Equation 1

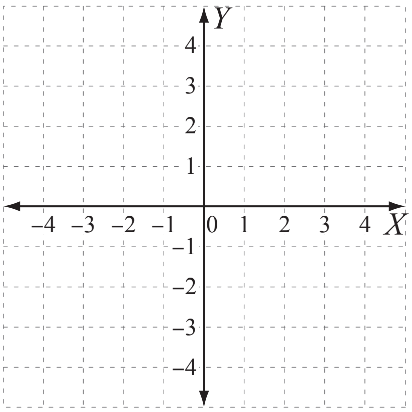
$\Rightarrow$

$\Rightarrow$

Equation 2

$\Rightarrow$

$\Rightarrow$



d) Solve the simultaneous equations $2x + y = 3$ and $x - 2y = 4$ by graphing their lines on the Cartesian plane.

Equation 1

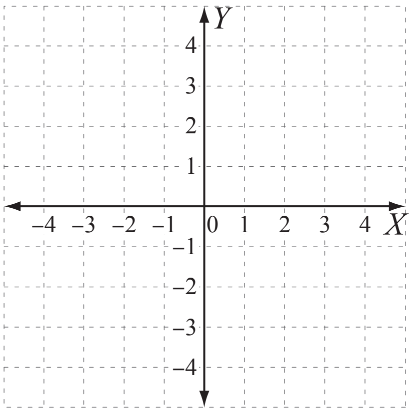
$\Rightarrow$

$\Rightarrow$

Equation 2

$\Rightarrow$

$\Rightarrow$



Skill 21.17 Calculating the distance between two points.

Mauve 11 22 33 44
Lime 11 22 33 44

- Identify (x_1,y_1) and (x_2,y_2) as the coordinates of the given points.
- Write the formula for the distance between two points.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

- Substitute the values of x_1, x_2, y_1 and y_2 into the formula.

Q. Use $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between the points (−4,1) and (−6,−3). [Leave the answer in surd form.]

A.
$(x_1,y_1)=(-4,1) \Rightarrow x_1=-4 \text{ and } y_1=1$
$(x_2,y_2)=(-6,-3) \Rightarrow x_2=-6 \text{ and } y_2=-3$
$(x_2-x_1)^2=(-6-{}^{-}4)^2=(-2)^2=4$
$(y_2-y_1)^2=(-3-1)^2=(-4)^2=16$
$d=\sqrt{4+16}=\sqrt{\mathbf{20}}$

a) Use $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between the points (5,3) and (−4,2).
[Leave the answer in surd form.]

$x_1=5,\ y_1=3,\ x_2=-4,\ y_2=2$
$(x_2-x_1)^2=(-4-5)^2=(-9)^2=81$
$(y_2-y_1)^2=(2-3)^2=(-1)^2=1$
$d=\sqrt{81+1}=\sqrt{82}$

b) Use $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between the points (−1,7) and (2,−3).
[Leave the answer in surd form.]

$x_1=\quad,\ y_1=\quad,\ x_2=\quad,\ y_2=$
$d=\quad=$

c) Use $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between the points (0,3) and (5,−2).
[Leave the answer in surd form.]

$x_1=\quad,\ y_1=\quad,\ x_2=\quad,\ y_2=$
$d=\quad=$

d) Use $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between the points (−4,0) and (−3,1).
[Leave the answer in surd form.]

$x_1=\quad,\ y_1=\quad,\ x_2=\quad,\ y_2=$
$d=\quad=$

e) Use $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between the points (−2,−2) and (0,4).
[Leave the answer in surd form.]

$x_1=\quad,\ y_1=\quad,\ x_2=\quad,\ y_2=$
$d=\quad=$

f) Use $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ to find the distance between the points (8,0) and (0,8).
[Leave the answer in surd form.]

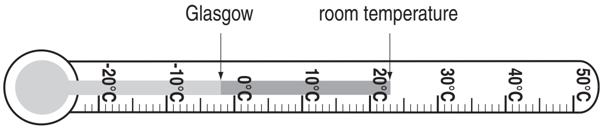
$x_1=\quad,\ y_1=\quad,\ x_2=\quad,\ y_2=$
$d=\quad=$

22. [Units of Measurement / Time]

Skill 22.1 Reading scales.

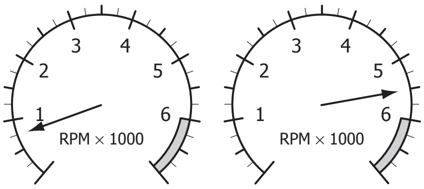
- Consider the unit of measurement and the value of each scale marking.

Q. What is the difference in temperature between the room and Glasgow?



A. *Each marking represents 1°C.*
room temperature = 23°C
Glasgow = –2°C
Difference = 23°C – (–2°C)
= 23°C + 2°C
*= **25°C***

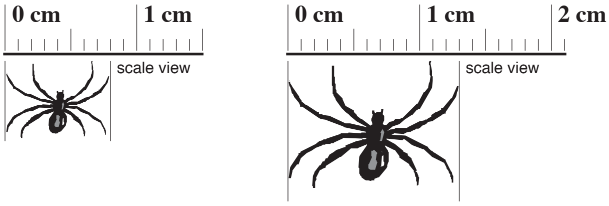
a) What is the difference in revolutions per minute (RPM) between the two vehicles?



5.5 × 1000 – 0.75 × 1000

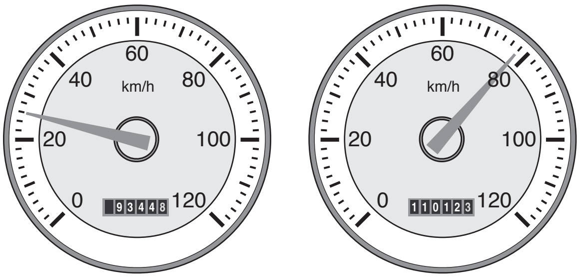
= *5500 – 750* = 4750 RPM

b) How many centimetres is the width difference between the two spiders?



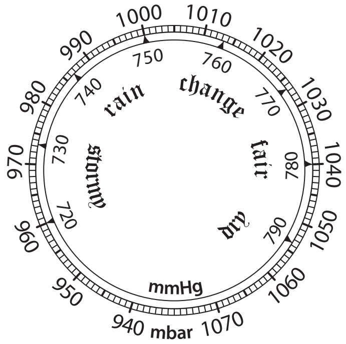
=

c) What is the difference in speed between the two vehicles?

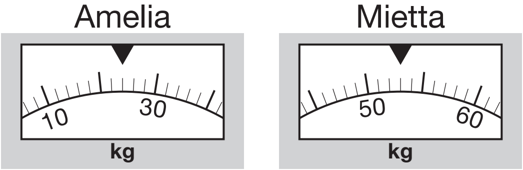


km/h

d) How many millimetres of mercury (mmHg) equal 980 millibars (mbar) of pressure?

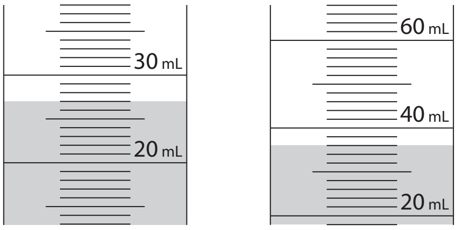


e) How much heavier is Mietta than Amelia?



kg

f) How much more water is in the second cylinder?



mL

Skill 22.2 Choosing appropriate units and measurements.

- Compare the length, area, mass or capacity to that of common objects (ruler, tennis court, bag of flour, carton of milk) or any standard units you know, chosen because they are sensible and accurate.
  Examples: Carpenters measure wood lengths in millimetres.
  Height of a person is measured in centimetres.
  Mountain heights are measured in metres.

Q. The diameter of a snowflake could most reasonably be described as:

A) 0.01 cm
B) 1 cm
C) 10 cm

A. *Convert difficult measurements to a unit you can visualise:*

A) 0.01 cm = 0.1 mm ⇒ too small
B) 1 cm ⇒ reasonable
C) 10 cm ⇒ too large

The answer is ***B***.

a) The most appropriate unit for measuring the weight of a truck is:

A) tonnes
B) kilograms
C) grams per cubic centimetre
D) grams

A

b) The most appropriate unit for measuring the width of a book is:

A) square millimetres
B) metres
C) millimetres
D) square centimetres

c) The most appropriate unit for measuring the mass of a 20 cent coin is:

A) tonnes
B) kilograms
C) grams
D) milligrams

d) The most appropriate unit for measuring the area of a football ground is:

A) square centimetres
B) square kilometres
C) hectares
D) square metres

e) Choose the most reasonable weight of a BBQ gas cylinder.

A) 50 kg
B) 8.5 kg
C) 1 kg

f) Choose the most reasonable capacity of a green wheelie rubbish bin.

A) 240 L
B) 24 L
C) 2400 L

g) Choose the most reasonable weight of a standard red house brick.

A) 6 kg
B) 3 kg
C) 0.5 kg

h) Choose the most reasonable capacity of a hen's egg.

A) 45 mL
B) 20 mL
C) 5 mL

i) Choose the most reasonable volume of water used in a 3 minute shower.

A) 5 L
B) 500 L
C) 50 L

j) Choose the most reasonable surface area of skin on an adult human.

A) 1.7 m^2
B) 0.17 m^2
C) 17 m^2

Skill 22.3 Working with measurement prefixes.

- Consider the value of each metric prefix. (see Glossary or Maths Facts, page 455)

Q. Which metric prefix is used to describe 1 000 000 standard units?

A. ***Mega***

a) The symbol 'm' represents which metric prefix of 0.001 in value?

milli

b) The symbol 'c' represents which metric prefix of 0.01 in value?

c) The symbol 'k' represents which metric prefix of 1000 in value?

d) The symbol 'd' represents which metric prefix of one tenth in value?

e) Which metric prefix is used to describe one millionth of a unit?

f) Which metric prefix is used to describe 1 000 000 000 standard units?

g) Which metric prefix is used to describe one hundredth of a unit?

h) Which metric prefix is used to describe one thousandth of a unit?

i) Which number represents the metric prefix 'kilo'?

A) 0.01
B) $\frac{1}{10000}$
C) $\frac{1}{1000}$
D) 1000

j) Which number represents the metric prefix 'Mega'?

A) $\frac{1}{10000}$
B) 1 000 000
C) 100 000
D) $\frac{1}{1000000}$

k) Which number represents the metric prefix 'Giga'?

A) 0.000 000 000 1
B) $\frac{1}{1000000}$
C) 1 000 000 000
D) 1 000 000

l) Which number represents the metric prefix 'milli'?

A) $\frac{1}{10000}$
B) 0.01
C) 0.0001
D) $\frac{1}{1000}$

Skill 22.4 Measuring with precision and tolerating error.

- Calculate the minimum accepted quantity by subtracting the tolerance from the normal quantity.
- Calculate the maximum accepted quantity by adding the tolerance to the normal quantity.

To calculate the tolerance interval of a measurement:

- Halve the sum (find the average) of the highest and lowest values.
- Halve the difference between the highest or lowest values.
- Add or subtract (±) this difference to the average.

Q. Regulations require potato chips bags to have a weight of 150 ± 2 g. What is the minimum acceptable weight?

A. *tolerance = 2 g*
minimum accepted weight = 150 – 2 = ***148 g***

a) Regulations require potato bags to have a weight of 3 ± 0.1 kg. What is the minimum acceptable weight?

2.9 kg

b) FIFA require soccer balls to have a circumference of 69 ± 1 cm. What is the maximum acceptable circumference?

cm

c) Regulations require cricket balls to have a diameter of 22.65 ± 0.25 cm. What is the maximum acceptable diameter?

cm

d) Regulations require certain cars' petrol tanks to have a capacity of 56 ± 0.5 L. What is the minimum acceptable capacity?

L

e) Regulations require CDs to have a diameter of 120 ± 0.5 mm. What is the minimum acceptable diameter?

mm

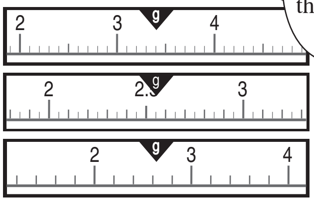
f) A healthy human's body temperature must measure 37 ± 0.8°C. What is the maximum acceptable temperature for a healthy body?

°C

g) Match the weights to the instruments based on the precision of their scales.

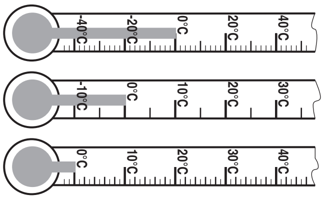
precision: the smallest unit on the scale

A) 0.2 g
B) 0.05 g
C) 0.1 g



h) Match the temperatures to the thermometers based on the precision of their scales.

A) 2°C
B) 1°C
C) 5°C



i) 'Beer is fermented between 19°C and 23°C.' Choose the description for the temperature tolerance suggested by this statement.

A) 19 ± 4°C
B) 23 ± 4°C
C) 21 ± 2°C

$\frac{19+23}{2} = 21$ *and* $\frac{23-19}{2} = 2$

21 ± 2°C ⇒

j) 'A softball must weigh between 177.2 g and 198.4 g.' Choose the description for the mass tolerance given this statement.

A) 198.4 ± 21.2 g
B) 187.8 ± 10.6 g
C) 177.2 ± 21.2 g

Skill 22.5 Calculating elapsed time and reading timetables.

Hint: When calculating elapsed time from am to pm, or pm to am, first find the time to midnight or midday.

Q. How long is the flight from Singapore to London?

[Hint: Singapore time is 8 hours ahead of London time.]

Flights Out: Melbourne to London - Saturday 9 Feb 08

From		To		Flight	Duration
14:00	Melbourne	15:20	Sydney	◣ QF438	27h 20m
17:50	Sydney	**06:20**	London ^	◣ QF31	
17:10	Melbourne	21:30	Singapore	◣ QF9	22h 50m
22:45	Singapore	**05:00**	London ^	■ QF3345	

^ = next day

A. *Singapore departure time = 22:45*

(London time = 22:45 less 8 h = 14:45)

Consider time difference

Flight time (Using London time) = 14:45 to 05:00

14:45 to 24:00 = 9 h 15 min

Compare times from one zone

First find time to midnight

9 h 15 min + 5 h

= ***14 h 15 min***

a) How many minutes from 8:30 pm until 2:10 am the next day?

First find time to midnight

8:30 pm to 12:00 = 3 h 30 min

3 h 30 min + 2 h 10 min = 5 h 40 min

300 min + 40 min = 340 min

b) How many minutes from 2:45 am until 3:20 pm the same day?

= min

c) Express in minutes:

3 hours and 52 minutes =

min

d) Express in seconds:

5 minutes and 14 seconds =

s

e) At 0520 hours on a Friday in Brisbane, what day and time is it at the Vatican given that the Vatican is 9 hours behind Brisbane time?

=

f) Greta departs Canberra on Monday at 1000 hours and arrives in Los Angeles (LA) on Monday at 1015 hours. If LA time is 17 hours behind Canberra, how long was the flight?

= h min

g) If it was 2:45 pm on the 9th of March 2010, how long would have to wait until the next high tide at Mooloolaba Beach?

Mooloolaba Beach (QLD) Tide data:

Friday 9th March 2010		Saturday 10th March 2010	
05:00 am	0.62 m Low	05:45 am	0.73 m Low
09:46 am	1.37 m High	11:20 am	1.25 m High
04:55 pm	0.43 m Low	05:29 pm	0.5 m Low
11:33 pm	1.51 m High		

h min

h) What is the latest tram you can take from Melbourne University to get to South Melbourne Beach by 6:30 am?

Monday to Friday East Coburg to South Melbourne Beach

Route 1 via Brunswick > Carlton > City > Sth Melbourne

Stop		AM	AM	AM	AM	AM	AM	AM	AM
135	East Coburg - Bell St						5:40	5:50	6:00
112	Elgin St & Lygon St	4:59	5:11	5:35	5:46	5:56	6:06	6:16	6:26
1	Melbourne University	5:01	5:13	5:25	5:37	5:48	5:58	6:08	6:18
13	Federation Square	5:12	5:24	5:36	5:48	5:59	6:09	6:16	6:29
14	Arts Centre	5:14	5:26	5:38	5:50	6:01	6:11	6:21	6:31
16	Southbank Blvd & St Kilda Rd	5:15	5:27	5:39	5:51	6:02	6:12	6:22	6:32
32	South Melbourne Beach	5:27	5:39	5:51	6:03	6:14	6:24	6:34	6:54

Skill 22.6 Converting units of measurement for length.

Mauve 1 1 2 2 **3** 3 4 4
Lime 1 1 2 **2** 3 3 4 4

- Find the conversion factor. (see Maths Facts, page 455)

Q. The lake path around Canberra's Lake Burley Griffin measures 21.25 km. Express the length of the path in metres.

A. *1 km = 1000 m* Conversion factor

21.25 km × 1000

= *21 250 m*

a) Convert 630 millimetres to centimetres. mm to cm: ÷ 10

630 mm ÷ 10 = 63 cm

b) Convert 645 centimetres to millimetres.

c) Express in millimetres:

1 m and 12 mm =

d) Express in metres:

3 m and 2100 cm =

e) Write in centimetres:

2 m and 760 mm =

f) A world pole vault record was set in 1994 by Sergei Bubka of 6.14 m. Is this record <, = or > 6014 mm?

g) Mike Powell holds the world long jump record of 8.95 m. Is this record <, = or > 8950 cm?

h) Which distance is greater?

A) running 2 heats and the final in the 200 m
B) swimming 0.7 km

i) Which basketball organisation has their 3 point throw line further from the ring?

A) National Basketball Association - 7.24 m
B) International Basketball Federation - 625 cm

j) The blood vessels of a typical adult are approximately 160 000 000 m long. If it is 40 000 km around the equator, how many times would a person's blood vessels stretch around the earth?

k) Place in descending order:

301 cm, 3.1 m and 3001 mm convert all to mm

l) Place in ascending order:

5900 cm, 5.9 km and 590 m

Skill 22.7 Converting units of measurement for mass.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find the conversion factor. (see Maths Facts, page 455)

Q. The ‘Mogul’ emerald weighs 43.56 g.
Express this weight in milligrams.

A. *1 g = 1000 mg* Conversion factor

43.56 g × 1000

= ***43 560 mg***

a) Convert 7500 milligrams to grams. mg to g: ÷ 1000

7500 mg ÷ 1000 = 7.5 g

b) Convert 0.001 kilograms to grams.

c) Express in kilograms:

1300 g =

d) Write in kilograms:

0.08 tonnes and 800 g =

e) Weight lifter Antonio Krastev in 1987 lifted 216 kg in the ‘snatch’. Is this world record <, = or > 216 000 g?

f) The 4 cables of the Brooklyn Bridge can together sustain a load of about 44 000 tonnes. What load can 1 cable sustain in kg?

kg

g) The ‘Hand of Faith’ gold nugget weighs 27.2 kg. Express this weight in milligrams (mg).

h) If a heavier car is a safer car, which car is safer?

A) 1996 Holden HR weighing 1.178 tonnes
B) 2006 Holden Calais weighing 1642 kg

i) The Olympic flyweight boxing class is between 48 kg and 51 kg. Express this weight difference in grams.

j) A baseball weighs 142 g. If a baseball bat weighs 5 times as much as the ball, how much does the bat weigh in kilograms?

k) Place in ascending order:

2 kg, 2002 g and 0.02 tonne convert all to g

l) Place in descending order:

55 000 mg, 550 g and 5.5 kg

Skill 22.8 Converting units of measurement for capacity and cubic volume.

Mauve 11 22 33 44
Lime 11 22 33 44

- Find the conversion factor. (see Maths Facts, page 455)

Q. Convert 0.0006 megalitres to litres.

A. *1 ML = 1 000 000 L* Conversion factor

0.0006 ML × 1 000 000

= 600 L

a) Convert 50 millilitres to litres. mL to L: ÷ 1000

50 mL ÷ 1000 = 0.05 L

b) Convert 15 000 000 litres to megalitres (ML).

c) Express in millilitres:

3 L and 75 mL =

d) Express in litres:

3 L and 600 000 mL =

e) Express in litres:

3 ML and 200 L =

f) Express in millilitres:

1 L and 32 mL =

g) The average total lung capacity of a healthy teenager is 5800 mL. Express this in litres.

h) If a cup holds 250 mL, how many cups would you need to fill a 1.25 L bottle?

i) Moscow's biggest fountain in Manezhnaya Square holds 780 m^3 of water. Is this <, = or > 78 000 000 cm^3?

j) The human body carries approximately 4700 cubic centimetres of blood. Is this <, = or > 4.7 m^3?

k) An orange when squeezed provided 67.5 mL of juice and a grapefruit 0.25 L. Find the difference in millilitres.

l) Place in order from smallest to largest:

0.0068 ML, 68 L and 680 000 mL

convert all to L

Skill 22.9 Converting units of measurement for area.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find the conversion factor. (see Maths Facts, page 455)

Q. The area of the Bayeux Tapestry is 35 m^2. Express this area in square centimetres.

A. *1 m^2 = 10 000 cm^2* Conversion factor

35 m^2 × 10 000

$= \mathbf{350\,000\ cm^2}$

a) Convert 44 cm^2 to mm^2. cm^2 to mm^2: × 100

44 cm^2 × 100 = ... 4400 mm^2

b) Express in square millimetres:

25 cm^2 and 500 mm^2 =

....

c) The surface area of the lungs of a human is 160 m^2. Is this area <, = or > 160 000 cm^2?

....

d) The area of a championship billiard table is 3.76 m^2. Express this area in square millimetres.

....

e) Auckland, New Zealand, has an area of approximately 650 km^2. Is this <, = or > 65 000 000 m^2?

....

f) Uluru National Park has an area of approximately 132 500 hectares. Express this area in km^2.

....

g) Graceland, Elvis Presley's estate, is 5.5 hectares. Is this <, = or > 55 000 m^2?

....

h) The area of Trafalgar Square is 0.121 km^2. Express this area in square metres.

....

i) The Philippines has an area of 30 million hectares. Indonesia is approximately 1 920 000 km^2. Which country is the biggest?

....

j) The soccer goal area between the posts, the ground and the crossbar is approximately 178 000 cm^2. Express this area in m^2.

....

k) Place in order from largest to smallest:

2 cm^2, 0.02 m^2 and 2000 mm^2

....

....

l) Place in order from largest to smallest:

700 cm^2, 0.7 m^2 and 7 000 000 mm^2

....

....

Skill 22.10 Converting between units of measurement for capacity and cubic volume.

- Find the conversion factor. (see Maths Facts, page 455)

Q. The volume of a cement truck is 6.3 m^3. After 3150 L of cement is unloaded, how many litres of cement are left?

A. $1\ m^3 = 1000\ L$ Conversion factor

$6.3\ m^3 \times 1000$

$= 6300\ L$

$6300 - 3150$

$= \mathbf{3150\ L}$

cm^3 to L: ÷ 1000

a) Convert 500 000 cubic centimetres to litres.

$500\,000 \div 1000 =$ 500 L

b) Change 2.3 litres to cubic centimetres.

c) Express in litres:

24 m^3 =

d) Write in millilitres:

30 000 cm^3 =

e) Express in litres:

2 L and 4000 cm^3 =

f) Write in litres:

3000 L and 500 000 cm^3 =

g) The capacity of a cement mixer is 350 L. How many cubic centimetres is this?

h) What volume of milk in cubic metres could be in a milk tanker with capacity of 26 million millilitres?

i) The dosage of medicine is 5 mL. How many cubic millimetres of volume would this equal?

j) A sprinkler uses 250 L of water every 15 minutes. How many cubic metres of water would be used after 1 hour?

k) Place in descending order:
45 m^3, 4500 L and 45 000 mL

convert all to L

l) Place in ascending order:
850 mL, 8.5 L and 85 cm^3

23. [Perimeter / Area]

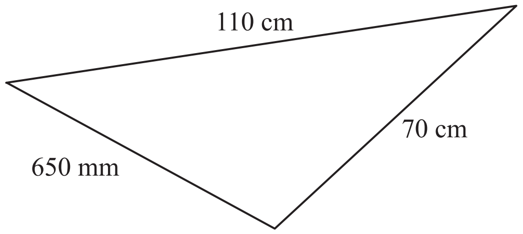
Skill 23.1 Calculating the perimeter of polygons (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Convert all measurements to the same unit.
- Find and label the length of all sides.
- Add together all side lengths.

Hints: Sides marked with a dash (|) are of equal length.
Sides marked with two dashes (||) are of equal length etc.

Q. Find the perimeter of the scalene triangle in centimetres.



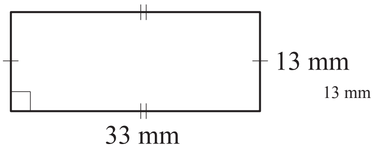
A. $650\text{ mm} = 650 \div 10\text{ cm}$
$= 65\text{ cm}$

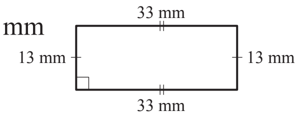
Convert mm to cm

$P = 65\text{ cm} + 110\text{ cm} + 70\text{ cm}$

$P = \mathbf{245\ cm}$

a) Find the perimeter of the rectangle.

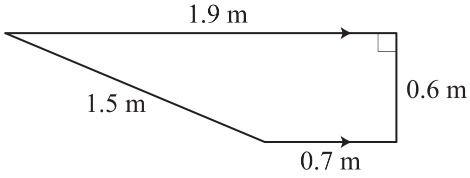




$P = 33 + 33 + 13 + 13$

$= 66 + 26$ = ☐ mm

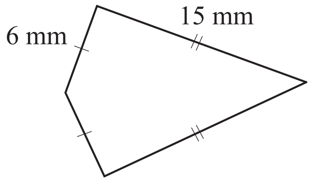
b) Find the perimeter of the trapezium.



$P = 1.5 +$

= = ☐ m

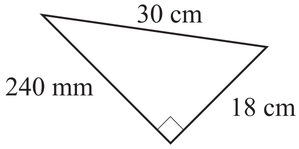
c) Find the perimeter of the kite.



$P =$

= = ☐ mm

d) Find the perimeter of the right-angled triangle in millimetres.



$P =$ = ☐

e) What is the perimeter of a regular heptagon with sides measuring 14 m?

$P =$

= = ☐ m

f) What is the perimeter in centimetres of a rhombus with a side length measuring 125 mm?

$P =$ = ☐

Skill 23.1 Calculating the perimeter of polygons (2).

g) What is the perimeter in centimetres of an isoceles triangle with congruent sides of 15 m and the other side measuring 1.5 m?

$P =$ $=$ ☐

h) Find the perimeter in centimetres of a parallelogram with side lengths measuring 202 cm and 100 mm.

$P =$ $=$ ☐

i) The smallest ever postage stamp came from Colombia. Rectangular, it measured 7.85 mm by 9.4 mm. What was its perimeter in cm?

$P =$

$=$ $=$ ☐

j) An Australian $20 note measures 14.4 cm by 6.5 cm. What is its perimeter in millimetres?

$P =$

$=$ $=$ ☐

k) Lisa's backyard is a rectangle measuring 28 m in length and 12 m in width. What will the perimeter of the backyard be?

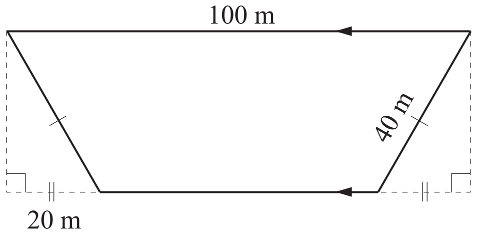
$P =$

$=$ $=$ ☐ m

l) Find the perimeter in centimetres of a kite with side lengths measuring 180 cm and 750 mm.

$P =$ $=$ ☐

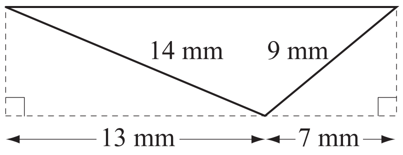
m) Find the perimeter of the trapezium.



$P =$

$=$ $=$ ☐ m

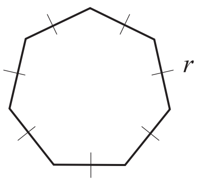
n) Find the perimeter of the triangle.



$P =$

$=$ $=$ ☐ mm

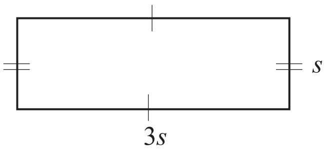
o) Write an algebraic expression for the perimeter P of the heptagon. [Express the answer in terms of r.]



$P =$

$=$ $P =$

p) Write an algebraic expression for the perimeter P of the rectangle. [Express the answer in terms of s.]



$P =$

$=$ $P =$

Skill 23.2 Calculating the perimeter of composite shapes.

Mauve 11 22 33 44
Lime 11 22 33 44

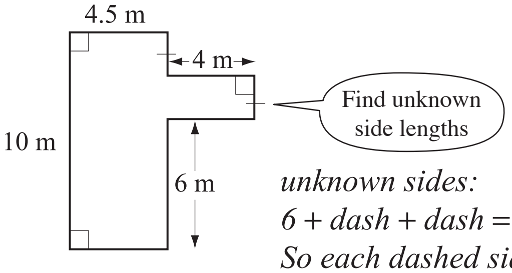
- Find and label the length of all sides.
- Add together all side lengths.

Hints: Sides marked with a dash ( ) are of equal length.
Sides marked with two dashes ( ) are of equal length etc.

OR

- Manipulate shapes to become rectangles by pushing out inverted corners.

Q. Find the perimeter of the shape.

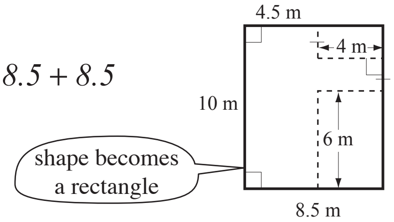


unknown sides:
6 + dash + dash = 10 m
So each dashed side = 2 m

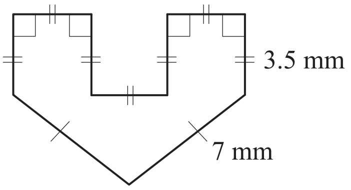
A. $P = 10 + 4.5 + 2 + 4 + 2 + 4 + 6 + 4.5$
$= 14.5 + 8 + 10 + 4.5$
$= \mathbf{37\ m}$

OR

$P = 10 + 10 + 8.5 + 8.5$
$= 20 + 17$
$= \mathbf{37\ m}$



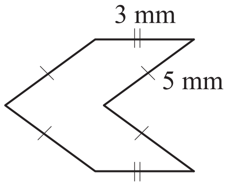
a) Find the perimeter of the shape.



P = 3.5 + 7 + 7 + 3.5 + 3.5 + 3.5 + 3.5 + 3.5 + 3.5

= 14 + 24.5 = ☐ mm

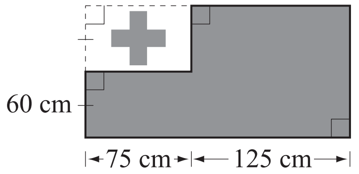
b) Find the perimeter of the shape.



P =

= = ☐ mm

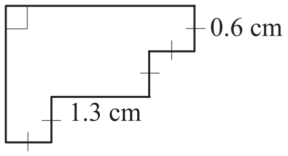
c) Find the perimeter in centimetres around the coloured background of this Tongan flag.



P =

= = ☐ cm

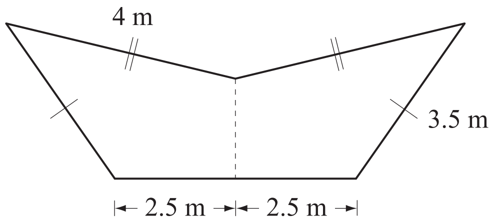
d) Find the perimeter of the shape.



P =

= = ☐ cm

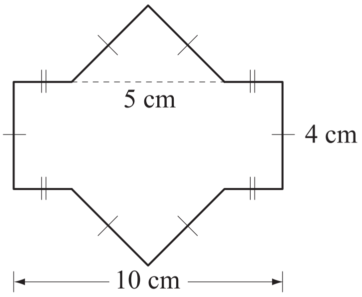
e) Find the perimeter of the shape.



P =

= = ☐ m

f) Find the perimeter of the shape.



P =

= = ☐ cm

Skill 23.3 Calculating the circumference of circles.

Mauve 11 22 33 **4**4
Lime 11 22 **3**3 44

- Substitute known values into the formula.
  Hints: The diameter of a circle is equal to twice the radius.
  π (pi) gets its value because the diameter of any circle fits approximately 3.14 times around the circumference.

Circumference of a circle

$C = 2 \times \pi \times \text{radius}$

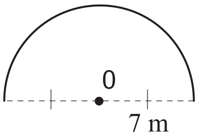
$C = 2\pi r$

OR $C = \pi \times \text{diameter}$

$C = \pi d$

where $\pi \approx 3.14\ldots$ or $\frac{22}{7}$

Q. Using $C = 2\pi r$ where $\pi \approx \frac{22}{7}$, find the length of the semicircle.



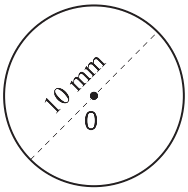
A. $C = 2\pi r$

$= 2 \times \frac{22}{\cancel{7}} \times \cancel{7}$ Simplify: ÷ 7

$= 44$

$\frac{1}{2}C = \frac{1}{2} \times 44 = \mathbf{22\ m}$

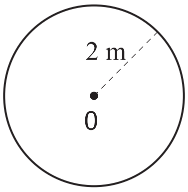
a) Using $C = 2\pi r$ where $\pi \approx 3.14$, find the circumference of the circle.



$C = \pi d$ where $d = 10$

$= 10 \times 3.14 \quad = \boxed{\qquad \text{mm}}$

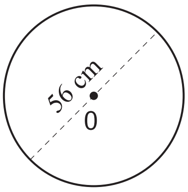
b) Using $C = 2\pi r$ where $\pi \approx 3.14$, find the circumference of the circle.



$C = 2\pi r$

$= \qquad = \boxed{\qquad \text{m}}$

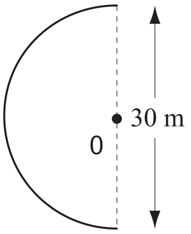
c) Using $C = 2\pi r$ where $\pi \approx \frac{22}{7}$, find the circumference of the circle.



$C =$

$= \qquad = \boxed{\qquad \text{cm}}$

d) Using $\pi \approx 3.14$ find the length of the semicircle.



$C =$

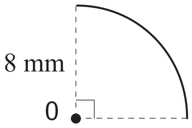
$\frac{1}{2}C = \qquad = \boxed{\qquad \text{m}}$

e) The diameter of a circular discus is 2.5 m. Using $\pi \approx 3.14$ what is the circumference?

$C =$

$= \qquad = \boxed{\qquad \text{m}}$

f) Using $\pi \approx 3.14$ find the length of the quarter circle.



$C =$

$\frac{1}{4}C = \qquad = \boxed{\qquad \text{mm}}$

Skill 23.4 Calculating the perimeter of composite circular shapes (1).

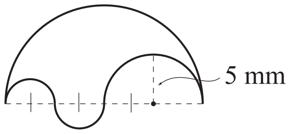
Mauve 11 22 33 44
Lime 11 22 33 44

Circumference of a circle

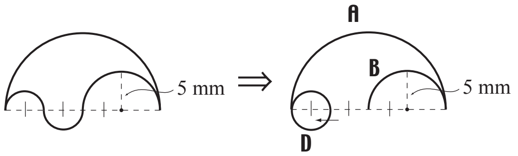
$C = 2\pi r = \pi d$

- Find and label the length of all sides.
- Break the shape into workable parts.
- For circular shapes substitute known values into the formula for the circumference:
  Hint: Consider 2 congruent semicircles equal 1 full circle.
- Add together all side lengths.
  Hints: Sides marked with a dash (|) are of equal length.
  Sides marked with two dashes (||) are of equal length etc.

Q. Find the perimeter of the shape.
(Use $\pi \approx 3.14$)



A.



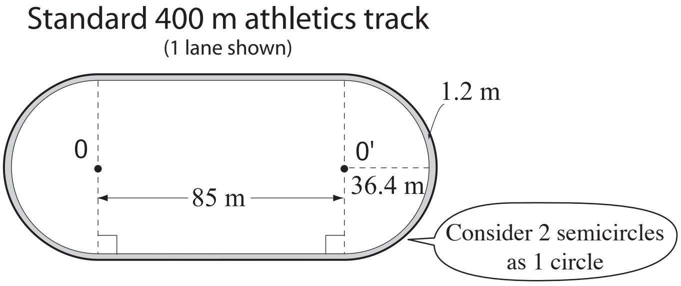
semicircle A
$C = 2\pi r$ where $r = 10$
$= 2 \times 3.14 \times 10 = 62.8$
A $= 62.8 \div 2 = 31.4$

semicircle B
$C = 2\pi r$ where $r = 5$
$= 2 \times 3.14 \times 5 = 31.4$
B $= 31.4 \div 2 = 15.7$

circle D
$C = \pi d$ where $d = 5$
D $= 3.14 \times 5 = 15.7$

shape $= 31.4 + 15.7 + 15.7$
$= \mathbf{62.8\ mm}$

a) Using $C = 2\pi r$ where $\pi \approx 3.14$, find the perimeter around the outside of the first lane of an athletics track.



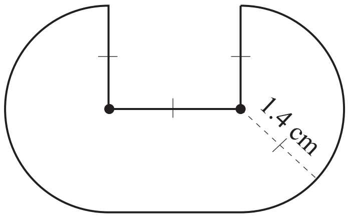
$C = 2\pi r$ where $r = 36.4 + 1.2 = 37.6$

$C = 2 \times 3.14 \times 37.6 = 236.128$

$85 + 85 = 170$

$P = 236.128 + 170 =$ 406.128 m

b) Find the perimeter of the shape.
(Use $\pi \approx \frac{22}{7}$)

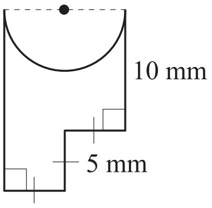


$C = 2\pi r$

$P =$ $=$ cm

Skill 23.4 Calculating the perimeter of composite circular shapes (2).

c) Using $C = 2\pi r$ where $\pi \approx 3.14$, find the perimeter of the shape.



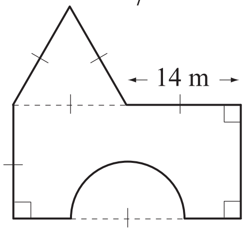
$C = 2\pi r$ where $r =$

$C =$

$P =$

$=$ $=$ $\boxed{\quad \text{mm}}$

d) Find the perimeter of the shape.
(Use $\pi \approx \frac{22}{7}$)

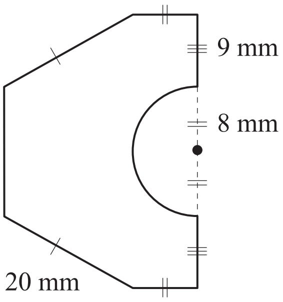


$C = 2\pi r$

$P =$

$=$ $=$ $\boxed{\quad \text{m}}$

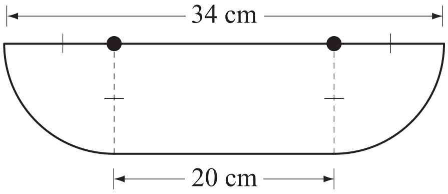
e) Using $C = 2\pi r$ where $\pi \approx 3.14$, find the perimeter of the shape.



$P =$

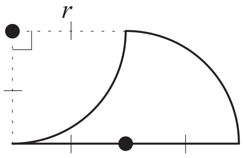
$=$ $=$ $\boxed{\quad \text{mm}}$

f) Find the perimeter of the shape.
(Use $\pi \approx \frac{22}{7}$)



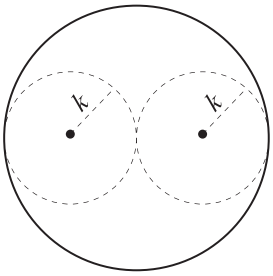
$\boxed{\quad \text{cm}}$

g) Write an algebraic expression for the perimeter P of the shape. [Express the answer in terms of r and π.]



$\boxed{P =}$

h) Write an algebraic expression for the circumference P of the outer circle.
[Express the answer in terms of k and π.]



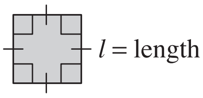
$\boxed{P =}$

Skill 23.5 Calculating the area of squares and rectangles.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

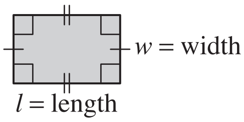
- Substitute known values into the appropriate formula.

Area of a square



$A = l \times l$
$= l^2$

Area of a rectangle



$A = l \times w$
$= lw$

Q. A boxing ring is a square with side length 5.2 m. What is the area of the ring?

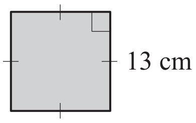
A. $A = l^2$
$= 5.2 \times 5.2\ m$
$= \mathbf{27.04\ m^2}$

a) What is the area of a rectangular billiard table with a length of 3.7 m and a width of 1.9 m?

$A = l \times w$

$= 3.7 \times 1.9$ = m^2

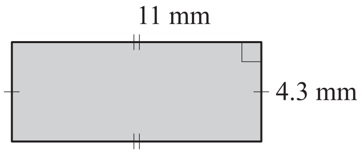
b) Find the area of the square.



$A =$

$=$ = cm^2

c) Find the area of the rectangle.



$A =$

$=$ = mm^2

d) A baseball diamond is a square of side length of approximately 27 m. What is its area?

$A =$

$=$ = m^2

e) The rectangular grounds of the Taj Mahal are 360 m long and 260 m wide. What is its area?

$A = l \times w$

$=$ = m^2

f) A rectangular badminton court measures approximately 13.5 m long and 6 m wide. What is its area?

$A =$

$=$ = m^2

g) What is the perimeter of a square with an area of 400 cm^2?

length =

$P =$ = cm

h) Paddy's rectangular iPod screen has an area of 720 mm^2. What is the perimeter of the screen, if the length measures 30 mm?

width =

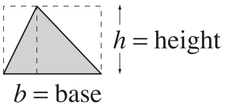
$P =$ = mm

Skill 23.6 Calculating the area of triangles.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

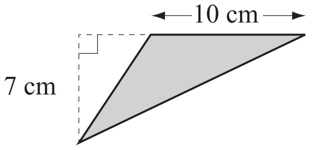
- Substitute known values into the formula:

Area of a triangle



$$A = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2}bh$$

Q. Find the area of the scalene triangle.

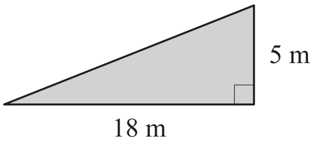


A. $A = \frac{1}{2}bh$

$= \frac{1}{\cancel{2}_{1}} \times \cancel{10}^{5} \times 7$ Simplify: ÷ 2

$= \boldsymbol{35\ cm^2}$

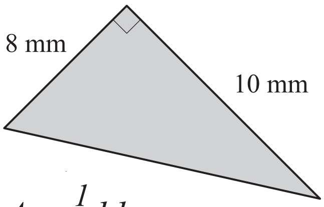
a) Find the area of the right-angled triangle.



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 5$ = m^2

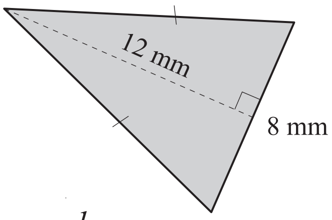
b) Find the area of the right-angled triangle.



$A = \frac{1}{2}bh$

= = mm^2

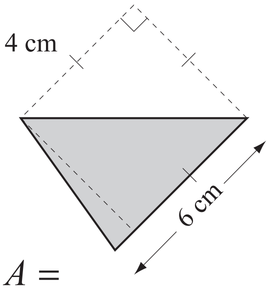
c) Find the area of the isoceles triangle.



$A = \frac{1}{2}bh$

= = mm^2

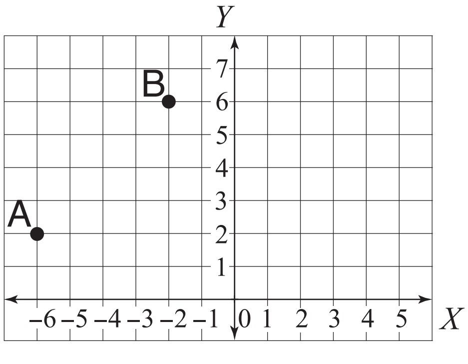
d) Find the area of the scalene triangle.



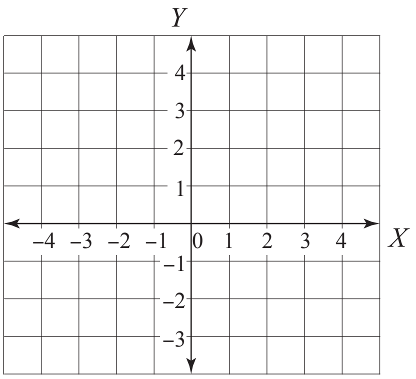
$A =$

= = cm^2

e) Plot the points A(−6,2), B(−2,6) and C(5,2) and use them to find the area of ΔABC.



f) Plot the points A(−2,3), B(3,3) and C(−2,−3) and use them to find the area of ΔABC.

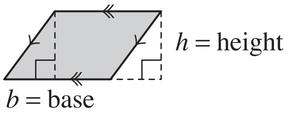


Skill 23.7 Calculating the area of parallelograms.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

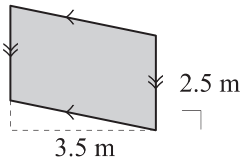
- Substitute known values into the formula.

Area of a parallelogram



$$A = b \times h$$
$$= bh$$

Q. Find the area of the parallelogram.

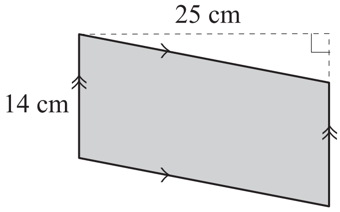


A. $A = bh$

$= 2.5 \times 3.5\ m$

$= \mathbf{8.75\ m^2}$

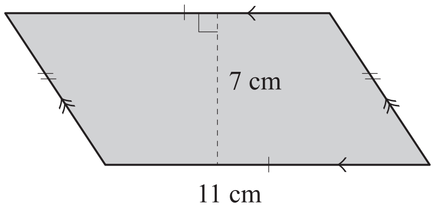
a) Find the area of the parallelogram.



$A = bh$ ……………………

$= 14 \times 25$ …………… = cm^2

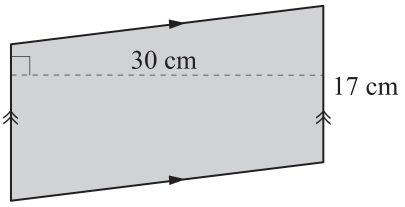
b) Find the area of the parallelogram.



$A =$ ……………………

$=$ …………… = cm^2

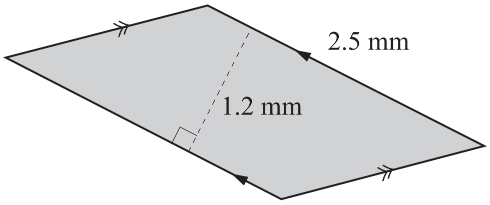
c) Find the area of the parallelogram.



$A =$ ……………………

$=$ …………… = cm^2

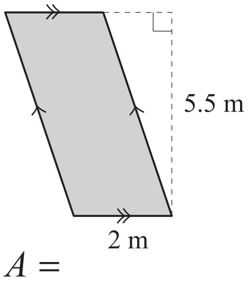
d) Find the area of the parallelogram.



$A =$ ……………………

$=$ …………… = mm^2

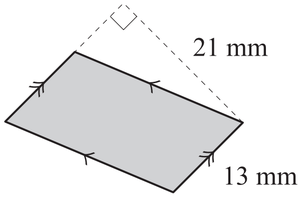
e) Find the area of the parallelogram.



$A =$ ……………………

$=$ …………… = m^2

f) Find the area of the parallelogram.



$A =$ ……………………

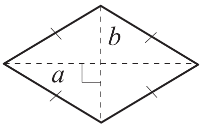
$=$ …………… = mm^2

Skill 23.8 Calculating the area of rhombi and kites.

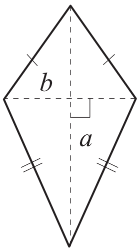
Mauve 11 22 33 44
Lime 11 22 33 44

- Substitute known values into the formula.

Area of a rhombus or kite



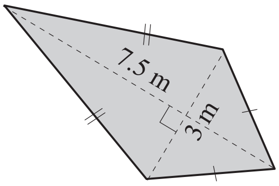
OR



$A = \frac{1}{2} \times a \times b$ (where a is the long diagonal and b is the short diagonal)

$= \frac{1}{2} ab$

Q. Find the area of the kite.

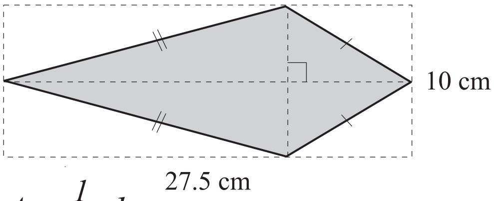


A. $A = \frac{1}{2}ab$

$= \frac{1}{2} \times 7.5 \times 3$

$= \mathbf{11.25\ m^2}$

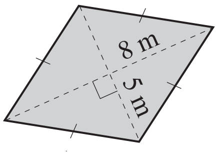
a) Find the area of the kite.



$A = \frac{1}{2}ab$

$= \frac{1}{2} \times 27.5 \times 10$ = cm^2

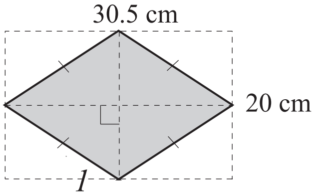
b) Find the area of the rhombus.



$A = \frac{1}{2}ab$

= = m^2

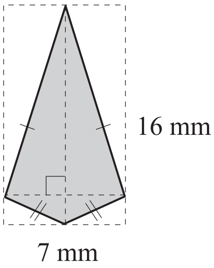
c) Find the area of the rhombus.



$A = \frac{1}{2}ab$

= = cm^2

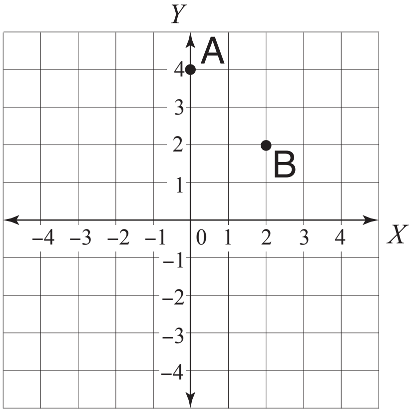
d) Find the area of the kite.



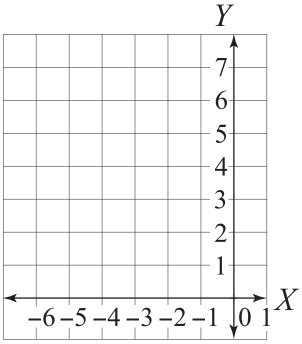
$A =$

= = mm^2

e) Plot the points A(0,4), B(2,2), C(0,–3) and D(–2,2) and use them to find the area of the kite ABCD.



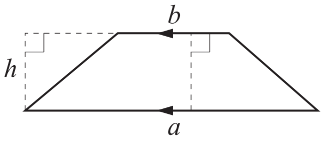
f) Plot the points A(–4,6), B(–2,3), C(–4,0) and D(–6,3) and use them to find the area of the rhombus ABCD.



Skill 23.9 Calculating the area of trapeziums.

- Substitute known values into the formula.

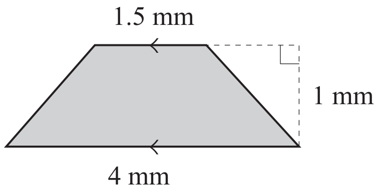
Area of a trapezium



$$A = \frac{1}{2} \times (a + b) \times \text{height}$$ (where a and b are the parallel side lengths)

$$= \frac{1}{2}(a + b)h$$

Q. Find the area of the trapezium.



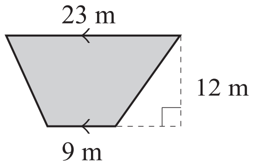
A. $A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (4 + 1.5) \times 1$

$= \frac{1}{2} \times 5.5$

$= \mathbf{2.75\ mm^2}$

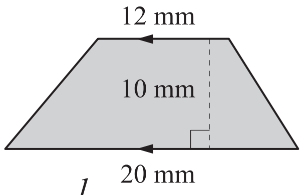
a) Find the area of the trapezium.



$A = \frac{1}{2}(a + b)h = \frac{1}{2} \times (23 + 9) \times 12$

$= \frac{1}{2} \times 32 \times 12$ = m^2

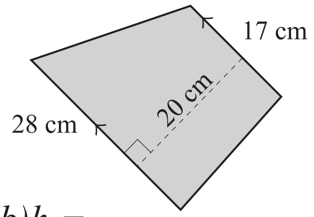
b) Find the area of the trapezium.



$A = \frac{1}{2}(a + b)h =$

$=$ $=$ mm^2

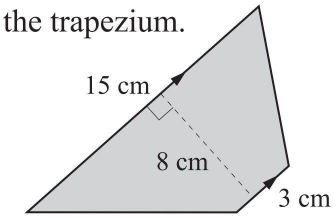
c) Find the area of the trapezium.



$A = \frac{1}{2}(a + b)h =$

$=$ $=$ cm^2

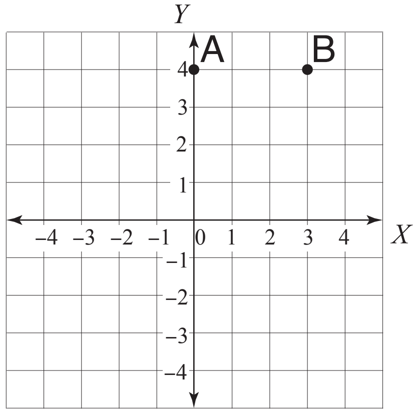
d) Find the area of the trapezium.



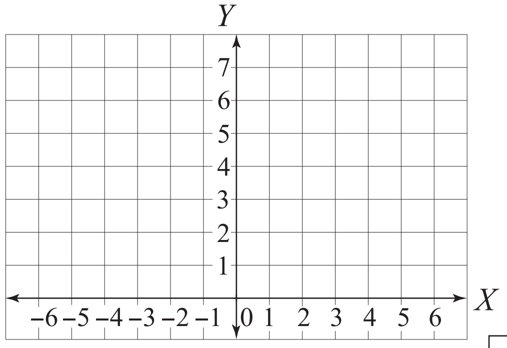
$A =$ $=$

$=$ $=$ cm^2

e) Plot the points A(0,4), B(3,4), C(3,–2) and D(–4,–2) and use them to find the area of the trapezium ABCD.



f) Plot the points A(–4,5), B(4,6), C(4,1) and D(–4,4) and use them to find the area of the trapezium ABCD.

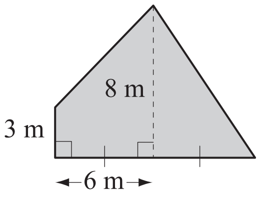


Skill 23.10 Calculating the area of composite shapes (1).

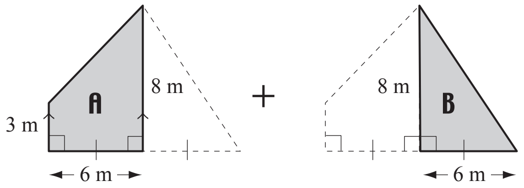
Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find and label the length of all sides.
- Break the shape into workable parts.
- Where possible substitute values into a known area formula.
  (see skill 23.5, page 265 to skill 23.9, page 269)
- Add or subtract the area totals where necessary.

Q. Find the area of the polygon.



A.



$$\textbf{A} = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2} \times (3+8) \times 6$$
$$= \frac{1}{2} \times 11 \times 6 = 33$$

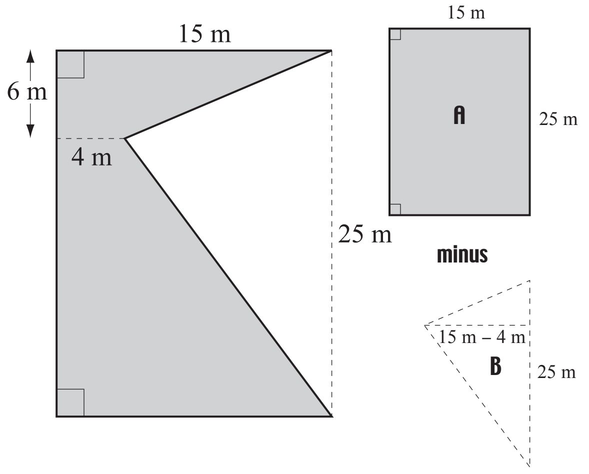
trapezium A

$$\textbf{B} = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 6 \times 8$$
$$= \frac{1}{2} \times 48 = 24$$

triangle B

shape $= A + B = 33 + 24 =$ ***57 m²***

a) Find the area of the shape.



A $= lw$ *(a rectangle)*

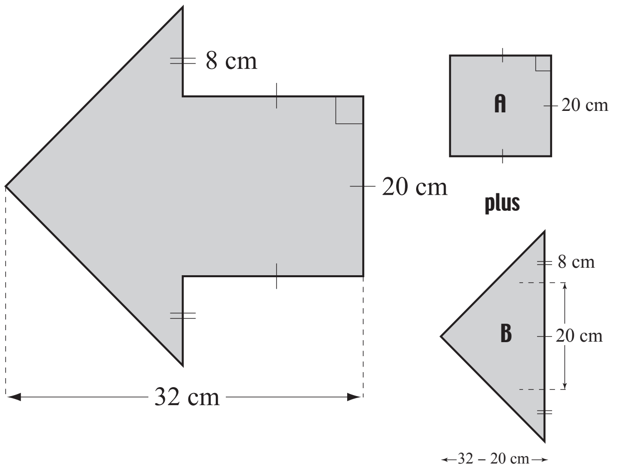
$= 25 \times 15 = 375$

B $= \frac{1}{2}bh$ *(a triangle)*

$= \frac{1}{2} \times 25 \times 11 = \frac{1}{2} \times 275 = 137.5$

shape $= 375 - 137.5 =$ $\boxed{237.5 \text{ m}^2}$

b) Find the area of the polygon.



A $= l^2$

$=$

B $=$

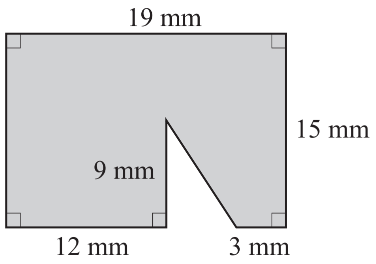
$=$ $=$ $=$

shape $=$ $=$ $\boxed{\quad \text{cm}^2}$

Skill 23.10 Calculating the area of composite shapes (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

c) Find the area of the shape.



A =

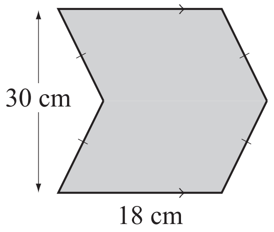
=

B =

=

shape = = mm^2

d) Find the area of the polygon.



A =

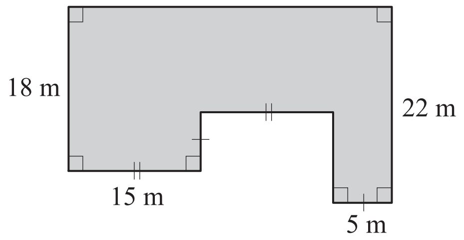
=

B =

=

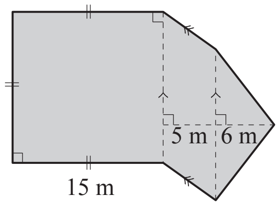
shape = = cm^2

e) Find the area of the shape.



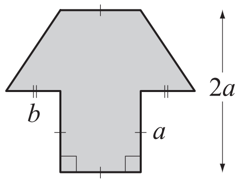
shape = = m^2

f) Find the area of the polygon.



shape = = m^2

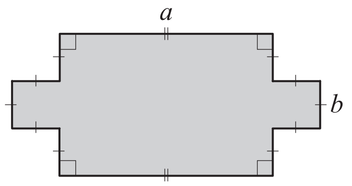
g) Write an algebraic expression for the area A of the shape. [Express the answer in terms of a and b.]



A =

A =

h) Write an algebraic expression for the area A of the shape. [Express the answer in terms of a and b.]



A =

A =

Skill 23.11 Calculating the area of circles.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Substitute known values into the formula:
  Hint: The diameter of a circle is equal to twice the radius.
  Pi (π) gets its value because the diameter of any circle fits approximately 3.14 times around the circumference.

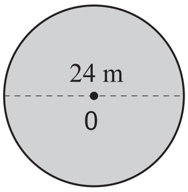
Area of a circle

$A = \pi \times \text{radius} \times \text{radius}$

$A = \pi r^2$

where $\pi \approx 3.14...$ or $\frac{22}{7}$

Q. Using $A = \pi r^2$ where $\pi \approx 3.14$, find the area of the circle.



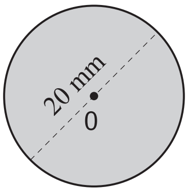
A. $A = \pi r^2$ *where* $d = 24$, *so* $r = 12$

$= 3.14 \times 12 \times 12$

$= 3.14 \times 144$

$= \mathbf{452.16\ m^2}$

a) Using $A = \pi r^2$ where $\pi \approx 3.14$, find the area of the circle.

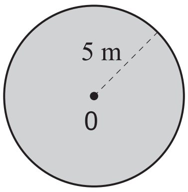


$A = \pi r^2$ *where* $d = 20$ *so* $r = 10$ *mm*

$= 3.14 \times 10 \times 10$

$= 3.14 \times 100$ = ☐ mm^2

b) Using $A = \pi r^2$ where $\pi \approx 3.14$, find the area of the circle.

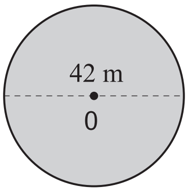


$A = \pi r^2$

=

= ……… = ☐ m^2

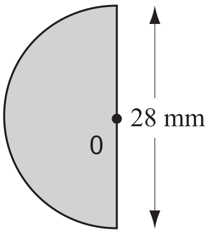
c) Using $A = \pi r^2$ where $\pi \approx \frac{22}{7}$, find the area of the circle.



………

……… = ☐ m^2

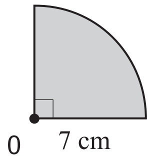
d) Using $\pi \approx \frac{22}{7}$ find the area of the semicircle.



………

……… = ☐ mm^2

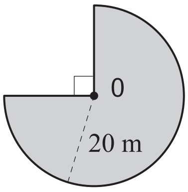
e) Using $\pi \approx \frac{22}{7}$ find the area of the quarter circle.



………

……… = ☐ cm^2

f) Using $\pi \approx 3.14$ find the area of the shape.



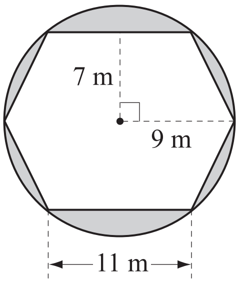
………

……… = ☐ m^2

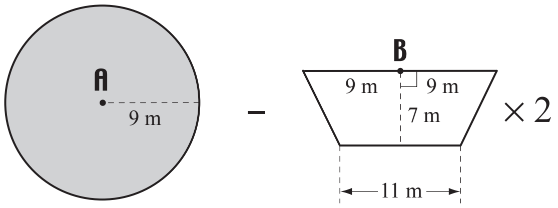
Skill 23.12 Calculating the area of composite circular shapes (1).

- Find and label the length of all sides.
- Break the shape into workable parts.
- Where possible substitute values into a known area formula.
  (see skills 23.5 to skill 23.9, pages 265 to 269 and skill 23.11, page 272)
- Add or subtract the area totals where necessary.

Q. Use $A = \pi r^2$ where $\pi \approx 3.14$, to find the area of the shaded shape.



A.



$$\mathbf{A} = \pi r^2 \textit{ where } r = 9$$
$$= 3.14 \times 9 \times 9$$
$$= 3.14 \times 81$$
$$= 254.34$$

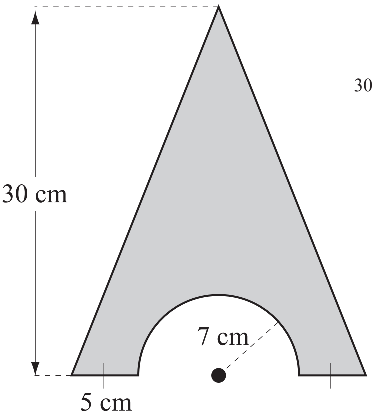
circle A

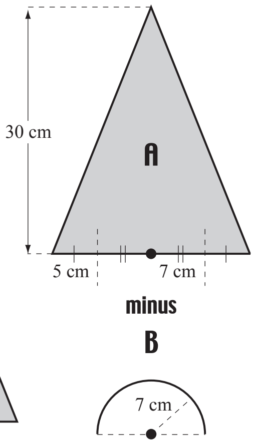
$$A = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2} \times (18 + 11) \times 7 = \frac{1}{2} \times 203$$
$$\mathbf{B} = \frac{1}{2} \times 203 \times 2 = 203$$

trapezium ×2 B

shape $= A - B = 254.34 - 203 =$ **51.34 m^2**

a) Use $A = \pi r^2$ where $\pi \approx \frac{22}{7}$, to find the shaded area.





$\mathbf{A} = \frac{1}{2}bh$ (a triangle)

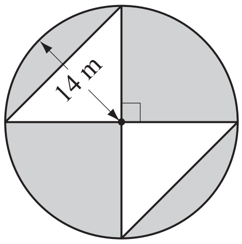
$= \frac{1}{2} \times (5 + 7 + 7 + 5) \times 30 = 360$

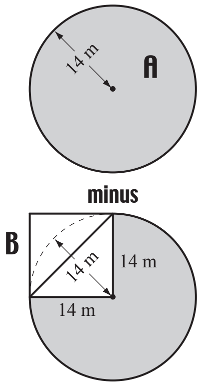
$\mathbf{B} = \frac{1}{2}\pi r^2,\ r = 7$ (a semicircle)

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 11 \times 7 = 77$

shape $= 360 - 77$ = ☐ cm^2

b) Use $A = \pi r^2$ where $\pi \approx \frac{22}{7}$, to find the shaded area.





$\mathbf{A} = \pi r^2,\ r = 14$ (a circle)

= =

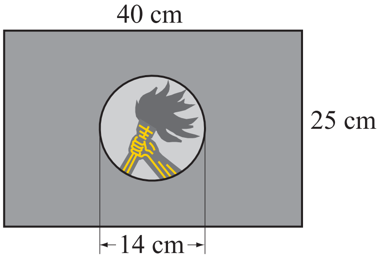
$\mathbf{B} = l^2$ (a square)

= =

shape = = ☐ m^2

Skill 23.12 Calculating the area of composite circular shapes (2).

c) Use $A = \pi r^2$ where $\pi \approx \frac{22}{7}$, to find the area of the background colour of the flag of Zaire, without the central circle.



=

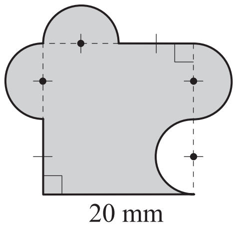
= =

=

= =

shape = = $\boxed{\quad \text{cm}^2}$

d) Use $A = \pi r^2$ where $\pi \approx 3.14$, to find the area of the shaded shape.



=

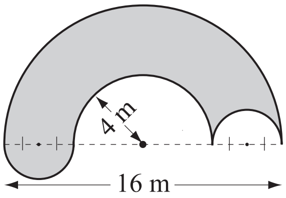
= =

=

= =

shape = = $\boxed{\quad \text{mm}^2}$

e) Use $A = \pi r^2$ where $\pi \approx 3.14$, to find the area of the shaded shape.



=

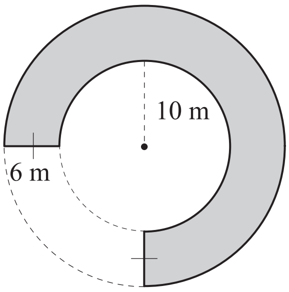
= =

=

= =

shape = = $\boxed{\quad \text{m}^2}$

f) Use $A = \pi r^2$ where $\pi \approx 3.14$, to find the area of the shaded shape.



=

= =

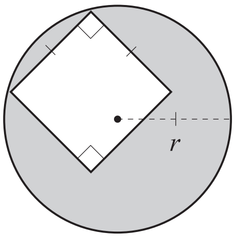
=

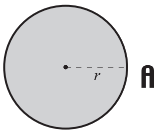
= =

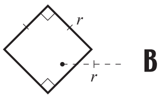
shape = = $\boxed{\quad \text{m}^2}$

Skill 23.12 Calculating the area of composite circular shapes (3).

g) Write an algebraic expression for the area A of the shaded shape. [Express the answer in terms of r and π.]







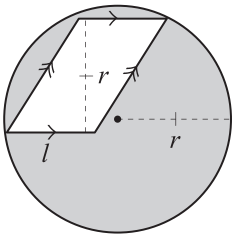
A = πr^2 *(a circle)*

B = l^2 *(a square)*

= r^2 OR $r^2(\pi - 1)$

shape = **A** – **B** $A = \pi r^2 - r^2$

h) Write an algebraic expression for the area A of the shaded shape. [Express the answer in terms of r, l and π.]



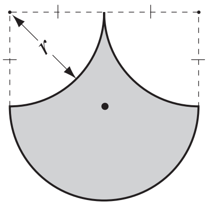
A = πr^2

B =

=

shape = $A =$

i) Write an algebraic expression for the area A of the shaded shape. [Express the answer in terms of r and π.]



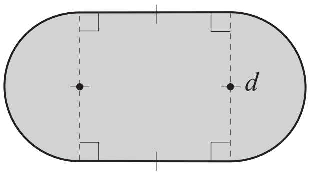
=

=

=

shape = $A =$

j) Write an algebraic expression for the area A of the shaded shape. [Express the answer in terms of d and π.]



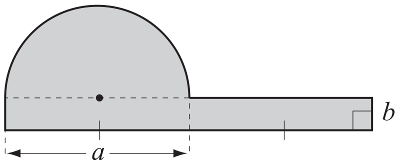
=

=

=

shape = $A =$

k) Write an algebraic expression for the area A of the shaded shape. [Express the answer in terms of a, b and π.]



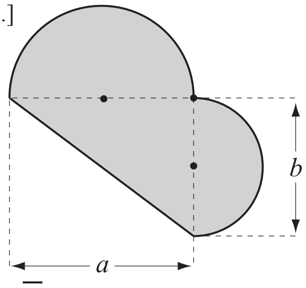
$A_{rectangle} =$

$A_{semicircle} =$

=

shape = $A =$

l) Write an algebraic expression for the area A of the shaded shape. [Express the answer in terms of a, b and π.]



$A_{triangle} =$

$A_{semicircle\ 1} =$

$A_{semicircle\ 2} =$

shape = $A =$

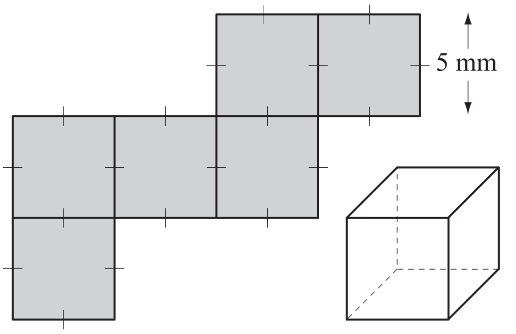
24. [Surface Area]

Skill 24.1 Calculating the total surface area (*TSA*) of rectangular prisms and cubes using nets (1).

Mauve **1** 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find any unknown side lengths.
- Calculate the area of each face as shown on the net.
  Hint: Rectangular prisms have 6 faces of 3 different sizes: base and top (2)
  front and back (2)
  other faces (2)
- Add together the area of all faces.
  Hints: Sides marked with a dash (|) are of equal length.
  Sides marked with two dashes (||) are of equal length etc.

Q. Find the total surface area of the cube by finding the area of its net.



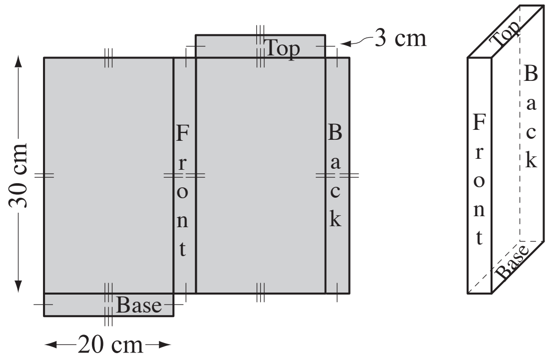
A. *Area of square face* $= 5 \text{ mm} \times 5 \text{ mm}$

$= 25 \text{ mm}^2$

TSA $= 25 \text{ mm}^2 \times 6$ A cube has 6 identical faces

$= \mathbf{150 \text{ mm}^2}$

a) Find the total surface area of the rectangular prism by finding the area of its net.



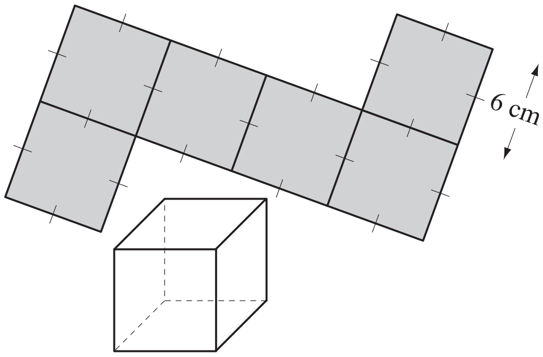
Area: base & top $= 2 \times 20 \times 3 = 120$

Area: front & back $= 2 \times 30 \times 3 = 180$

Area: 2 other faces $= 2 \times 30 \times 20 = 1200$

TSA $= 120 + 180 + 1200 =$ ☐ cm^2

b) Find the total surface area of the cube by finding the area of its net.



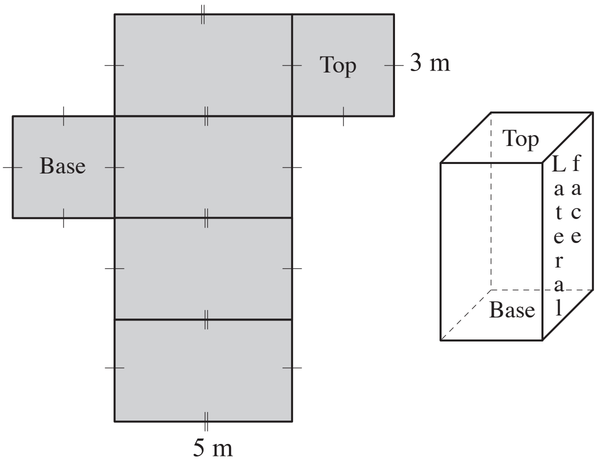
Area of 1 face =

TSA = = ☐ cm^2

Skill 24.1 Calculating the total surface area (*TSA*) of rectangular prisms and cubes using nets (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

c) Find the total surface area of the square prism by finding the area of its net.

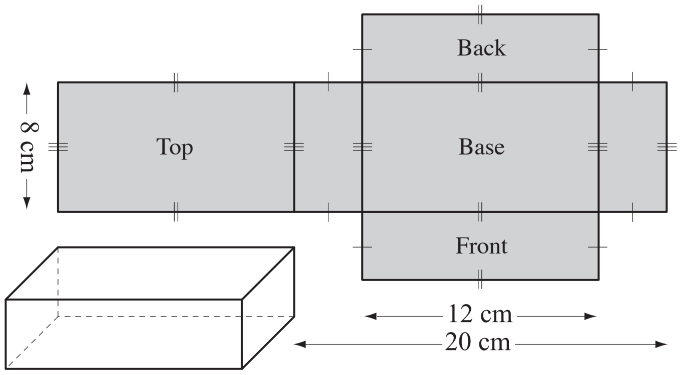


Area: base & top = ...

Area: 4 lateral faces = ...

TSA = ... = $\boxed{\quad m^2}$

d) Find the total surface area of the rectangular prism by finding the area of its net.



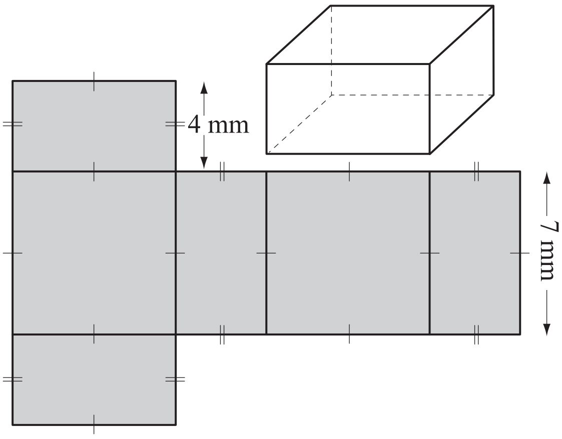
Area: base & top = ...

Area: front & back = ...

Area: 2 other faces = ...

TSA = ... = $\boxed{\quad cm^2}$

e) Find the total surface area of the square prism by finding the area of its net.

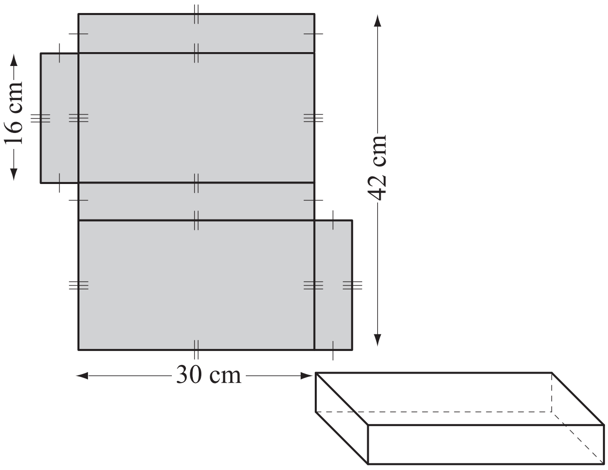


....

....

TSA = ... = $\boxed{\quad mm^2}$

f) Find the total surface area of the rectangular prism by finding the area of its net.



....

....

....

TSA = ... = $\boxed{\quad cm^2}$

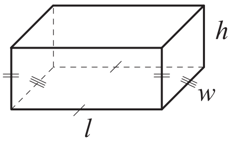
Skill 24.2 Calculating the total surface area (*TSA*) of rectangular prisms.

Mauve 11 22 33 44
Lime 11 22 33 44

rectangular prism

$TSA = 2(\text{length} \times \text{width}) + 2(\text{length} \times \text{height}) + 2(\text{width} \times \text{height})$

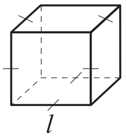
$TSA = 2lw + 2lh + 2wh = 2(lw + lh + wh)$



cube

$TSA = 6(\text{length} \times \text{length})$

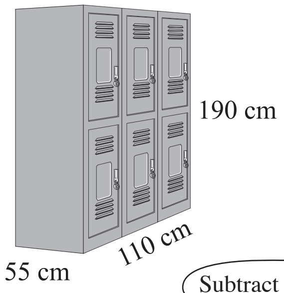
$TSA = 6l^2$



Q. Lewis wants to make a box, with a lid, for his card collection. The box needs a base of 11 cm by 20 cm and must be 12 cm high. How much wood does Lewis need?

A. $TSA = 2 \times (11 \times 20 + 11 \times 12 + 20 \times 12)$
$= 2 \times (220 + 132 + 240)$
$= 2 \times 592$
$= \mathbf{1184\ cm^2}$

a) The locker block needs to be resurfaced. What is the surface area of this rectangular prism disregarding its base?



Subtract 1 base area

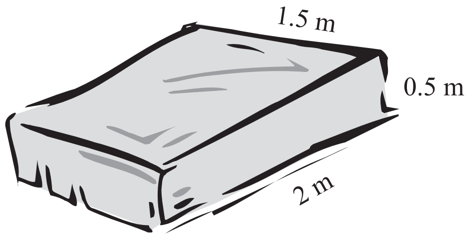
$TSA = lw + 2lh + 2wh$

$= 110 \times 55 + 2 \times 110 \times 190 + 2 \times 55 \times 190$

$= 6050 + 2 \times 20\,900 + 2 \times 10\,450$

$= 6050 + 41\,800 + 20\,900 =$ ………… cm^2

b) Zoe's mattress was torn in removal. What is the minimum amount of mattress ticking needed to re-cover the mattress?



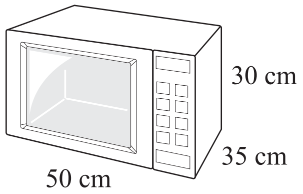
$TSA = 2(lw + lh + wh)$

= …………

= …………

= ………… = ………… m^2

c) Find the total surface area of the microwave.



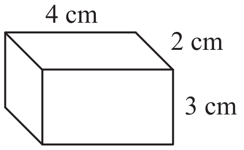
$TSA =$ …………

= …………

= …………

= ………… = ………… cm^2

d) The total surface area of the rectangular prism is 52 m^2. What is the *TSA* if all the dimensions are doubled?



$TSA =$ …………

= …………

= …………

= ………… = ………… cm^2

Skill 24.3 Calculating the total surface area (*TSA*) of rectangular composite solids (1).

- Find any unknown side lengths.
- Calculate the area of each face.
- Add together the area of all faces.

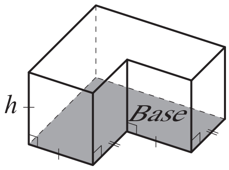
OR

- Identify the base by finding the two, identical parallel faces.
  Hint: A prism does not necessarily sit on its base.
- Substitute values into the formula:

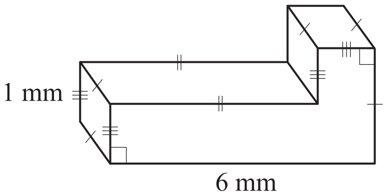
rectangular composite solid

TSA = Perimeter of base × height + 2 × Area of base

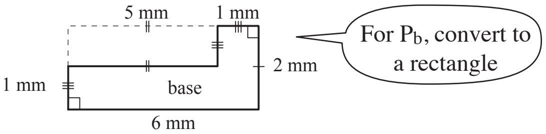
$TSA = P_b h + 2A_b$



Q. Find the total surface area of the prism.



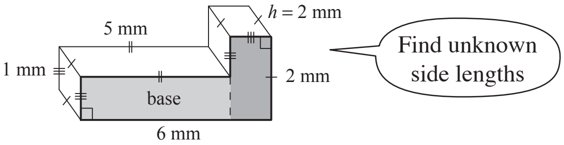
A.



$P_b = 6 + 1 + 5 + 1 + 1 + 2 = 16$

OR

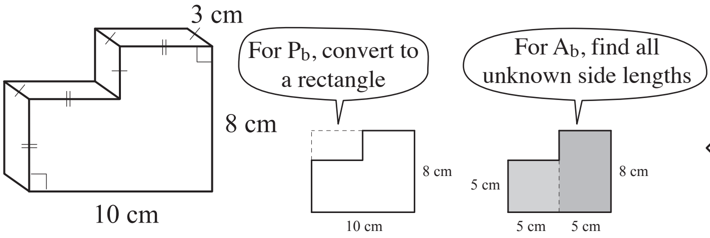
$P_b = 6 + 6 + 2 + 2 = 16$



$A_b = 5 \times 1 + 2 \times 1$
$= 5 + 2 = 7$

$TSA = P_b h + 2A_b$
$= 16 \times 2 + 2 \times 7$
$= 32 + 14 = \mathbf{46\ mm^2}$

a) Find the total surface area of the prism.



$P_b = 10 + 10 + 8 + 8 = 36$

$A_b = 5 \times 5 + 5 \times 8 = 25 + 40 = 65$

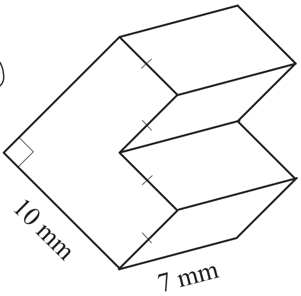
$TSA = P_b h + 2A_b$

Use TSA formula for a prism

$= 36 \times 3 + 2 \times 65$

$= 108 + 130$ = cm^2

b) Find the total surface area of the prism.



$P_b =$

$A_b =$

$TSA = P_b h + 2A_b$

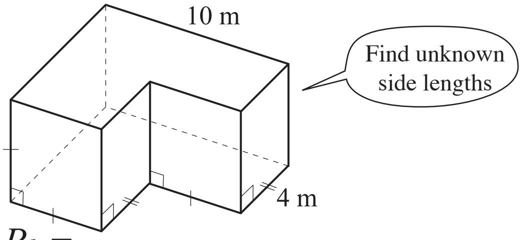
=

= = mm^2

Skill 24.3 Calculating the total surface area (*TSA*) of rectangular composite solids (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

c) Find the total surface area of the prism.



$P_b =$

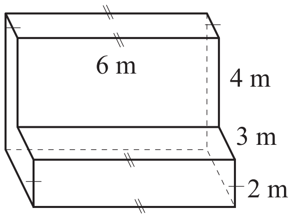
$A_b =$

$TSA = P_bh + 2A_b$

$=$

$=$ $\quad=$ m^2

d) Find the total surface area of the prism.



$P_b =$

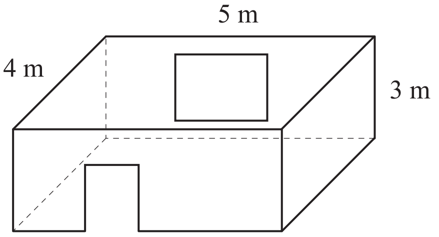
$A_b =$

$TSA = P_bh + 2A_b$

$=$

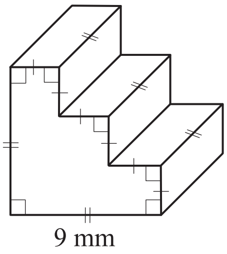
$=$ $\quad=$ m^2

e) A window 2 m by 1.5 m and a doorway 2m by 0.8 m are in the plan for this room. Find the total area of the inside walls to be painted.



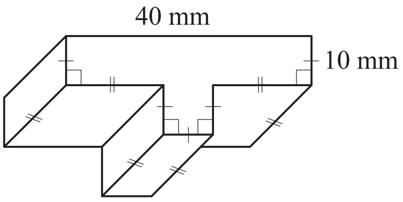
$TSA =$ $\quad=$ m^2

f) Find the total surface area of the prism.



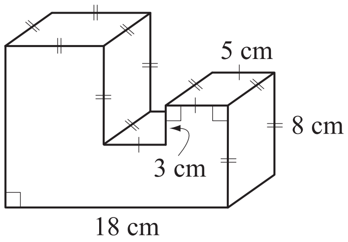
$TSA =$ $\quad=$ mm^2

g) Find the total surface area of the prism.



$TSA =$ $\quad=$ mm^2

h) Find the total surface area of the prism.



$TSA =$ $\quad=$ cm^2

Skill 24.4 Calculating the total surface area (*TSA*) of triangular prisms (1).

Mauve 11 22 33 44
Lime 11 22 33 44

- Find any unknown side lengths.
- Calculate the area of each face.
- Add together the area of all faces.

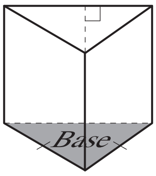
OR

- Substitute values into the formula:

triangular prism

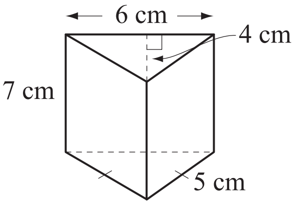
$TSA =$ Perimeter of base $\times$ height $+ 2 \times$ Area of base

$TSA = P_b h + 2A_b$



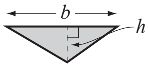
Hint: Do not confuse the height needed to calculate the area of the triangular base, with the height (h) of the prism.

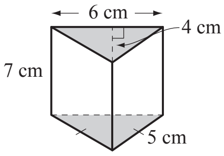
Q. Find the total surface area of the triangular prism.



A. $P_b = 6 + 5 + 5 = 16$

$A_b = \frac{1}{2} bh$





$= \frac{1}{2} \times (6 \times 4) = 12$

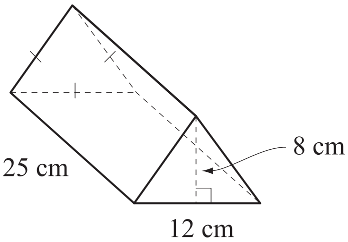
$TSA = P_b h + 2A_b$

$= 16 \times 7 + 2 \times 12$

$= 112 + 24$

$= \mathbf{136\ cm^2}$

a) Find the total surface area of the triangular prism.



$P_b = 12 + 12 + 12 = 36$

First find the perimeter and area of the base

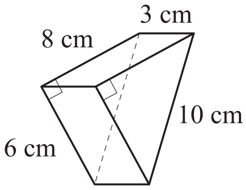
$A_b = \frac{1}{2} \times (12 \times 8) = 48$

$TSA = P_b h + 2A_b$

$= 36 \times 25 + 2 \times 48$

$= 900 + 96$ $=$ ☐ cm^2

b) Find the total surface area of the triangular prism.



$P_b =$

$A_b =$

$TSA =$

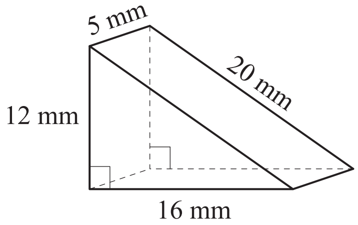
$=$

$=$ $=$ ☐ cm^2

Skill 24.4 Calculating the total surface area (*TSA*) of triangular prisms (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

c) Find the total surface area of the triangular prism.



$P_b =$

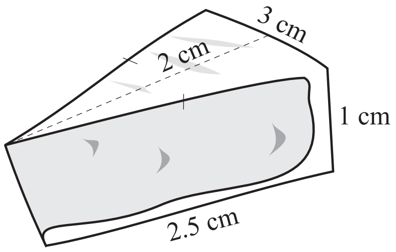
$A_b =$

$TSA = P_b h + 2A_b$

$=$

$=$ $=$ $\boxed{\quad \text{mm}^2}$

d) Find the total surface area of the triangular prism shaped slice of cheese.



$P_b =$

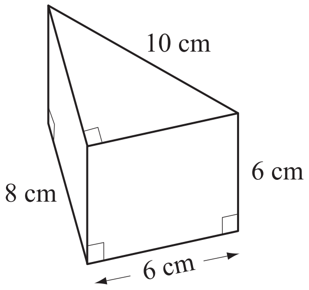
$A_b =$

$TSA = P_b h + 2A_b$

$=$

$=$ $=$ $\boxed{\quad \text{cm}^2}$

e) Find the total surface area of the triangular prism.



$P_b =$

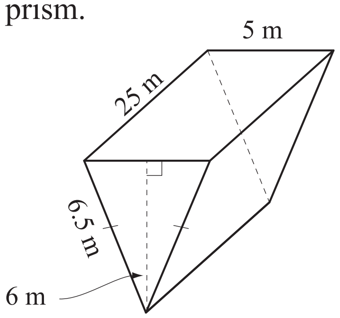
$A_b =$

$TSA =$

$=$

$=$ $=$ $\boxed{\quad \text{cm}^2}$

f) Find the total surface area of the triangular prism.



$P_b =$

$A_b =$

$TSA =$

$=$

$=$ $=$ $\boxed{\quad \text{m}^2}$

Skill 24.5 Calculating the total surface area (*TSA*) of pyramids (1).

- Find any unknown side lengths.
- Calculate the area of each face.
- Add together the area of all faces.

OR

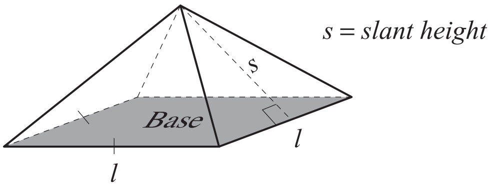
- Substitute values into the formulas:

regular square pyramid

$TSA = \text{Area of square base} + 4 \times \text{Area of triangle}$

$TSA = A_b + 4 \times \frac{1}{2} ls$

$TSA = l^2 + 2ls$

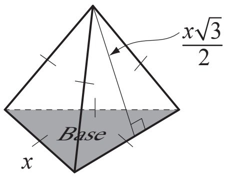


regular triangular pyramid (regular tetrahedron)

$TSA = 4 \times \text{Area of equilateral triangle}$

$TSA = 4 \times \frac{1}{2} x \times \frac{x\sqrt{3}}{2}$

$TSA = x^2\sqrt{3}$

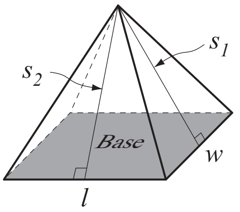


rectangular pyramid

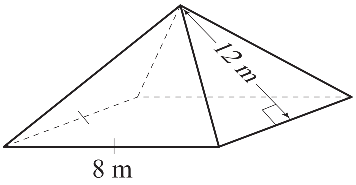
$TSA = \text{Area of base} + 2 \times \text{Area of triangles left \& right} + 2 \times \text{Area of triangles front \& back}$

$TSA = B + 2 \times \frac{1}{2} ws_1 + 2 \times \frac{1}{2} ls_2$

$TSA = lw + ws_1 + ls_2$



Q. Find the total surface area of the regular square pyramid.



A. $TSA = l^2 + 2ls$ where $l = 8$ and $s = 12$

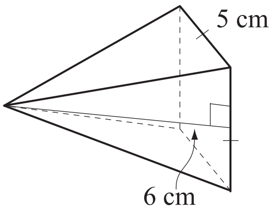
$= 8 \times 8 + 2 \times 8 \times 12$

$= 64 + 16 \times 12$

$= 64 + 192$

$= \mathbf{256\ m^2}$

a) Find the total surface area of the regular square pyramid.

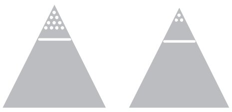


$TSA = l^2 + 2ls$ where $l = 5$ and $s = 6$

$= 5 \times 5 + 2 \times 5 \times 6$

$= 25 + 60$ = cm^2

b) Find the total surface area of one of the salt and pepper shakers given that they are regular square pyramids of base side length 3 cm and slant height 4 cm.

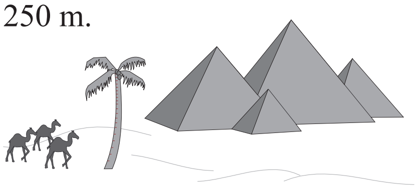


$TSA = l^2 + 2ls$

= cm^2

Skill 24.5 Calculating the total surface area (*TSA*) of pyramids (2).

c) Find the total surface area of the largest regular square pyramid below. It has a base side length of 200 m and slant height of 250 m.

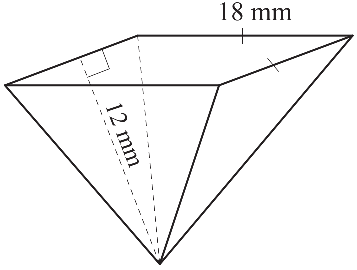


TSA = ...

....

.... = m^2

d) Find the total surface area of the regular square pyramid.

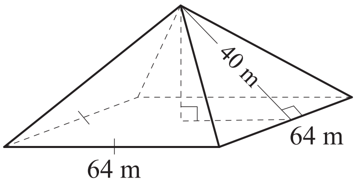


TSA = ...

....

.... = mm^2

e) Find the surface area of the regular square pyramid.



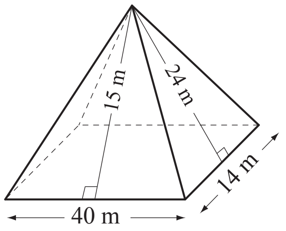
TSA = ...

....

....

.... = m^2

f) Find the surface area of the rectangular pyramid.



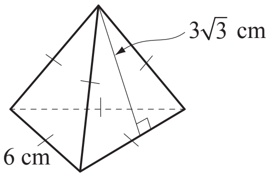
TSA = ...

....

....

.... = m^2

g) Find the surface area of the regular tetrahedron. [Give your answer as a surd.]



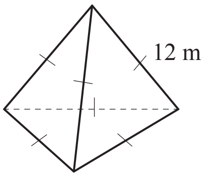
TSA = ...

....

....

.... = cm^2

h) Find the surface area of the regular tetrahedron. [Give your answer as a surd.]



TSA = ...

....

....

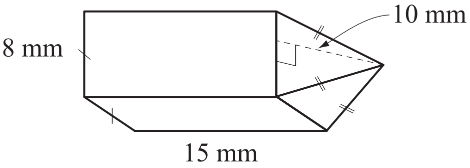
.... = m^2

Skill 24.6 Calculating the total surface area of composite solids (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Break the solid into workable parts.
- Calculate the total surface area of each solid. (see skills 24.2, page 279 and 24.3, page 280)
- Add the results.

Q. Find the total surface area of the obelisk.



A. *TSA regular square pyramid* (without base)

l = 8 (length)
s = 10 (slant height)

$= 2ls$ where $l = 8$ and $s = 10$

$= 2 \times 8 \times 10$

$= 160$

TSA square prism (without base)

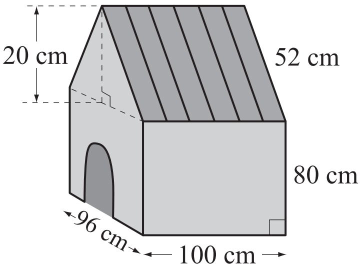
$= 4lh + l^2$ where $l = 8$ and $h = 15$

$= 4 \times (8 \times 15) + 8 \times 8$

$= 4 \times 120 + 64 = 544$

TSA obelisk $= 160 + 544 =$ **704 mm^2**

a) Disregarding the entrance, find the total surface area of the doghouse, excluding its floor.



TSA roof prism $= 2 \times 100 \times 52 + 2 \times \frac{1}{2} \times 96 \times 20 =$ ....

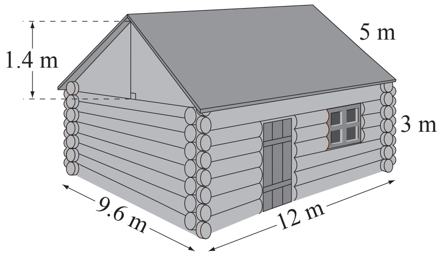
= ....

TSA base prism $= 2 \times 100 \times 80 + 2 \times 96 \times 80 =$ ....

= ....

TSA house = ... = cm^2

b) Disregarding the door and windows, find the total surface area of the log cabin, excluding its floor.



TSA roof prism = ...

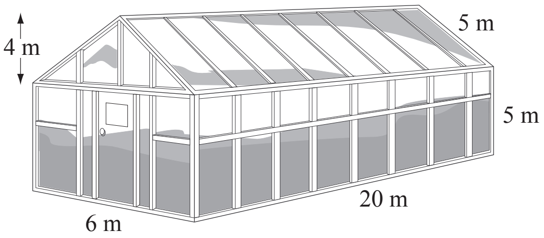
= ....

TSA base prism = ...

= ....

TSA cabin = ... = m^2

c) Find the total surface area of the glass house, excluding its floor.



TSA roof prism = ...

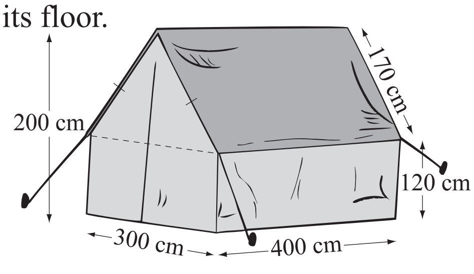
= ....

TSA base prism = ...

= ....

TSA house = ... = m^2

d) Find the total surface area of the tent canvas excluding its floor.



....

....

....

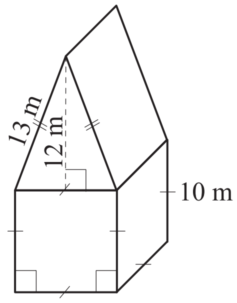
....

.... = cm^2

Skill 24.6 Calculating the surface area of composite solids (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

e) Find the total surface area of the solid.



Roof

$P_b = 36$ $\quad A_b = \frac{1}{2} \times 10 \times 12 = 60$

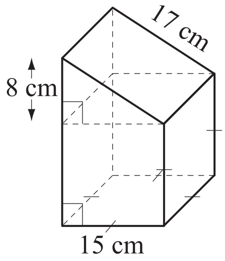
TSA prism =

TSA prism – face =

TSA cube – face =

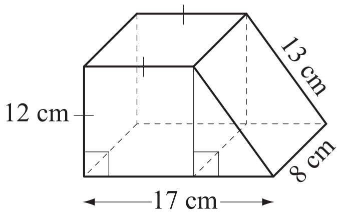
TSA solid = = m^2

f) Find the total surface area of the solid.



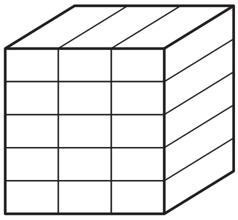
TSA = = cm^2

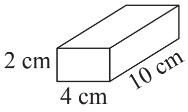
g) Find the total surface area of the solid.



TSA = = cm^2

h) Bernie bought a rectangular box containing 15 tightly packaged erasers. What is the total surface area of the box?



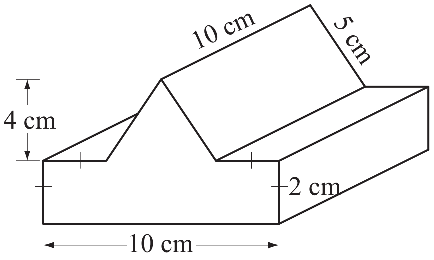


TSA = = cm^2

Skill 24.6 Calculating the surface area of composite solids (3).

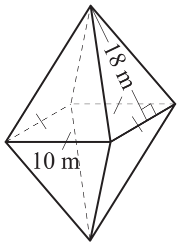
Mauve 1 1 2 2 3 **3** 4 4
Lime 1 1 2 2 3 3 **4** 4

i) Find the total surface area of the prism.



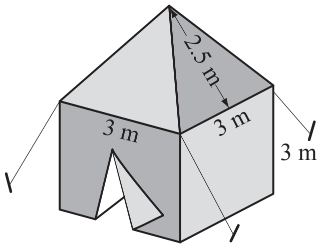
$TSA =$ = cm^2

j) Find the total surface area of the octahedron.



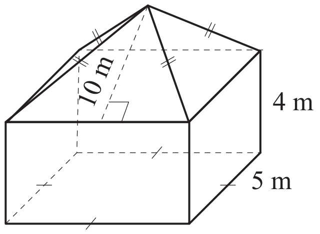
$TSA =$ = m^2

k) Disregarding the entrance, find the total surface area of the marquee canvas excluding its floor.



$TSA =$ = m^2

l) Find the total surface area of the obelisk.



$TSA =$ = m^2

Skill 24.7 Calculating the total surface area (*TSA*) of basic 3-dimensional round shapes (1).

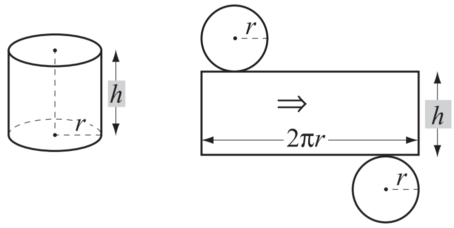
Mauve 11 22 33 **44**
Lime 11 22 **33** 44

- Substitute values into the formulas:

cylinder

$TSA = 2\pi r^2 + 2\pi rh$

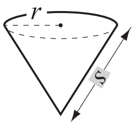
$TSA = 2\pi r(r + h)$



cone

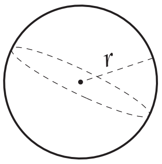
$TSA = \pi r^2 + \pi rs$

$TSA = \pi r(r + s)$

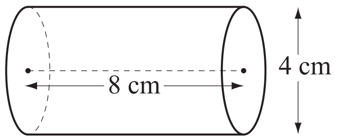


sphere

$TSA = 4\pi r^2$



Q. Using $TSA = 2\pi r(r + h)$ and $\pi \approx 3.14$, find the total surface area of the cyclinder.



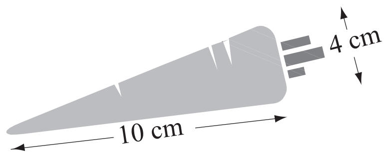
A. $TSA = 2\pi r(r + h)$ *where* $r = 2$ *and* $h = 8$

$= 2 \times 3.14 \times 2 \times (2 + 8)$

$= 12.56 \times 10$

$= \mathbf{125.6\ cm^2}$

a) Use $TSA = \pi r(r + s)$ and $\pi \approx 3.14$, to find the total surface area of the conical carrot.

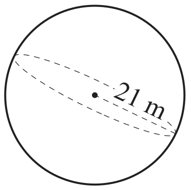


$TSA = \pi r(r + s)$ *where* $r = 2$ *and* $s = 10$

$= 3.14 \times 2 \times (2 + 10)$

$= 6.28 \times 12$ $=$ $\boxed{75.36\ \text{cm}^2}$

b) Using $TSA = 4\pi r^2$ and $\pi \approx \frac{22}{7}$, find the total surface area of the sphere.



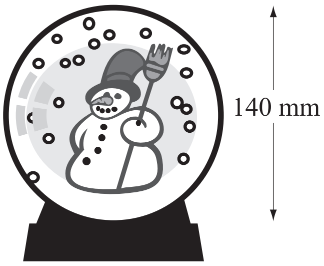
$TSA =$ ....

$=$ ....

$=$ .... $=$ $\boxed{\quad\ \text{m}^2}$

Skill 24.7 Calculating the total surface area (*TSA*) of basic 3-dimensional round shapes (2).

c) Using $TSA = 4\pi r^2$ and $\pi \approx \frac{22}{7}$, find the total surface area of the snow globe.

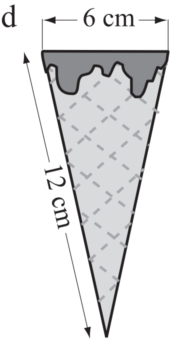


TSA = ...

= ....

= .... = $\boxed{\quad\text{mm}^2}$

d) Use $TSA = \pi r(r + s)$ and $\pi \approx 3.14$ to find how much area still needs to be covered in chocolate to cover the whole cone only on the outside, given that 40 cm^2 have been covered so far.

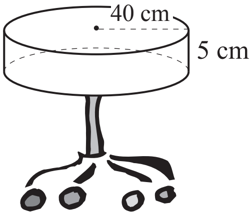


TSA = ...

= ....

= .... = $\boxed{\quad\text{cm}^2}$

e) Using $TSA = 2\pi r(r + h)$ and $\pi \approx 3.14$, find the total surface area of the cyclindrical stool seat.

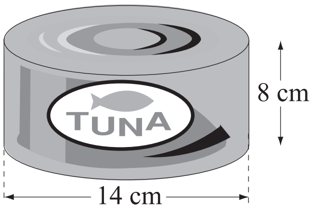


TSA = ...

= ....

= .... = $\boxed{\quad\text{cm}^2}$

f) Using $TSA = 2\pi r(r + h)$ and $\pi \approx \frac{22}{7}$, find the total surface area of the can of tuna.

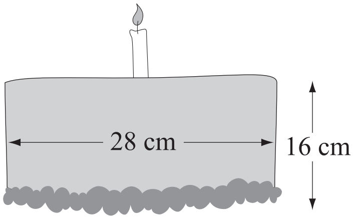


TSA = ...

= ....

= .... = $\boxed{\quad\text{cm}^2}$

g) Using *TSA* of a cylinder $= 2\pi r(r + h)$ and $\pi \approx \frac{22}{7}$, find the total surface area of the icing.
[N.B. The base of the cake is not iced.]



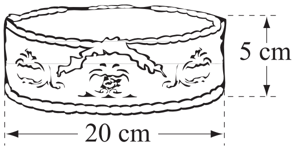
TSA = ...

= ....

= ....

= .... = $\boxed{\quad\text{cm}^2}$

h) This wedding cake is covered in white icing, except for the base. Using $\pi \approx 3.14$ find the total surface area of the white icing.



TSA = ...

= ....

= ....

= .... = $\boxed{\quad\text{cm}^2}$

Skill 24.8 Calculating the total surface area (*TSA*) of more complex 3-dimensional round shapes.

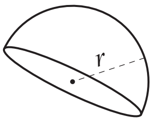
Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Substitute values into the appropriate formula.
- Adapt formulas where necessary.

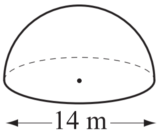
hemisphere

$$TSA = \frac{4\pi r^2}{2} + \pi r^2$$

$$TSA = 3\pi r^2$$



Q. Using $\pi \approx \frac{22}{7}$ find the total surface area of the hemisphere.



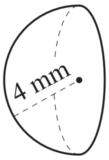
A. *TSA* = $3\pi r^2$ *where r = 7 m*

$= 3 \times \frac{22}{\cancel{7}_1} \times \overset{1}{\cancel{7}} \times 7$

$= 66 \times 7$

$= \mathbf{462\ m^2}$

a) Using the total surface area of a sphere *TSA* = $4\pi r^2$ and $\pi \approx 3.14$, find the total surface area of the hemisphere.

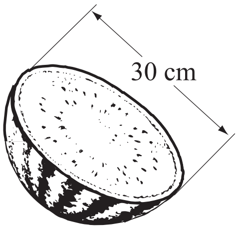


TSA = $3\pi r^2$

= $3 \times 3.14 \times 4 \times 4$

= 9.42×16 = ☐ mm²

b) The total surface area of a sphere is *TSA* = $4\pi r^2$. Using $\pi \approx 3.14$ find the total surface area of the watermelon half.

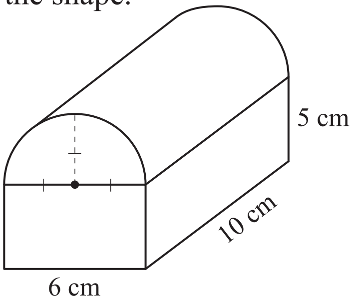


TSA =

=

= = ☐ cm²

c) Use $\pi \approx 3.14$ to find the total surface area of the shape.



TSA prism =

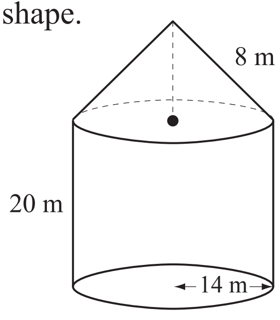
=

TSA cylinder half =

=

TSA = = ☐ cm²

d) Use $\pi \approx \frac{22}{7}$ to find the total surface area of the shape.



LA cone =

=

TSA cylinder =

=

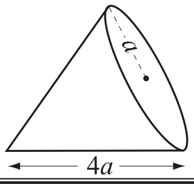
TSA = = ☐ m²

Skill 24.9 Expressing the total surface area (*TSA*) of 3-dimensional shapes in algebraic form.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Substitute values into the appropriate formula for total surface area.
(see skills 24.2 to 24.5, pages 279 to 284, skills 24.7, page 289 and 27.8, page 291)
- Adapt formulas where necessary.

Q. Write an algebraic expression for the total surface area *TSA* of the cone. [Express the answer in terms of a and π.]



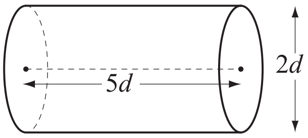
A. $TSA = \pi r(r + s)$ where $r = a$ and $s = 4a$

$= \pi \times a \times (a + 4a)$

$= \pi \times a \times 5a$

$= \mathbf{5\pi a^2}$

a) Write an algebraic expression for the total surface area *TSA* of the cylinder. [Express the answer in terms of d and π.]



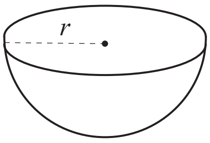
$TSA = 2\pi r(r + h)$ where $r = d$ and $h = 5d$

$= 2\pi d(d + 5d)$

$= 2\pi d \times 6d$

$TSA = 12\pi d^2$

b) Write an algebraic expression for the total surface area *TSA* of the hemisphere. [Express the answer in terms of r and π.]

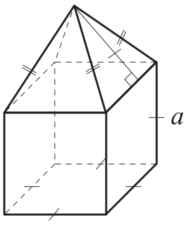


$TSA =$ ……………………………………

$=$ ……………………………………

$=$ …………………… $TSA =$

c) Write an algebraic expression for the total surface area *TSA* of the obelisk. [Express the answer in terms of a.]

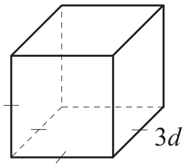


$TSA =$ ……………………………………

$=$ ……………………………………

$=$ …………………… $TSA =$

d) Write an algebraic expression for the total surface area *TSA* of the cube. [Express the answer in terms of d.]

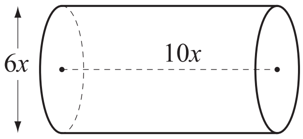


$TSA =$ ……………………………………

$=$ ……………………………………

$=$ …………………… $TSA =$

e) Write an algebraic expression for the total surface area *TSA* of the cylinder. [Express the answer in terms of x and π.]

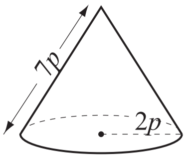


$TSA =$ ……………………………………

$=$ ……………………………………

$=$ …………………… $TSA =$

f) Write an algebraic expression for the total surface area *TSA* of the cone. [Express the answer in terms of p and π.]



$TSA =$ ……………………………………

$=$ ……………………………………

$=$ …………………… $TSA =$

25. [Volume]

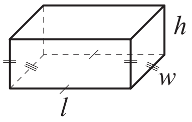
Skill 25.1 Calculating the volume of square and rectangular prisms.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

rectangular prism

$V = \text{length} \times \text{width} \times \text{height}$

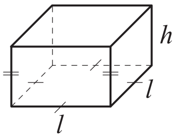
$V = lwh$



square prism

$V = \text{length} \times \text{width} \times \text{height}$

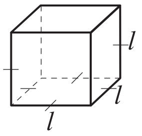
$V = l^2h$



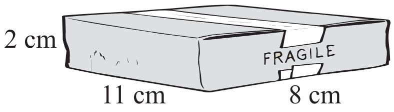
cube

$V = \text{length} \times \text{length} \times \text{length}$

$V = l^3$



Q. The parcel is a rectangular prism. What is the volume of the parcel?



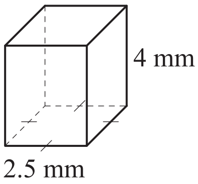
A. *$V = lwh$ where $l = 11$, $w = 8$ and $h = 2$*

$= 11 \times 8 \times 2$

$= 88 \times 2$

$= \mathbf{176\ cm^3}$

a) Find the volume of the square prism.

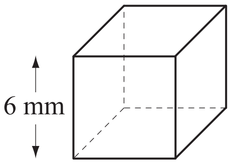


$V = l^2h$ where $l = 2.5$ and $h = 4$

$= 2.5 \times 2.5 \times 4$

$= 2.5 \times 10$ = ______ mm^3

b) Find the volume of the cube.

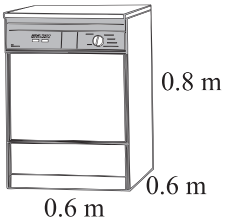


$V = l^3$

= ______

= ______ = ______ mm^3

c) Given that the dishwasher is a square prism, find its volume.

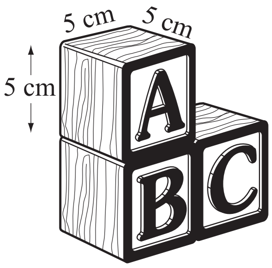


$V =$ ______

= ______

= ______ = ______ m^3

d) Find the volume of the building blocks stack.



$V =$ ______

= ______

= ______ = ______ cm^3

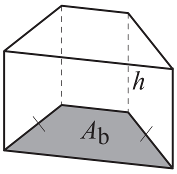
Skill 25.2 Calculating the volume of other prisms (1).

- Find any unknown side lengths.
- Calculate the area of the base. Use known formulae where possible.
  Hint: The height of a triangle is needed to calculate the area of a triangle and should not be confused with the height (h) of the prism.
- Substitute known values into the formula:

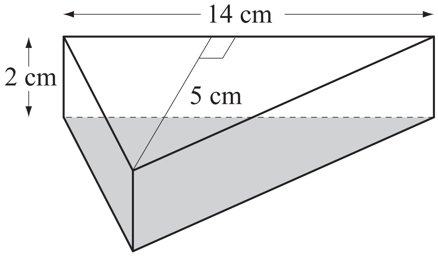
prism

$V =$ Area of base $\times$ height of prism

$V = A_b h$



Q. Find the volume of the triangular prism using $V = A_b h$.



A.

$V = A_b h$ h = height of prism

$A_b = \frac{1}{2} bh$ h = height of triangle

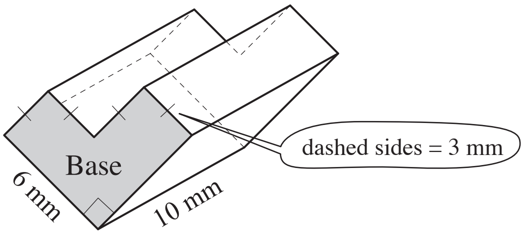
$= \frac{1}{\cancel{2}} \times \overset{7}{\cancel{14}} \times 5$ First calculate area of base

$= 7 \times 5 = 35$

$V = 35 \times 2$

$= \mathbf{70\ cm^3}$

a) Find the volume of the prism.

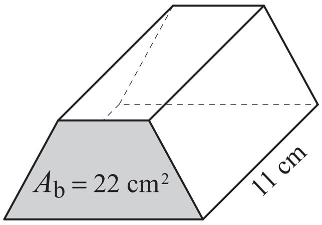


$V = A_b h$ where $h = 10$ mm

$A_b = 3 \times 3 + 6 \times 3 = 9 + 18 = 27$

$V = 27 \times 10$ = ______ mm^3

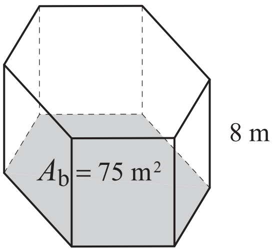
b) Using $V =$ Area of base (A_b) $\times$ height (h), find the volume of the prism.



$V = A_b h$

= ______ = ______ cm^3

c) Using $V = A_b h$ find the volume of the hexagonal prism.

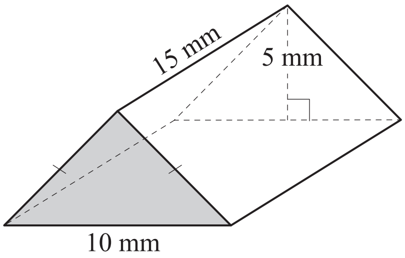


$V = A_b h$

= ______

= ______ = ______ m^3

d) Find the volume of the triangular prism using $V = A_b h$.



$V = A_b h$

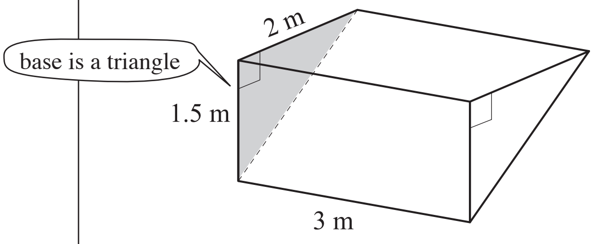
$A_b =$ ______

$V =$ ______ = ______ mm^3

Skill 25.2 Calculating the volume of other prisms (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

e) Find the volume of the prism.

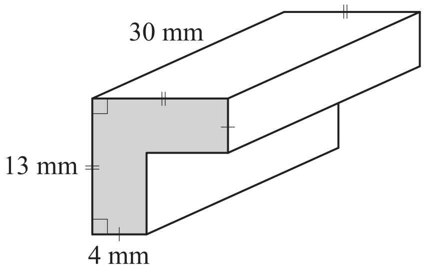


$V = A_b h$ First calculate area of base

$A_b = \frac{1}{2} \times 1.5 \times 2 = 1.5$

$V = 1.5 \times 3$ = m^3

f) Find the volume of the prism.

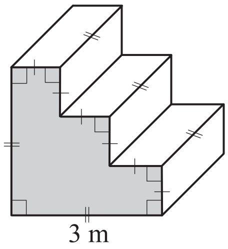


$V =$

$A_b =$

$V =$ = mm^3

g) Find the volume of concrete used to build the steps.

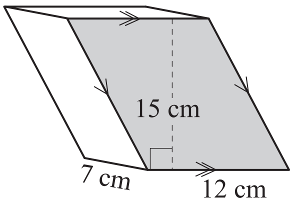


$V =$

$A_b =$

$V =$ = m^3

h) Find the volume of the prism.

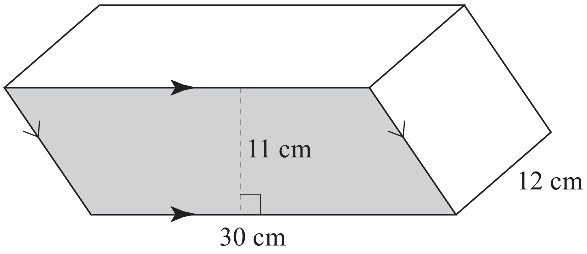


$V =$

$A_b =$

$V =$ = cm^3

i) Find the volume of the prism.

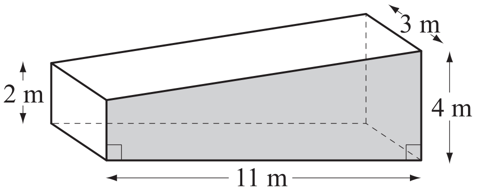


$V =$

$A_b =$

$V =$ = cm^3

j) Find the volume of the prism.



$V =$

$A_b =$

$V =$ = m^3

Skill 25.3 Calculating the volume of pyramids.

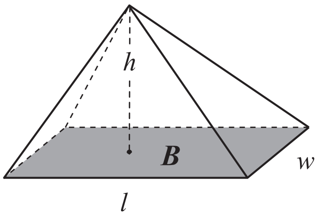
Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Substitute known values into the appropriate formula to find the area of the base.
- Substitute known values into the volume formula:

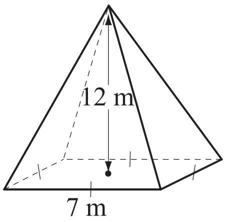
pyramid

$V = \frac{1}{3} \times \text{Area of base} \times \text{height of pyramid}$

$V = \frac{Bh}{3}$



Q. Find the volume of the square pyramid.



A. $V = \frac{A_b h}{3}$

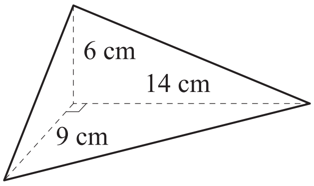
$A_b = 7 \times 7 = 49$

$V = \frac{49 \times \cancel{12}^{4}}{\cancel{3}}$ Simplify: ÷ 3

$= 49 \times 4$

$= \mathbf{196\ m^3}$

a) Using $V = \frac{A_b h}{3}$ find the volume of the triangular pyramid of height 6 cm.

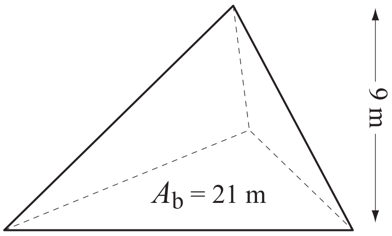


$V = \frac{A_b h}{3}$

$A_b = \frac{1}{2} bh = \frac{1}{\cancel{2}} \times 9 \times \cancel{14}^{7} = 63$

$V = \frac{63 \times \cancel{6}^{2}}{\cancel{3}}$ = cm^3

b) Using $V = \frac{A_b h}{3}$ find the volume of the triangular pyramid.

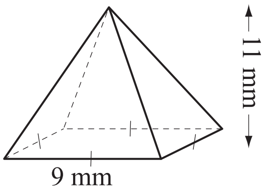


$V = \frac{A_b h}{3}$

$A_b =$

$V =$ = m^3

c) Find the volume of the square pyramid.

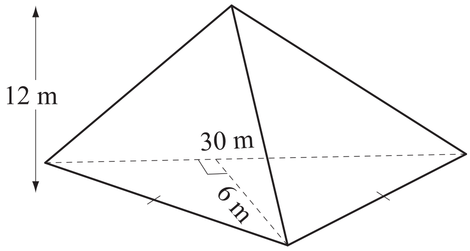


$V =$

$A_b =$

$V =$ = mm^3

d) Find the volume of the triangular pyramid.



$V =$

$A_b =$

$V =$ = m^3

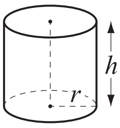
Skill 25.4 Calculating the volume of basic 3-dimensional round shapes.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Substitute known values into the appropriate formula:

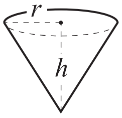
cylinder

$V = \pi r^2 h$



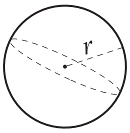
cone

$V = \dfrac{\pi r^2 h}{3}$

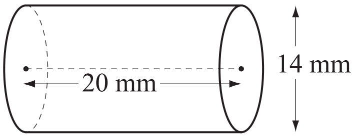


sphere

$V = \dfrac{4\pi r^3}{3}$



Q. Using $V = \pi r^2 h$ and $\pi \approx \frac{22}{7}$, find the volume of the cylinder.



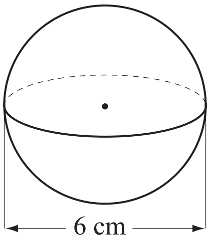
A. $V = \pi r^2 h$ *where* $r = 7$ *and* $h = 20$

$= \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 20$ Simplify: ÷ 7

$= 154 \times 20$

$= \mathbf{3080\ mm^3}$

a) Using $V = \frac{4\pi r^3}{3}$ and $\pi \approx 3.14$, find the volume of the sphere.

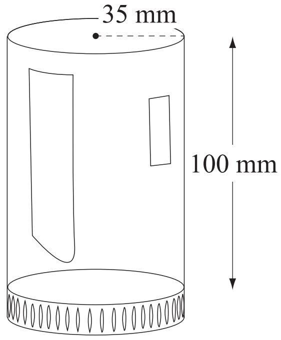


$V = \dfrac{4\pi r^3}{3}$ *where* $r = 3$ *cm*

$= \dfrac{4 \times 3.14 \times \cancel{3} \times 3 \times 3}{\cancel{3}}$ Simplify: ÷ 3

$= 36 \times 3.14 \qquad = \boxed{\qquad \text{cm}^3}$

b) Using $V = \pi r^2 h$ and $\pi \approx \frac{22}{7}$, find the maximum volume of the glass.

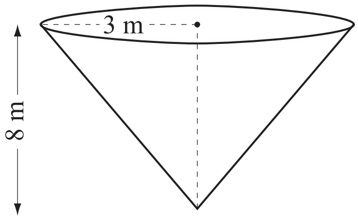


$V =$

$=$

$= \qquad = \boxed{\qquad \text{mm}^3}$

c) Using $V = \frac{\pi r^2 h}{3}$ and $\pi \approx 3.14$, find the volume of the cone.

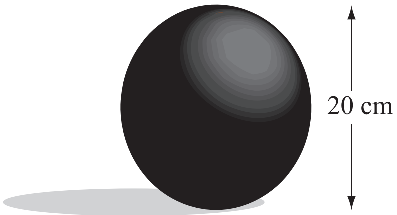


$V =$

$=$

$= \qquad = \boxed{\qquad \text{m}^3}$

d) Using $V = \frac{4\pi r^3}{3}$ and $\pi \approx 3.14$, find the volume of the sphere, correct to 2 decimal places.



$V =$

$=$

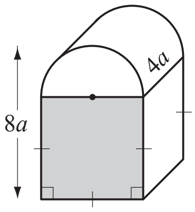
$= \qquad = \boxed{\qquad \text{cm}^3}$

Skill 25.5 Expressing the volume of 3-dimensional shapes in algebraic form.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 **3** 4 4

- Substitute values into the appropriate formula for volume. (see skills 25.1 to 25.4, pg. 293 to 297)
- Adapt the formulas where necessary.

Q. Write an algebraic expression for the volume V of the shape. [Express the answer in terms of a and π.]



A. $V_{sq.\ prism} = l^2h$ where $l = 5a$ and $h = 4a$

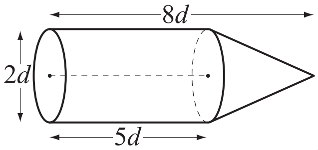
$= 5a \times 5a \times 4a = 100a^3$

$V_{half\ cyl.} = \frac{1}{2}\pi r^2 h$ where $r = 3a$ and $h = 4a$

$= \frac{1}{2}\pi \times 9a^2 \times 4a = 18\pi a^3$

$V_{shape} = 100a^3 + 18\pi a^3 = \mathbf{2a^3(50 + 9\pi)}$

a) Write an algebraic expression for the volume V of the shape. [Express the answer in terms of d and π.]



r = d and h = 5d

$V_{cyl} = \pi r^2 h = \pi \times d^2 \times 5d = 5\pi d^3$

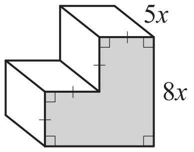
r = d and h = 3d

$V_{cone} = \frac{\pi r^2 h}{3} = \frac{1}{3} \times \pi \times d^2 \times 3d = \pi d^3$

$V_{shape} = 5\pi d^3 + \pi d^3 =$

$V =$

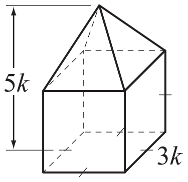
b) Write an algebraic expression for the volume V of the prism. [Express the answer in terms of x.]



$V =$

$V =$

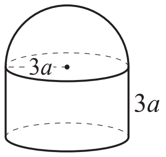
c) Write an algebraic expression for the volume V of the obelisk. [Express the answer in terms of k.]



$V =$

$V =$

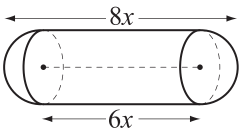
d) Write an algebraic expression for the volume V of the shape. [Express the answer in terms of a and π.]



$V =$

$V =$

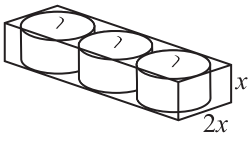
e) Write an algebraic expression for the volume V of the capsule. [Express the answer in terms of x and π.]



$V =$

$V =$

f) A rectangular box contains 3 identical candles placed with no room to move. Write an algebraic expression in terms of x and π for the volume of the box which is **not** occupied by the candles.



$V =$

$V =$

Skill 25.6 Calculating volume in relation to capacity.

- Substitute known values into the appropriate formula.
- Use the conversion factors between cubic units and capacity units:

Conversion Facts - CUBIC VOLUME to CAPACITY

$1000\text{ cm}^3 = 1\,000\text{ mL} = 1\text{ L}$

$1000\text{ L} = 1\text{ m}^3$

Q. A rectangular swimming pool is 20 m long and 12 m wide. If its average depth is 2 m, how many litres of water would you need to fill the pool? [Hint: $1000\text{ L} = 1\text{ m}^3$]

A. $V = lwh$ *where* $l = 20$, $w = 12$ *and* $h = 2$

$= 20 \times 12 \times 2$

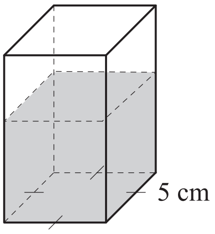
$= 20 \times 24$

$= 480\text{ m}^3$

Convert m^3 to L

$= \mathbf{480\,000\text{ L}}$

a) The vase has 0.5 litre of water in it. Find the depth of the water. [Hint: $1000\text{ cm}^3 = 1\text{ L}$]



Using 0.5 L = 500 mL

$V = l^2h$ *where* $l = 5$ *and* $V = 500$

$500 = 5 \times 5 \times h \Rightarrow 25h = 500$

$25h \div 25 = 500 \div 25$

$h = 20$ cm

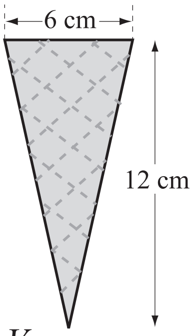
b) A rectangular fish tank with dimensions 20 cm by 15 cm by 10 cm is half full of water. How many millilitres of water would you need to fill the fish tank? [Hint: $1\text{ mL} = 1\text{ cm}^3$]

=

=

= mL

c) Using $V = \frac{\pi r^2 h}{3}$ and $\pi \approx 3.14$, find how much ice cream could fit exactly inside this cone. [Hint: $1\text{ mL} = 1\text{ cm}^3$]

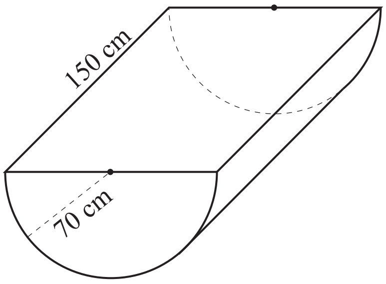


$V =$

=

= = mL

d) Using $\pi \approx \frac{22}{7}$ find the maximum volume of water the trough could hold. [Hint: $1000\text{ cm}^3 = 1\text{ L}$]



$V =$

=

= = L

Skill 25.7 Calculating volume in relation to length and area.

Mauve 11 22 33 44
Lime 11 22 33 **4**4

- Substitute known values into the appropriate formulas for area and volume.

Q. A rectangular prism with volume 216 cm^3 has a height of 6 cm and a width of 5 cm. Calculate the length of the prism.

A. $V = lwh$ *where* $V = 216$, $w = 5$ *and* $h = 6$

$216 = l \times 5 \times 6$

$30l = 216$

$l = 216 \div 30$ Divide 21.6 by 3

$l = \mathbf{7.2\ cm}$

a) A cube has a total surface area of 54 cm^2. What is the volume of the cube?

$TSA = 6l^2$ *and* $V = l^3$ In a cube: l = w = h

$54 = 6l^2$ *so* $l^2 = \frac{\cancel{54}}{\cancel{6}} = 9$ *and* $l = 3$

$V = 3^3 = \boxed{27\ \text{cm}^3}$

b) A rectangular prism with volume 189 mm^3 has a height of 3 mm and a length of 7 mm. Calculate the width of the prism.

mm

c) If a cube has a total surface area of 96 mm^2, what is the volume of the cube?

mm^3

d) If a cube has a total surface area of 150 cm^2, what is the volume of the cube?

cm^3

e) A rectangular long jump pit holds 13.5 m^3 of sand. If the pit is 9 m long and 3 m wide, how deep is the sand?

m

f) How many metal cubes of side length 4 mm need to be melted down to produce a single cube of side length 8 mm?

g) A rectangular fish tank can hold 30 000 cm^3 when full. If the tank is 20 cm wide and 30 cm long, how deep is the water?

cm

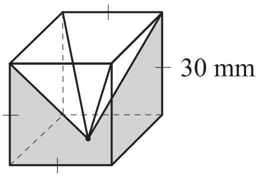
h) How many metal cubes of side length 3 cm need to be melted down to produce a single cube of side length 9 cm?

Skill 25.8 Calculating the volume of composite solids.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 **4**

- Substitute values into the appropriate formulas for volume.

Q. A 30 mm × 30 mm × 30 mm pyramid is removed from a 30 mm × 30 mm × 30 mm cube. Find the volume of the remaining shape.



A. V_1 of cube $= l^3$

V_2 of square pyramid $= \frac{A_b h}{3}$ where $A_b = l^2$ and $h = l$

$= \frac{l^3}{3}$

$V_1 - V_2 = l^3 - \frac{l^3}{3}$

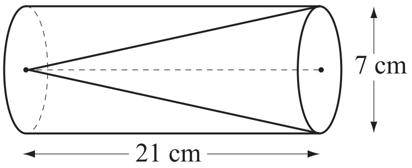
$V = \frac{2l^3}{3}$ where $l = 30$

$V = \frac{2 \times \overset{10}{\cancel{30}} \times 30 \times 30}{\cancel{3}_1}$ Simplify: ÷ 3

$V = 20 \times 900$

$= \mathbf{18\,000\ mm^3}$

a) How much less is the volume of the cone than the volume of the cylinder of the same height? (Use $\pi \approx \frac{22}{7}$)



V_1 of a cylinder $= \pi r^2 h$, V_2 of a cone $= \frac{\pi r^2 h}{3}$

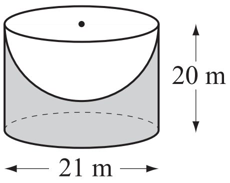
$V_1 - V_2 = \pi r^2 h - \frac{\pi r^2 h}{3}$

$= \frac{2\pi r^2 h}{3}$

$= \cancel{2} \times \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}} \times \overset{7}{\cancel{21}} \times \frac{1}{\cancel{3}}$ Simplify

$= 11 \times 7 \times 7$ = ... cm^3

b) A hemisphere of diameter 21 m is removed from this cylinder. Using $\pi \approx \frac{22}{7}$ find the volume of the remaining shape.



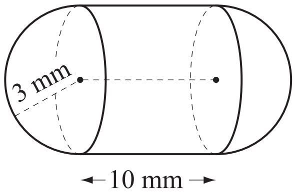
$V =$ ....

= ....

= ....

= .... = ... m^3

c) Using $\pi \approx 3.14$ find the volume of the shape.

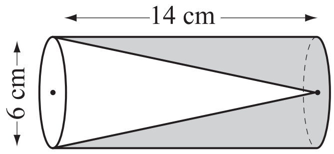


$V =$ ....

= ....

= .... = ... mm^3

d) A cone of diameter 6 cm and height 14 cm is removed from this cylinder. Find the volume of the remaining shape. (Use $\pi \approx \frac{22}{7}$)



$V =$ ....

= ....

= .... = ... cm^3

26. [Pythagoras / Trigonometry]

Skill 26.1 Solving simple quadratic equations.

Mauve 11 22 33 44
Lime 11 22 33 44

- Calculate the square numbers on the right-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.
- Estimate which positive number, when multiplied by itself, produces the number under the square root.
- Check your estimation by multiplying your guess by itself.
- If the number is a decimal number consider the position of the decimal point.

Q. Find the positive solution for a: $a^2 = 20^2 - 16^2$

A. $a^2 = 20^2 - 16^2$ $20^2 = 20 \times 20$ $16^2 = 16 \times 16$

$a^2 = 400 - 256$

$a^2 = 144$

$\sqrt{a^2} = \sqrt{144}$ $\sqrt{a^2} = a$

$a = \sqrt{12 \times 12}$

$a = \mathbf{12}$

a) Find the positive solution for c: $c^2 = 676$

$\sqrt{c^2} = \sqrt{676}$

$c = \sqrt{26 \times 26}$

$c =$ $\boxed{26}$

b) Find the positive solution for b: $b^2 = 441$

$b =$ ....

$b =$ ....

$b =$ .... $\boxed{}$

c) Find the positive solution for a: $a^2 = 225$

$a =$ ....

$a =$ ....

$a =$ .... $\boxed{}$

d) Find the positive solution for b: $b^2 = 1600$

$b =$ ....

$b =$ ....

$b =$ .... $\boxed{}$

e) Find the positive solution for c: $c^2 = 6.25$

$c =$ ....

$c =$ ....

$c =$ .... $\boxed{}$

f) Find the positive solution for a: $a^2 = 0.16$

$a =$ ....

$a =$ ....

$a =$ .... $\boxed{}$

g) Find the positive solution for c: $c^2 = 7^2 + 24^2$

$c^2 =$ ....

$c^2 =$ ....

$c =$ ....

$c =$ .... $\boxed{}$

h) Find the positive solution for a: $a^2 = 50^2 - 30^2$

$a^2 =$ ....

$a^2 =$ ....

$a =$ ....

$a =$ .... $\boxed{}$

i) Find the positive solution for b: $b^2 = 25^2 - 20^2$

$b^2 =$ ....

$b^2 =$ ....

$b =$ ....

$b =$ .... $\boxed{}$

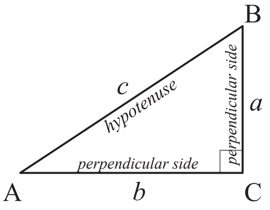
Skill 26.2 Recognising Pythagoras' theorem.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Determine which is the longest side of the right-angled triangle (hypotenuse).
  Hints: In a triangle, the vertices are labelled with capital letters.
  Any side length of a triangle is usually labelled with a lower case letter (the same as the letter at the opposite vertex or angle).
- Identify correct statements of Pythagoras' theorem or ones derived from it.

Pythagoras' Theorem: $a^2 + b^2 = c^2$

For any right-angled triangle, the square of the length of the hypotenuse (c) equals the sum of the squares of the lengths of the two perpendicular sides (a and b).



OR: $c^2 = a^2 + b^2$

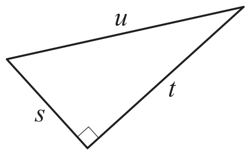
For any right-angled triangle, the square of the length of the hypotenuse (c) equals the sum of the squares of the lengths of the two perpendicular sides (a and b).

OR: $a^2 = c^2 - b^2$ **and** $b^2 = c^2 - a^2$

For any right-angled triangle, the square of the length of a perpendicular side equals the difference between the square of the length of the hypotenuse and the square of the length of the other perpendicular side.

Q. Which statements of Pythagoras' theorem are correct?

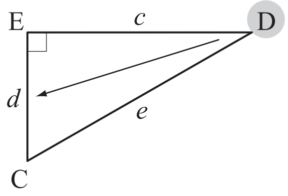
A) $t^2 + u^2 = s^2$
B) $u^2 = s^2 + t^2$
C) $s^2 = u^2 - t^2$



A. *Pythagoras' statements are:* $u^2 = s^2 + t^2$
or $s^2 + t^2 = u^2$
or $s^2 = u^2 - t^2$
or $t^2 = u^2 - s^2$

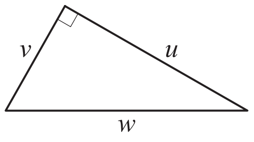
The correct statements are ***B*** *and* ***C****.*

a) Which letter marks the perpendicular side opposite to angle D in this right-angled triangle?



b) Which statements of Pythagoras' theorem are correct?

A) $w^2 = u^2 + v^2$
B) $u^2 = v^2 + w^2$
C) $v^2 = w^2 - u^2$



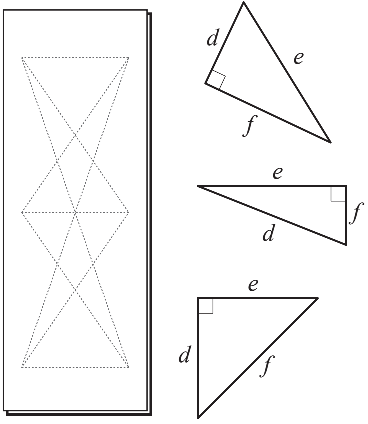
and

c) Connect the following Pythagoras' relationships to their corresponding diagram:

$f^2 = d^2 - e^2$

$e^2 = f^2 - d^2$

$e^2 = d^2 + f^2$

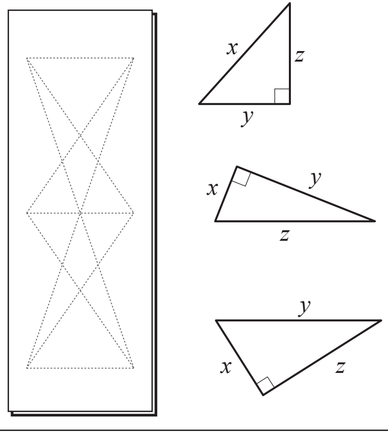


d) Connect the following Pythagoras' relationships to their corresponding diagram:

$x^2 + y^2 = z^2$

$z^2 = x^2 - y^2$

$y^2 = x^2 + z^2$



Skill 26.3 Solving more complex quadratic equations.

Mauve 11 2 2 33 44
Lime 11 22 33 44

- Calculate the square numbers on both sides of the equation.
- Isolate the pronumeral on the left-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.

Q. Find the positive solution for b:
$12^2 + b^2 = 15^2$

A.
$12^2 + b^2 = 15^2$
$144 + b^2 = 225$
$b^2 = 225 - 144$
$b^2 = 81$
$\sqrt{b^2} = \sqrt{81}$ — $81 = 9 \times 9$
$b = \mathbf{9}$

a) Find the positive solution for c: $12^2 + 16^2 = c^2$

$144 + 256 = c^2$
$c^2 = 400$
$\sqrt{c^2} = \sqrt{400}$
$c =$ 20

b) Find the positive solution for a: $a^2 + 15^2 = 17^2$

$a^2 +$
$a^2 =$
$\sqrt{a^2} =$
$a =$

c) Find the positive solution for b: $5^2 + b^2 = 13^2$

$25 + b^2 =$
$b^2 =$
$b =$

d) Find the positive solution for a: $a^2 + 20^2 = 25^2$

$a =$

e) Find the positive solution for b: $24^2 + b^2 = 25^2$

$b =$

f) Find the positive solution for c: $9^2 + 12^2 = c^2$

$c =$

g) Find the positive solution for c: $10^2 + 24^2 = c^2$

$c =$

h) Find the positive solution for b: $40^2 + b^2 = 50^2$

$b =$

i) Find the positive solution for c: $7^2 + 24^2 = c^2$

$c =$

Skill 26.4 Finding the hypotenuse when the other sides of a right-angled triangle are given.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

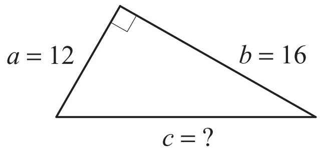
Pythagoras' Theorem

$a^2 + b^2 = c^2$

- Identify the given side lengths on the diagram.
- State Pythagoras' theorem.
- Substitute the values into Pythagoras' theorem.
- Isolate the unknown quantity on the left-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.

Hint: The most common triplets of numbers that make Pythagoras' theorem true are: (3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25). e.g. $3^2 + 4^2 = 5^2$ (Pythagorean triads)

Q. For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the hypotenuse.



A. *a = 12 and b = 16*

$a^2 + b^2 = c^2$

$12^2 + 16^2 = c^2$

$c^2 = 12^2 + 16^2$

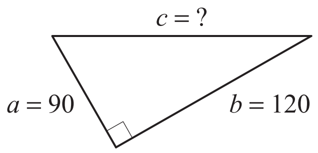
$c^2 = 144 + 256$

$c^2 = 400$

$\sqrt{c^2} = \sqrt{400}$

$c = \mathbf{20}$

a) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the hypotenuse.



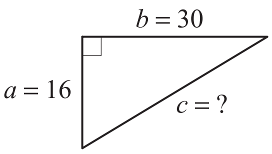
$90^2 + 120^2 = c^2$

$c^2 = 8100 + 14\,400$

$c^2 = 22\,500$

$\sqrt{c^2} = \sqrt{22\,500}$ $c =$

b) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the hypotenuse.

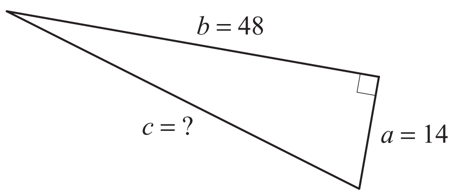


$c^2 =$

$c^2 =$

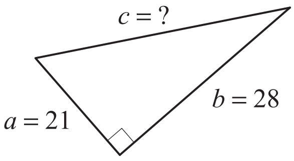
$c =$ $c =$

c) For this triangle use Pythagoras' theorem $c^2 = a^2 + b^2$. Find the length of the hypotenuse.



$c =$ $c =$

d) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the hypotenuse.



$c =$

Skill 26.5 Finding a perpendicular side when the other perpendicular side and the hypotenuse of a right-angled triangle are given.

Mauve 11 22 33 44
Lime 11 22 33 44

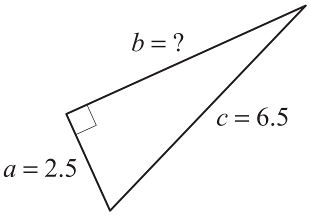
Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

- Identify the given side lengths on the diagram.
- State Pythagoras' theorem.
- Substitute the values into Pythagoras' theorem.
- Isolate the unknown quantity on the left-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.

Hint: The most common triplets of numbers that make Pythagoras' theorem true are: (3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25). e.g. $3^2 + 4^2 = 5^2$ (Pythagorean triads)

Q. For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled b.



A. *$a = 2.5$ and $c = 6.5$*

$a^2 + b^2 = c^2$

$2.5^2 + b^2 = 6.5^2$

$b^2 = 6.5^2 - 2.5^2$

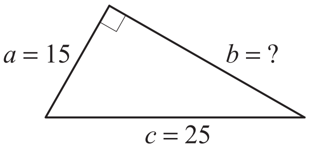
$b^2 = 42.25 - 6.25$

$b^2 = 36$

$\sqrt{b^2} = \sqrt{36}$

$b = \mathbf{6}$

a) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled b.



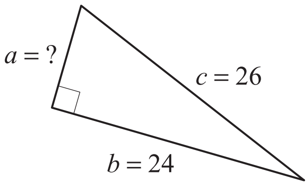
$15^2 + b^2 = 25^2$

$b^2 = 625 - 225$

$b^2 = 400$

$\sqrt{b^2} = \sqrt{400}$ $b =$

b) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled a.

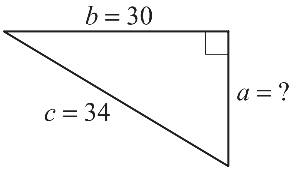


$a^2 =$

$a^2 =$

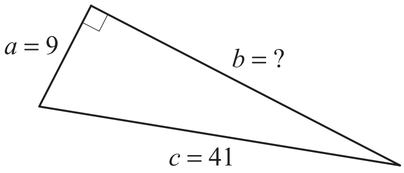
$a =$ $a =$

c) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled a.



$a =$ $a =$

d) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled b.

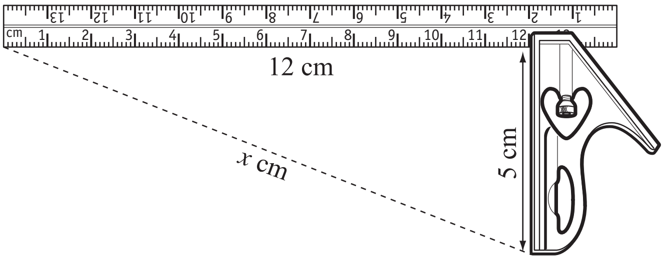


$b =$ $b =$

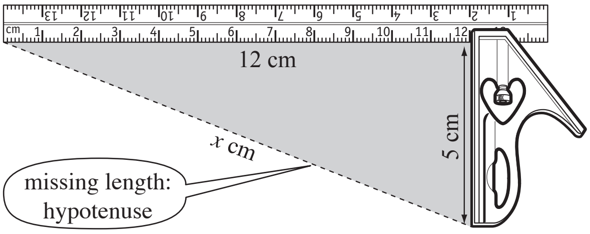
Skill 26.6 Applying Pythagoras' theorem (1).

- Locate or draw a right-angled triangle in the diagram.
- Identify the given side lengths in this right-angled triangle.
- Identify the required side length in this right-angled triangle and label it with a variable.
- Use Pythagoras' theorem to find the required side length.
  (see skills 26.4, page 306 and 26.5, page 307)

Q. Find the missing length in this diagram showing a T-square.



A.



$x^2 = 12^2 + 5^2$ Pythagoras' theorem

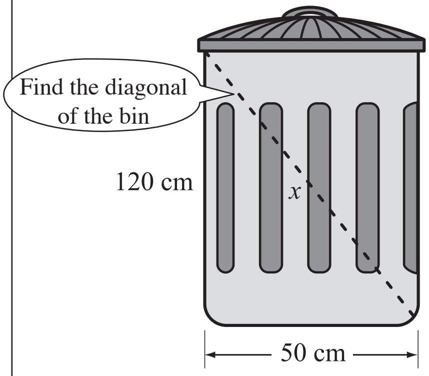
$x^2 = 144 + 25$

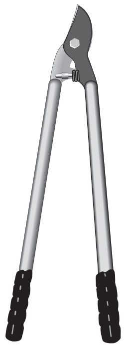
$x^2 = 169$

$x = \sqrt{169}$

$x = \mathbf{13}$

a) Would clipping shears, 125 cm long, fit inside this rubish bin with its lid on? [Objects not drawn to scale.]





$x^2 = 50^2 + 120^2$ Pythagoras' theorem

$x^2 = 2500 + 14\,400$

$x^2 = 16\,900$

$x = \sqrt{16\,900}$

$x = 130$

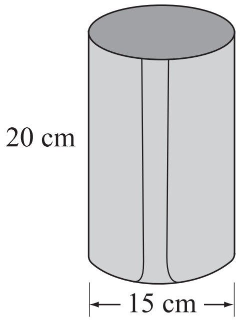
clipper = 125 cm

125 cm < 130 cm

clipper fits inside the bin

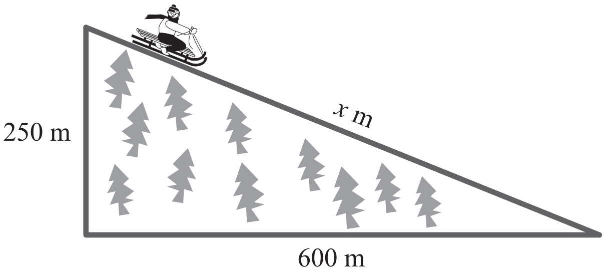
yes

b) Would a 26 cm long paint brush fit inside this tin with its lid on? [Objects not drawn to scale.]



Skill 26.6 Applying Pythagoras' theorem (2).

c) How far down this mountain slope is the sleigh descending?



$x^2 = 250^2 + 600^2$

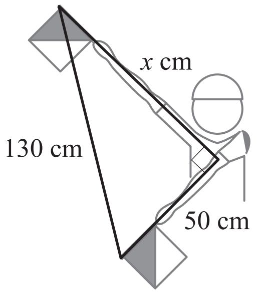
$x^2 =$

$x^2 =$

$x =$

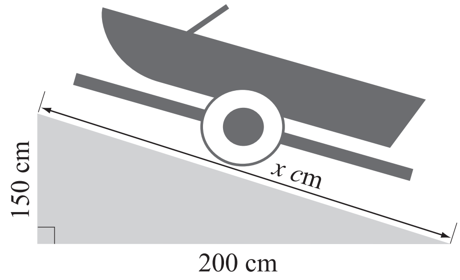
$x =$ ☐ m

d) What is the distance marked x on this diagram showing the semaphore which signals letter I?



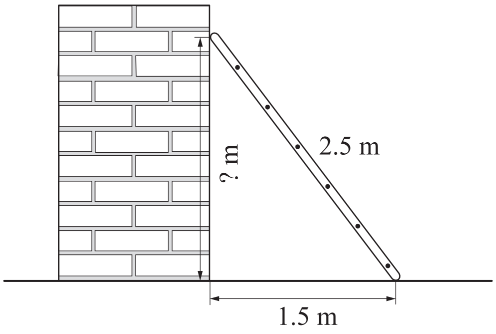
$x =$ ☐ cm

e) How long is the ramp on which the model boat descends?



$x =$ ☐ cm

f) A 2.5 m long ladder is leaning against a wall and its end is 1.5 m from the base of the wall. How high up the wall is the ladder reaching?



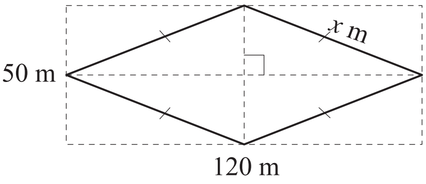
$x =$ ☐ m

Skill 26.7 Applying Pythagoras' theorem to find the perimeter of 2-dimensional shapes.

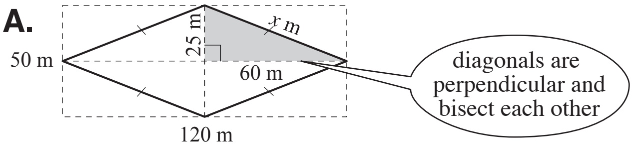
Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 3 4 4

- Highlight a right-angled triangle in the diagram.
- Identify the given side lengths in this right-angled triangle.
- Identify the missing side length in this right-angled triangle and label it with a variable.
- Use Pythagoras' theorem to find the missing side length. (see skills 26.4, page 306 and 26.5, page 307)
- Calculate the perimeter of the 2-dimensional shape. (see skill 23.1, page 259)

Q. Find the perimeter of this rhombus by first calculating the missing side length.



A.



diagonals are perpendicular and bisect each other

$x^2 = 25^2 + 60^2$ Pythagoras' theorem

$x^2 = 625 + 3600$

$x^2 = 4225$

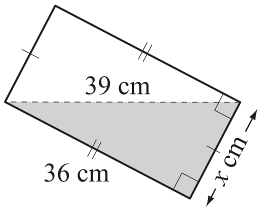
$x = \sqrt{25 \times 169}$

$x = 5 \times 13$

$x = 65$

$P = 4 \times 65 \text{ m} = \mathbf{260\ m}$

a) Find the perimeter of this rectangle by first calculating the missing side length.



$x^2 + 36^2 = 39^2$

$x^2 = 1521 - 1296$

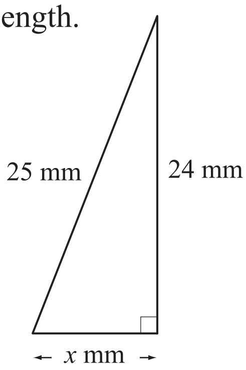
$x^2 = 225$

$x = \sqrt{225}$

$x = 15$

$P = 15 + 15 + 36 + 36$ = ☐ cm

b) Find the perimeter of this triangle by first calculating the missing side length.



$x^2 + 24^2 = 25^2$

$x^2 =$

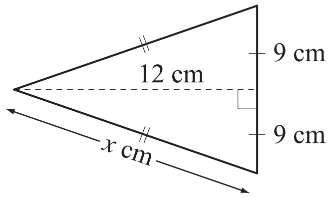
$x^2 =$

$x =$

$x =$

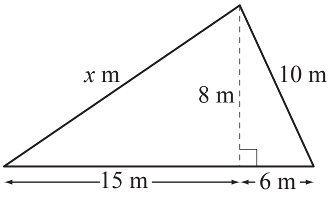
$P =$ = ☐ mm

c) Find the perimeter of this isosceles triangle by first calculating the missing side length.



$P =$ = ☐ cm

d) Find the perimeter of this triangle by first calculating the missing side length.

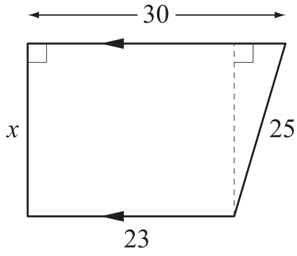


$P =$ = ☐ m

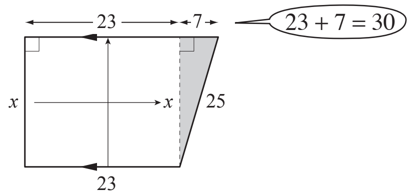
Skill 26.8 Applying Pythagoras’ theorem in a variety of 2-dimensional shapes.

- Highlight a right-angled triangle in the diagram.
- Identify the given lengths in this right-angled triangle.
- Identify the missing length in this right-angled triangle.
- Use Pythagoras’ theorem to find the missing length. (see skills 26.4, page 306 and 26.5, page 307)

Q. Find the missing length in this trapezium.



A.



$x^2 + 7^2 = 25^2$ Pythagoras’ theorem

$x^2 = 625 - 49$

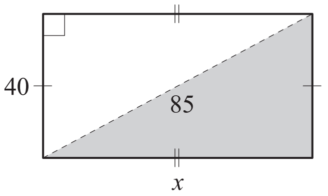
$x^2 = 576$

$x = \sqrt{576}$

$x = \mathbf{24}$

a) Find the missing length in this rectangle.

$x^2 + 40^2 = 85^2$ Pythagoras’ theorem



$x^2 = 7225 - 1600$

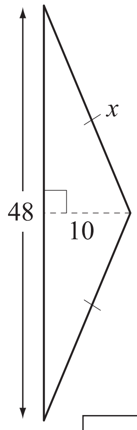
$x^2 = 5625$

$x = \sqrt{5625}$

$x = \sqrt{25 \times 225}$

$x = 5 \times 15$ =

b) Find the missing length in this triangle.



$x^2 =$

$x^2 =$

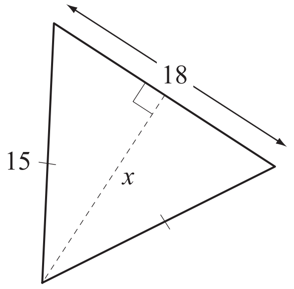
$x^2 =$

$x =$

$x =$

$x =$ =

c) Find the missing length in this triangle.



$x^2 =$

$x^2 =$

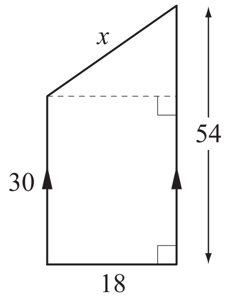
$x^2 =$

$x =$

$x =$

$x =$ =

d) Find the missing length in this trapezium.



$x^2 =$

$x^2 =$

$x^2 =$

$x =$

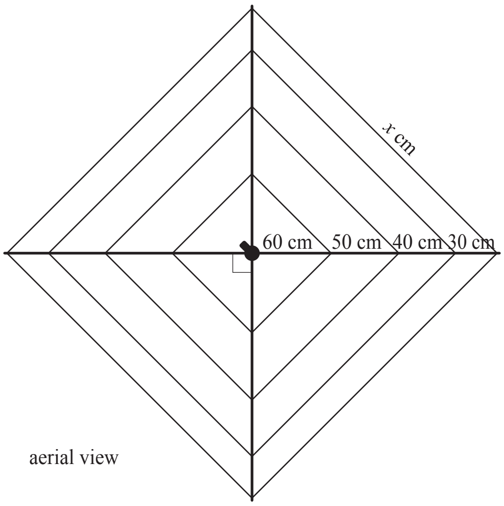
$x =$

$x =$ =

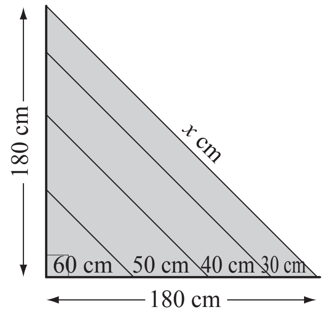
Skill 26.9 Finding a side length in isosceles right-angled triangles (1).

- Use Pythagoras' theorem in the isosceles right-angled triangle to find an unknown side length. (see skills 26.4, page 306 and 26.5, page 307)

Q. How much wire was used for the outside square of this clothes line?
[Leave your answer in surd form.]



A.



$x^2 = 180^2 + 180^2$ Pythagoras' theorem

$x^2 = 2 \times 180^2$

$x = \sqrt{2 \times 32\,400}$

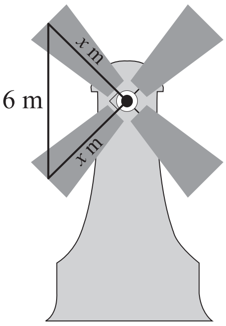
$x = \sqrt{64\,800}$

Perimeter wire $= 4x$

$= \mathbf{4 \times \sqrt{64800}\ cm}$

(approx. 1000 cm)

a) How long is each blade of this windmill, if they are all the same length and the distance between the tips of two consecutive blades is 6 m? [Leave your answer in surd form.]



$x^2 + x^2 = 6^2$ Pythagoras' theorem

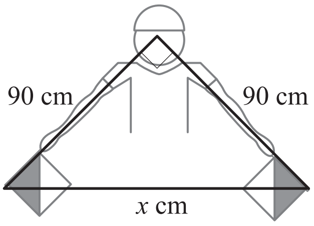
$2x^2 = 36$

$2x^2 \div 2 = 36 \div 2$

$x^2 = 18$

$x = \sqrt{18}$ ____ m

b) What is the distance between the flags when this semaphore is signalling letter N as shown in the diagram? [Leave your answer in surd form.]



$x^2 = 90^2 + 90^2$

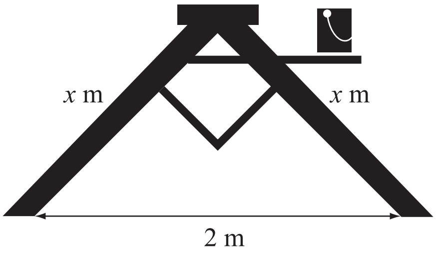
$x^2 = 2 \times$

$x =$

$x =$ ____ cm

Skill 26.9 Finding a side length in isosceles right-angled triangles (2).

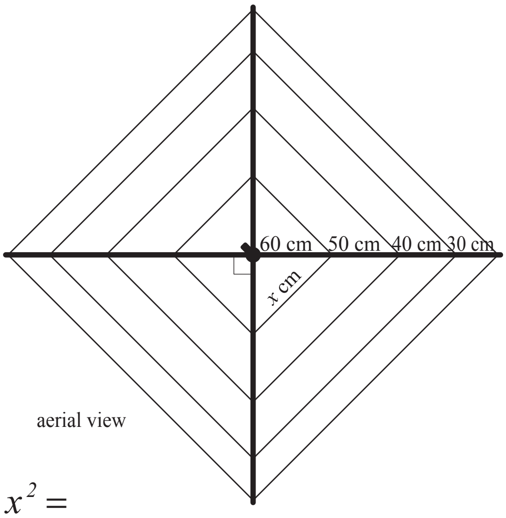
c) How long are this ladder's legs, if they are 2 m apart? [Leave your answer in surd form.]



$x^2 + x^2 = 2^2$

$x =$ m

d) How long is the inner wire on this clothes line? [Leave your answer in surd form.]



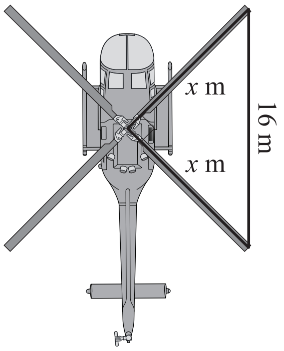
$x^2 =$

$x^2 =$

$x =$

$x =$ cm

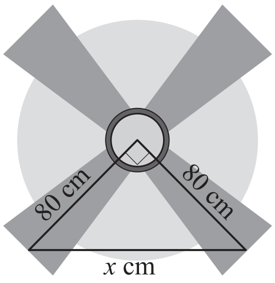
e) How long is each of these helicopter blades, if they are all the same length and the distance between the tips of two consecutive blades is 15 m? [Leave your answer in surd form.]



x^2

$x =$ m

f) Find the missing length in this diagram showing a ceiling fan. [Leave your answer in surd form.]

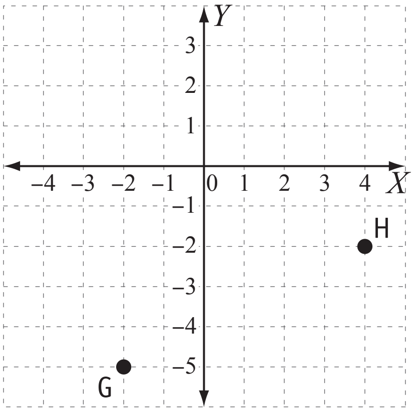


cm

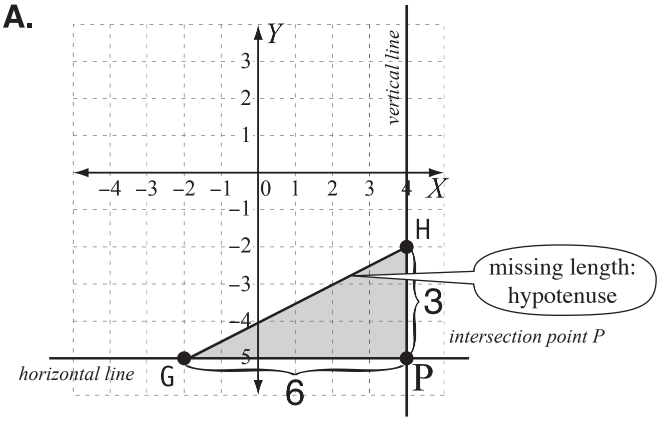
Skill 26.10 Applying Pythagoras' theorem to find the distance between two points located on a Cartesian plane (1).

- Draw a horizontal line through the first point.
- Draw a vertical line through the second point.
- Mark the point at the intersection of these lines.
- Join the three points (the two given points and the point at the intersection) to form a triangle.
- Count the units along the horizontal and vertical sides of the triangle.
- Use Pythagoras' theorem in this right-angled triangle to find the hypotenuse.
  (see skill 26.4, page 306)

Q. Find the distance GH in this Cartesian plane.
[Leave your answer in surd form.]



A.



$GH^2 = GP^2 + PH^2$ Pythagoras' theorem

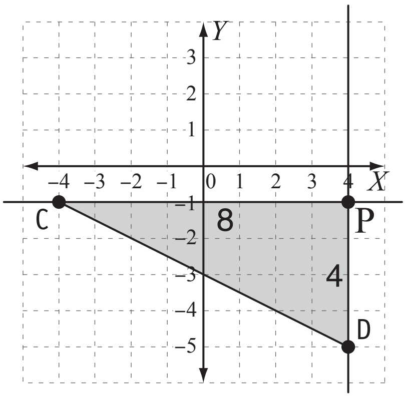
$GH^2 = 6^2 + 3^2$

$GH^2 = 36 + 9$

$GH^2 = 45$

$GH = \sqrt{\mathbf{45}}$

a) Find the distance CD in this Cartesian plane.
[Leave your answer in surd form.]



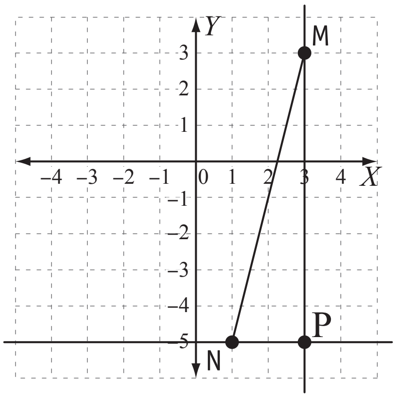
$CD^2 = CP^2 + PD^2$

$CD^2 = 8^2 + 4^2$

$CD^2 = 80$

$CD = \sqrt{80}$

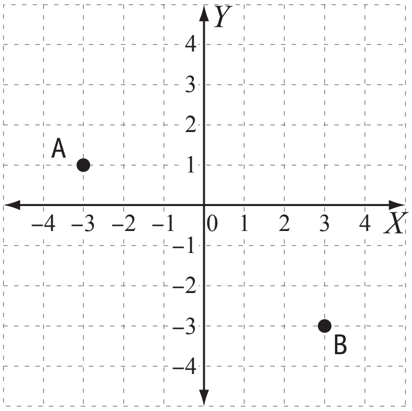
b) Find the distance MN in this Cartesian plane.
[Leave your answer in surd form.]



$MN^2 = MP^2 + PN^2$

Skill 26.10 Applying Pythagoras' theorem to find the distance between two points located on a Cartesian plane (2).

c) Find the distance AB in this Cartesian plane.
[Leave your answer in surd form.]



$AB^2 =$ ....

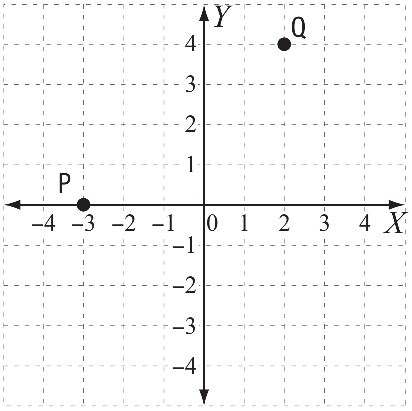
....

....

....

.... = ☐

d) Find the distance PQ in this Cartesian plane.
[Leave your answer in surd form.]



$PQ^2 =$ ....

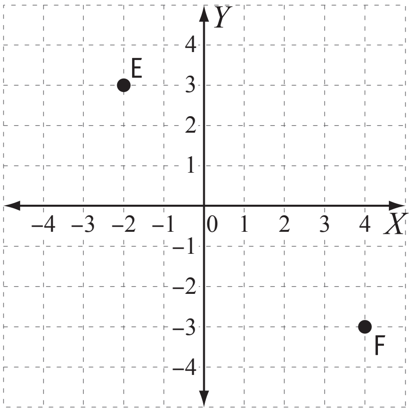
....

....

....

.... = ☐

e) Find the distance EF in this Cartesian plane.
[Leave your answer in surd form.]



$EF^2 =$ ....

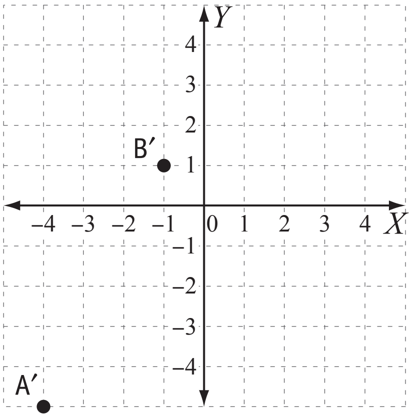
....

....

....

.... = ☐

f) Find the distance A′B′ in this Cartesian plane.
[Leave your answer in surd form.]



$A'B'^2 =$ ....

....

....

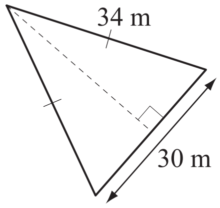
....

.... = ☐

Skill 26.11 Applying Pythagoras' theorem to find the area of 2-dimensional shapes.

- Highlight a right-angled triangle in the diagram.
- Identify the given side lengths in this right-angled triangle.
- Identify the missing side length in this right-angled triangle and label it with a pronumeral.
- Use Pythagoras' theorem to find the missing side length. (see skills 26.4, page 306 and 26.5, page 307)
- Calculate the area of the 2-dimensional shape. (see skills 23.5 to 23.7, pages 265 to 267)

Q. Find the area of this triangle.



A. Let x = perpendicular height

$x^2 = 34^2 - 16^2$

$x^2 = 1156 - 256$

$x^2 = 900$

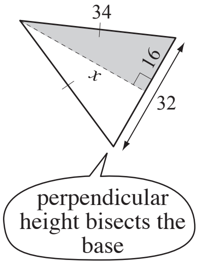
$x = \sqrt{900}$ 900 = 30 × 30

$x = 30$

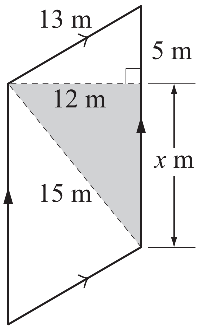
Area of triangle $= \frac{1}{2}bh$

$= \frac{1}{\cancelto{1}{2}} \times \cancelto{16}{32} \times 30$

$= \mathbf{480\ m^2}$



a) Find the area of the parallelogram by first calculating the missing side length.



$x^2 + 12^2 = 15^2$

$x^2 = 225 - 144$

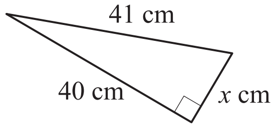
$x^2 = 81$

$x = \sqrt{81} \Rightarrow x = 9$

$base_{parallelogram} = 5 + 9 = 14$

$A_{parall.} = bh = 14 \times 12 =$ ☐ m^2

b) Find the area of the right-angled triangle by first calculating the missing side length.



$x^2 + 40^2 = 41^2$

$x^2 =$ ....

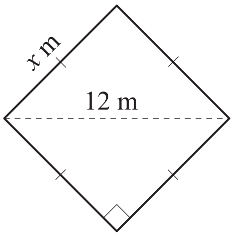
$x^2 =$ ....

$x =$ ....

$A_{triangle} =$ ....

$=$ .... $=$ ☐ cm^2

c) Find the area of the square by first calculating the missing side length.



$x^2 + x^2 = 12^2$

$2x^2 =$ ....

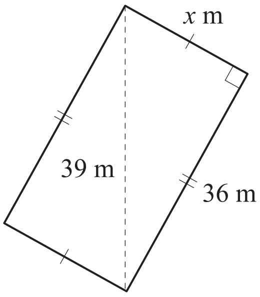
$x^2 =$ ....

$x =$ ....

$A_{square} =$ ....

$=$ .... $=$ ☐ m^2

d) Find the area of the rectangle by first calculating the missing side length.



....

....

....

....

$A_{rectangle} =$ ....

$=$ .... $=$ ☐ m^2

Skill 26.12 Recognising trigonometric functions (sine, cosine, tangent).

- Identify the hypotenuse of the triangle, and the opposite and adjacent sides corresponding to the marked angle (α - alpha, β - beta, θ - theta, etc).
- Label each side of the triangle with H, O and A.
- Decide which two sides of the triangle are given OR which side and angle of the triangle are given.
- Use one of the SOH - CAH - TOA relations to decide which trigonometric ratio can be used to find the unknown angle OR the unknown side.

Hint: Use the SOH - CAH - TOA rules to remember the trigonometric ratios.

Trigonometric ratio (function) sine

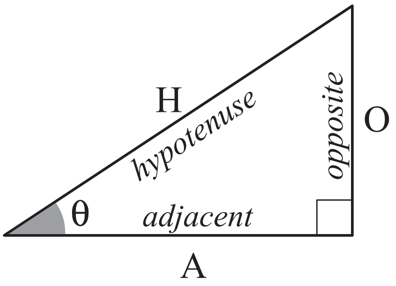
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{O}{H}$ **S**ine**O**pposite**H**ypotenuse **(SOH)**

Trigonometric ratio (function) cosine

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{A}{H}$ **C**osine**A**djacent**H**ypotenuse **(CAH)**

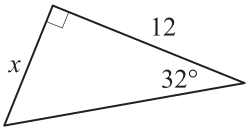
Trigonometric ratio (function) tangent

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \dfrac{O}{A}$ **T**angent**O**pposite**A**djacent **(TOA)**

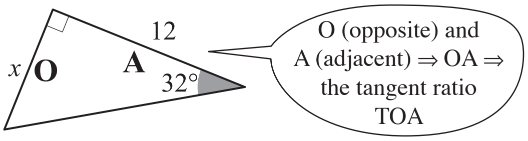


Q. Which trigonometric ratio would be used to find the unknown side x?

A) sine
B) cosine
C) tangent



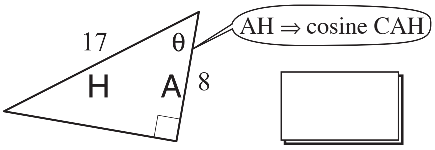
A.



The answer is ***C****.*

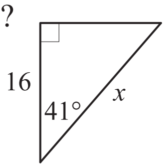
a) Which trigonometric ratio would be used to find angle θ?

A) sine
B) cosine
C) tangent



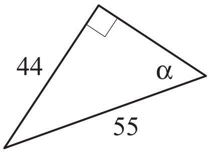
b) Which trigonometric ratio would be used to find the unknown side x?

A) sine
B) cosine
C) tangent



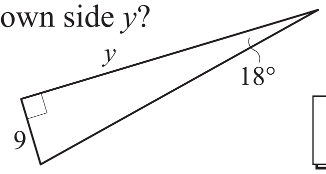
c) Which trigonometric ratio would be used to find angle α?

A) sine
B) cosine
C) tangent



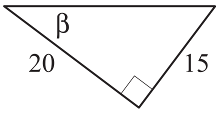
d) Which trigonometric ratio would be used to find the unknown side y?

A) sine
B) cosine
C) tangent



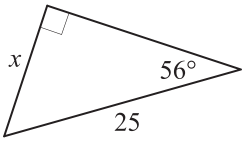
e) Which trigonometric ratio would be used to find angle β?

A) sine
B) cosine
C) tangent



f) Which trigonometric ratio would be used to find the unknown side x?

A) sine
B) cosine
C) tangent



Skill 26.13 Calculating the value of basic trigonometric ratios in right-angled triangles.

Mauve 11 22 33 44
Lime 11 22 33 44

- Mark the angle whose trigonometric ratio is required.
- Label each side of the triangle with H, O and A.
- Use one of the SOH - CAH - TOA relations to calculate the required trigonometric ratio.

Trigonometric ratio (function) sine

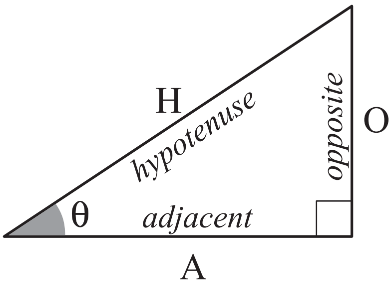
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{O}{H}$ **S**ine**O**pposite**H**ypotenuse **(SOH)**

Trigonometric ratio (function) cosine

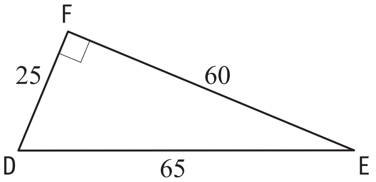
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{A}{H}$ **C**osine**A**djacent**H**ypotenuse **(CAH)**

Trigonometric ratio (function) tangent

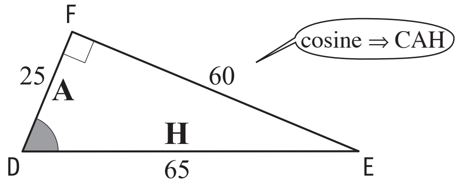
$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \dfrac{O}{A}$ **T**angent**O**pposite**A**djacent **(TOA)**



Q. Calculate the value of cos D in this triangle.

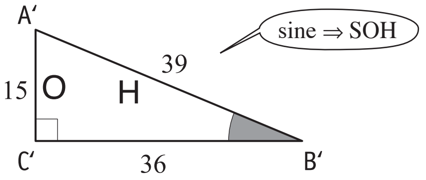


A.



$$\cos D = \frac{adjacent}{hypotenuse}$$
$$= \frac{25}{65} \quad \begin{matrix}\div 5\\ \div 5\end{matrix}$$
$$= \mathbf{\frac{5}{13}}$$

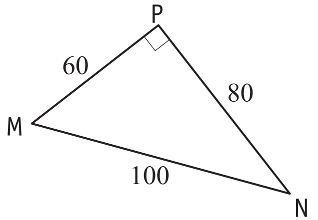
a) Calculate the value of sin B′ in this triangle.



$\sin B' \; \dfrac{O}{H} =$

$= \dfrac{15}{39} \begin{matrix}\div 3\\ \div 3\end{matrix}$ $=$ $\boxed{\quad}$

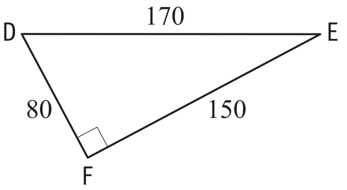
b) Calculate the value of tan M in this triangle.



$\tan M =$

$=$ $=$ $\boxed{\quad}$

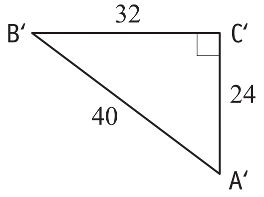
c) Calculate the value of cos E in this triangle.



$\cos E =$

$=$ $=$ $\boxed{\quad}$

d) Calculate the value of sin A′ in this triangle.



$\sin A' =$

$=$ $=$ $\boxed{\quad}$

Skill 26.14 Finding an unknown side of a right-angled triangle when a trigonometric ratio of an angle and another side of the triangle are given (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 **3** 4 4

- Label each side of the triangle with H, O and A.
- Use the SOH or CAH or TOA relation corresponding to the given trigonometric ratio value.
- Substitute the numeric values in the relation.
- Solve the equation for the unknown side length.

Trigonometric ratio (function) sine

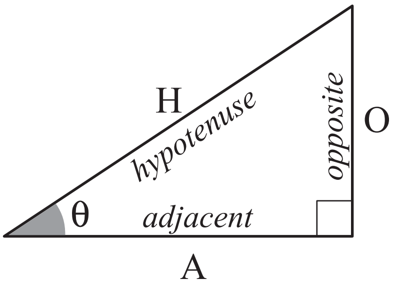
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \frac{O}{H}$ **S**ine**O**pposite**H**ypotenuse **(SOH)**

Trigonometric ratio (function) cosine

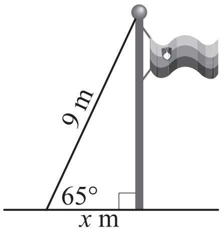
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \frac{A}{H}$ **C**osine**A**djacent**H**ypotenuse **(CAH)**

Trigonometric ratio (function) tangent

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \frac{O}{A}$ **T**angent**O**pposite**A**djacent **(TOA)**



Q. A 9 m support wire is attached to a flagpole and makes an angle of 65° with the ground. If cos 65° ≈ 0.42, find the approximate distance from the end of the wire to the base of the flagpole.



A. $\cos 65° = \frac{adjacent}{hypotenuse} = \frac{A}{H}$

$0.42 = \frac{x}{9}$ cosine ⇒ CAH

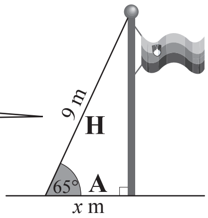
$\frac{42}{100} \times \frac{x}{9}$ Cross multiply

$42 \times 9 = 100 \times x$

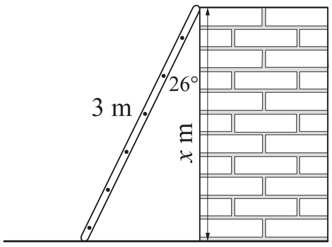
$100x = 378$

$x = 378 \div 100$

$\boldsymbol{x = 3.78\ m}$



a) A 3 m ladder is leaning against a wall and makes an angle of 26° with the wall. If cos 26° ≈ 0.89, how high up the wall is the top of the ladder?



$\cos 26° = \frac{adjacent}{hypotenuse}$

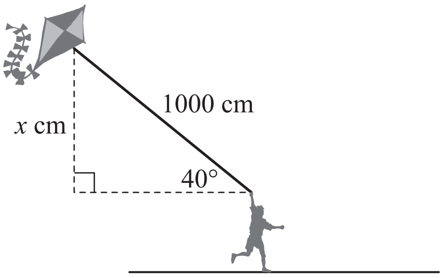
$\frac{89}{100} = \frac{x}{3}$ Cross multiply

$100x = 267$

$x = 267 \div 100$

$x =$ m

b) A kite's string makes an angle of 40° with the horizon. The string length is 1000 cm and sin 40°≈ 0.64. If the boy's height is 160 cm, how high above the ground is the kite flying?



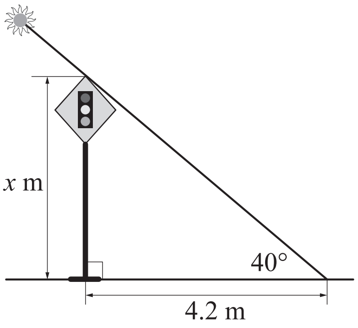
$\sin 40° =$

$x =$ $height =$ cm

Skill 26.14 Finding an unknown side of a right-angled triangle when a trigonometric ratio of an angle and another side of the triangle are given (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 **3** 4 4

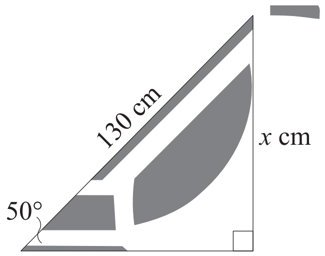
c) A road sign casts a shadow which is 4.2 m long when the sun is at an angle of 40° in the sky. If tan 40° ≈ 0.84, find the height of the road sign. [Give your answer correct to 2 decimal places.]



tan 40° =

$x =$ m

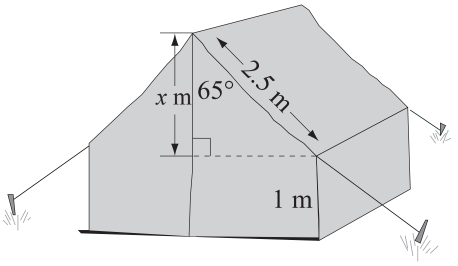
d) In this profile view the vacuum cleaner makes an angle of 50° with the ground. If sin 50°≈ 0.77, how high above the ground is the handle of the vacuum cleaner?



sin 50° =

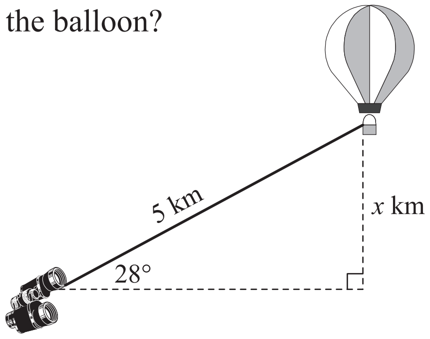
$x =$ cm

e) If cos 65° ≈ 0.42, what is the height of this tent above the ground?



$x =$ *height* = m

f) You are observing a hot air balloon which is 5 km away from you and makes an angle of 28° with your eye level. If sin 28°≈ 0.47, how high above eye level is the balloon?



$x =$ km

Skill 26.15 Calculating the value of trigonometric ratios in right-angled triangles by first applying Pythagoras' theorem (1).

Mauve 11 22 33 44
Lime 11 22 33 44

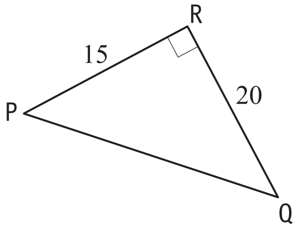
- Label each side of the triangle with H, O and A.
- Apply Pythagoras' theorem to calculate the unknown side length of the triangle. (see skills 26.4, page 306 and 26.5, page 307)
- Use one of the SOH - CAH - TOA relations to calculate the required trigonometric ratio.

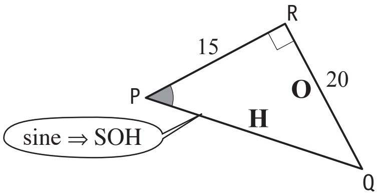
Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

$\sin\theta = \frac{O}{H}$ **(SOH)**	$\cos\theta = \frac{A}{H}$ **(CAH)**	$\tan\theta = \frac{O}{A}$ **(TOA)**

Q. Calculate the value of sin P in this triangle.
[Hint: Pythagoras' theorem will help.]





A. $\sin P = \frac{opposite}{hypotenuse} = \frac{RQ}{PQ}$ unknown PQ

$PQ^2 = PR^2 + RQ^2$ Pythagoras

$PQ^2 = 15^2 + 20^2$

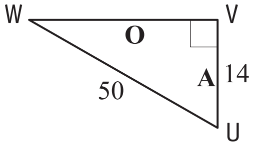
$PQ^2 = 225 + 400$

$PQ^2 = 625$

$PQ = 25$

$\sin P = \frac{RQ}{PQ} = \frac{20}{25} = \boldsymbol{\frac{4}{5}}$

a) Calculate the value of tan U in this triangle.
[Hint: Pythagoras' theorem will help.]



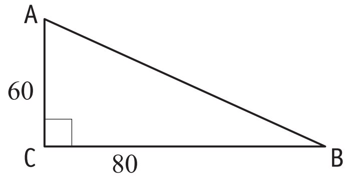
$\tan U = \frac{opposite}{adjacent} = \frac{VW}{UV}$ tangent ⇒ TOA

$VW^2 = UW^2 - UV^2$

$VW^2 = 2500 - 196 = 2304 \Rightarrow VW = 48$

$\tan U = \frac{VW}{UV} = \frac{48}{14}$ =

b) Calculate the value of sin B in this triangle.
[Hint: Pythagoras' theorem will help.]



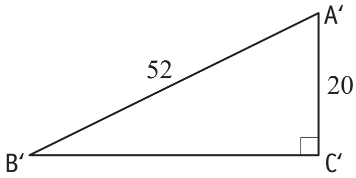
$\sin B =$

$AB^2 =$

$AB^2 =$ $\Rightarrow AB =$

$\sin B =$ =

c) Calculate the value of cos B′ in this triangle.
[Hint: Pythagoras' theorem will help.]



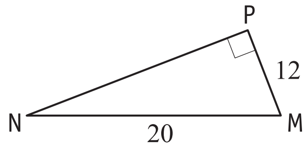
$\cos B' =$

$B'C'^2 =$

$B'C'^2 =$ $\Rightarrow B'C' =$

$\cos B' =$ =

d) Calculate the value of cos N in this triangle.
[Hint: Pythagoras' theorem will help.]



$\cos N =$

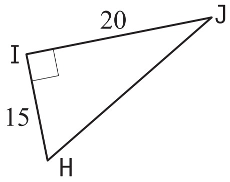
$NP^2 =$

$NP^2 =$ $\Rightarrow NP =$

$\cos N =$ =

Skill 26.15 Calculating the value of trigonometric ratios in right-angled triangles by first applying Pythagoras' theorem (2).

e) Calculate the value of cos J in this triangle.
[Hint: Pythagoras' theorem will help.]



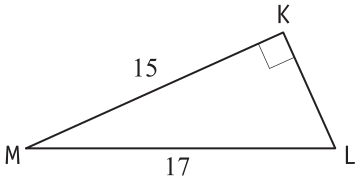
cos J =

HJ^2 =

HJ^2 = $\Rightarrow$ *HJ =*

cos J = =

f) Calculate the value of cos L in this triangle.
[Hint: Pythagoras' theorem will help.]



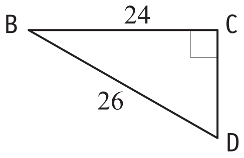
cos L =

KL^2 =

KL^2 = $\Rightarrow$ *KL =*

cos L = =

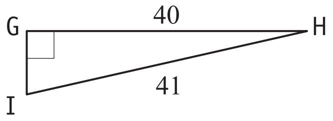
g) Calculate the value of tan D in this triangle.
[Hint: Pythagoras' theorem will help.]



tan D =

=

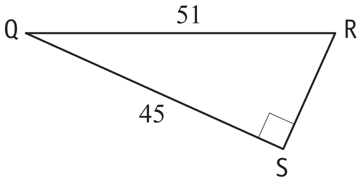
h) Calculate the value of tan H in this triangle.
[Hint: Pythagoras' theorem will help.]



tan H =

=

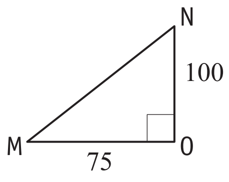
i) Calculate the value of sin Q in this triangle.
[Hint: Pythagoras' theorem will help.]



sin Q =

=

j) Calculate the value of sin M in this triangle.
[Hint: Pythagoras' theorem will help.]



sin M =

=

27. [Angles]

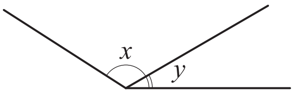
Skill 27.1 Choosing the correct terms related to angles.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Consider the definitions and properties of a variety of angles.
  (see Glossary or Maths Facts, page 455)
  Hints: An angle can be classified according to its size (acute, right, obtuse, straight and reflex).
  Two angles can be classified according to their position in relation to one another (adjacent, supplementary, complementary or vertically opposite).

Q. Which would describe the pair of angles marked x and y in this diagram?

A) vertically opposite
B) supplementary
C) adjacent

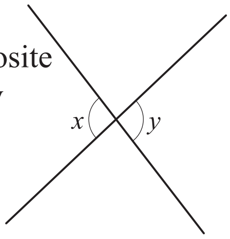


A. *A) vertically opposite ⇒ equal angles*
(x and y are not equal) false
B) supplementary ⇒ angles add to 180°
(x and y add to less than 180°) false
C) adjacent ⇒ angles share the vertex and an arm true

The answer is ***C***.

a) Which would describe the pair of angles marked x and y in this diagram?

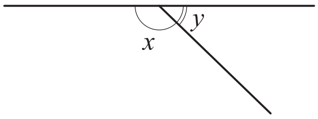
A) right
B) vertically opposite
C) supplementary



B

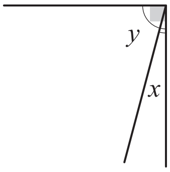
b) Which would describe the pair of angles marked x and y in this diagram?

A) straight
B) supplementary
C) acute



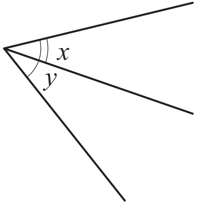
c) Which would describe the pair of angles marked x and y in this diagram?

A) reflex
B) right
C) complementary



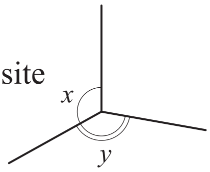
d) Which would describe the pair of angles marked x and y in this diagram?

A) acute
B) obtuse
C) complementary



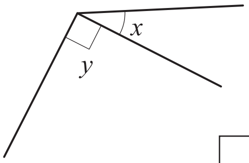
e) Which would describe the pair of angles marked x and y in this diagram?

A) supplementary
B) obtuse
C) vertically opposite



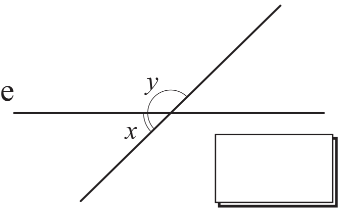
f) Which would describe the pair of angles marked x and y in this diagram?

A) complementary
B) right
C) adjacent



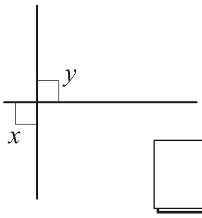
g) Which would describe the pair of angles marked x and y in this diagram?

A) supplementary
B) acute
C) vertically opposite



h) Which would describe the pair of angles marked x and y in this diagram?

A) straight
B) complementary
C) right



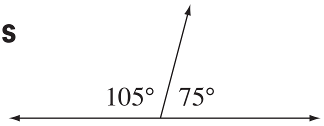
Skill 27.2 Finding the complement and the supplement of a given angle.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Use the properties of angles.
- Write an equation involving the unknown angle x°.
- Solve the equation for x°.

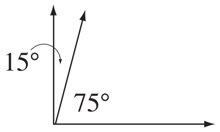
Supplementary Angles

Add to 180°

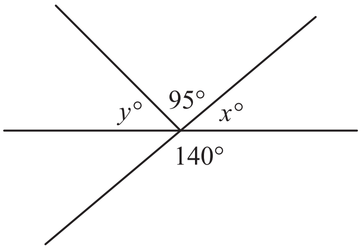


Complementary Angles

Add to 90°



Q. Find the values of x° and y°.



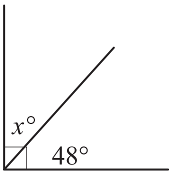
A. *x° and 140° are supplementary:*

$$x^\circ + 140^\circ = 180^\circ$$
$$x^\circ + 140^\circ - 140^\circ = 180^\circ - 140^\circ$$
$$x^\circ = \mathbf{40^\circ}$$

y°, 95° and x° are supplementary:

$$y^\circ + 95^\circ + 40^\circ = 180^\circ$$
$$y^\circ + 135^\circ - 135^\circ = 180^\circ - 135^\circ$$
$$y^\circ = \mathbf{45^\circ}$$

a) Find the value of x°.

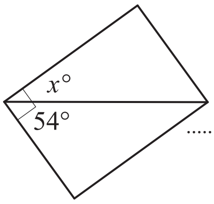


$x^\circ + 48^\circ = 90^\circ$

$x^\circ + 48^\circ - 48^\circ = 90^\circ - 48^\circ$

$x^\circ =$ ☐

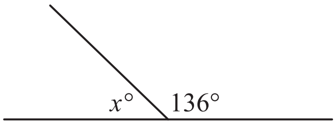
b) Find the value of x°.



$x^\circ + 54^\circ = 90^\circ$

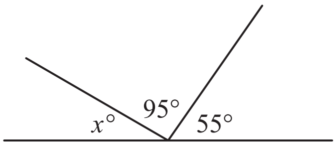
$x^\circ =$ ☐

c) Find the value of x°.



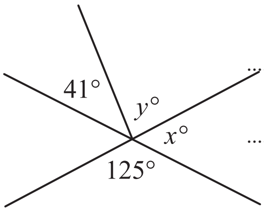
$x^\circ =$ ☐

d) Find the value of x°.



$x^\circ =$ ☐

e) Find the values of x° and y°.

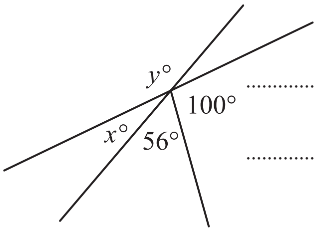


$x^\circ + 125^\circ = 180^\circ$

$x^\circ =$ ☐

$y^\circ =$ ☐

f) Find the values of x° and y°.



$x^\circ + 156^\circ = 180^\circ$

$x^\circ =$ ☐

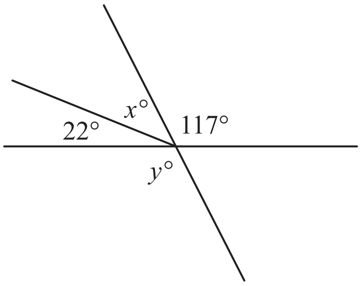
$y^\circ =$ ☐

Skill 27.3 Working with vertically opposite angles.

Mauve 1 1 **2** 2 3 3 4 4
Lime **1** 1 2 2 3 3 4 4

- Use the definition of vertically opposite angles. (see Glossary, page 443 and Maths Facts, page 455)
- Consider complementary and supplementary angles. (see skill 27.2, page 324)

Q. Find the values of $x°$ and $y°$.



A. *$y°$ and 117° are vertically opposite:*

$$y° = \mathbf{117°}$$

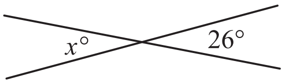
$x°$, 22° and 117° are supplementary:

$$x° + 22° + 117° = 180°$$

$$x° + 139° - 139° = 180° - 139°$$

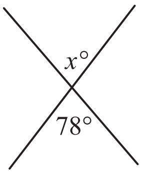
$$x° = \mathbf{41°}$$

a) Find the value of $x°$.

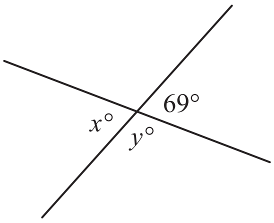


26°

b) Find the value of $x°$.



c) Find the values of $x°$ and $y°$.



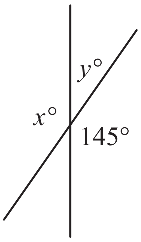
$x° =$ 69°

$y° + 69° = 180°$

$y° + 69° - 69° = 180° - 69°$

$y° =$

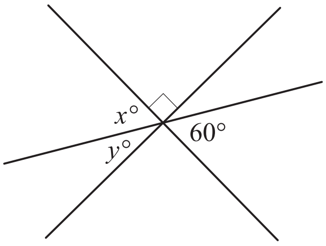
d) Find the values of $x°$ and $y°$.



$x° =$

$y° =$

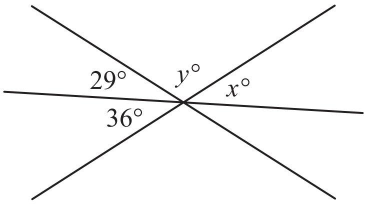
e) Find the values of $x°$ and $y°$.



$x° =$

$y° =$

f) Find the values of $x°$ and $y°$.



$x° =$

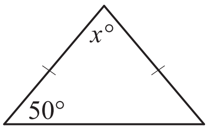
$y° =$

Skill 27.4 Working with angles in a triangle.

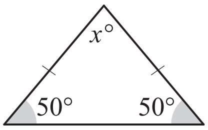
Mauve 1 1 2 **2** 3 3 4 4
Lime 1 **1** 2 2 3 3 4 4

- Use the properties of triangles.
- Write an equation involving the unknown angle x°.
- Solve the equation for x°.

Q. Find the value of x°.



A.



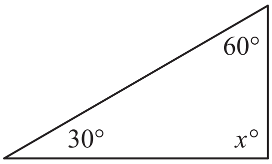
Isosceles triangle ⇒ base angles are equal

$$x^\circ + 50^\circ + 50^\circ = 180^\circ$$

$$x^\circ + 100^\circ - 100^\circ = 180^\circ - 100^\circ$$

$$x^\circ = \mathbf{80^\circ}$$

a) Find the value of x°.

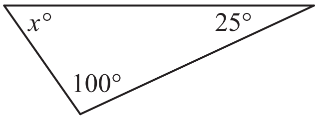


$$x^\circ + 90^\circ = 180^\circ$$

$$x^\circ + 90^\circ - 90^\circ = 180^\circ - 90^\circ$$

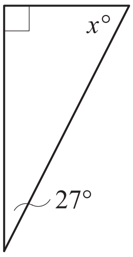
$x^\circ =$

b) Find the value of x°.



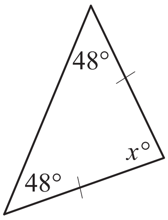
$x^\circ =$

c) Find the value of x°.



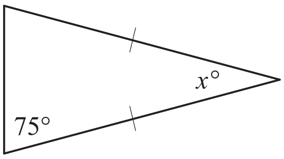
$x^\circ =$

d) Find the value of x°.



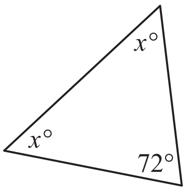
$x^\circ =$

e) Find the value of x°.



$x^\circ =$

f) Find the value of x°.



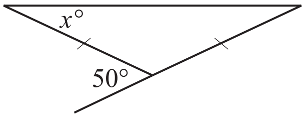
$x^\circ =$

Skill 27.5 Finding the exterior angle of a triangle.

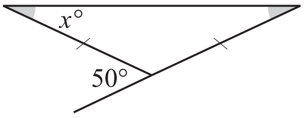
Mauve 1 1 2 2 **3** 3 4 4
Lime 1 1 **2** 2 3 3 4 4

- Use the property:
  - an exterior angle of a triangle is equal to the sum of the two opposite interior angles of the triangle.
- Write an equation involving the unknown angle x°.
- Solve the equation for x°.

Q. Find the value of x°.



A.



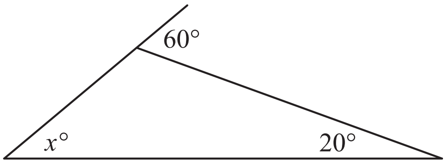
Isosceles triangle ⇒ base angles are equal

$$x^\circ + x^\circ = 50^\circ$$

$$2x^\circ \div 2 = 50^\circ \div 2$$

$$x^\circ = \mathbf{25^\circ}$$

a) Find the value of x°.

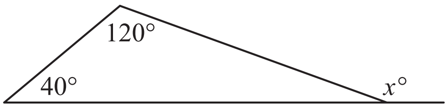


$$x^\circ + 20^\circ = 60^\circ$$

$$x^\circ + 20^\circ - 20^\circ = 60^\circ - 20^\circ$$

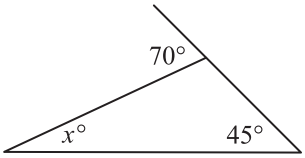
$x^\circ =$

b) Find the value of x°.



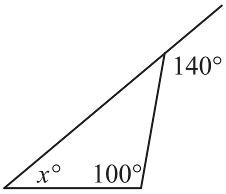
$x^\circ =$

c) Find the value of x°.



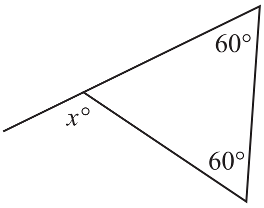
$x^\circ =$

d) Find the value of x°.



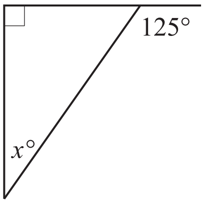
$x^\circ =$

e) Find the value of x°.



$x^\circ =$

f) Find the value of x°.

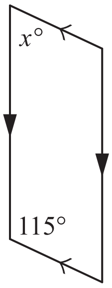


$x^\circ =$

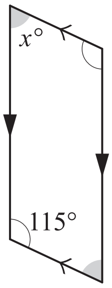
Skill 27.6 Working with angles in a quadrilateral.

- Use the property:
  - the sum of the interior angles of any quadrilateral is 360°.
- Write an equation involving the unknown angle x°.
- Solve the equation for x°.

Q. Find the value of x°.



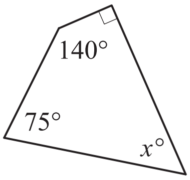
A.



Parallelogram ⇒ opposite angles are equal

$$2x^\circ + 2 \times 115^\circ = 360^\circ$$
$$2x^\circ + 230^\circ - 230^\circ = 360^\circ - 230^\circ$$
$$2x^\circ = 130^\circ$$
$$2x^\circ \div 2 = 130^\circ \div 2$$
$$x^\circ = \mathbf{65^\circ}$$

a) Find the value of x°.

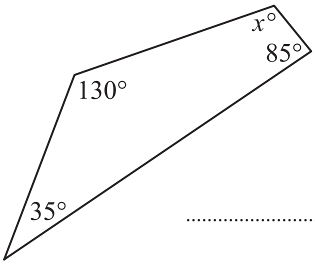


$$x^\circ + 90^\circ + 140^\circ + 75^\circ = 360^\circ$$

$$x^\circ + 305^\circ - 305^\circ = 360^\circ - 305^\circ$$

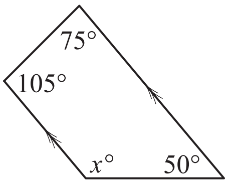
$x^\circ =$

b) Find the value of x°.



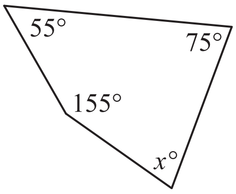
$x^\circ =$

c) Find the value of x°.



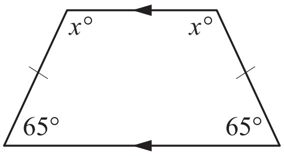
$x^\circ =$

d) Find the value of x°.



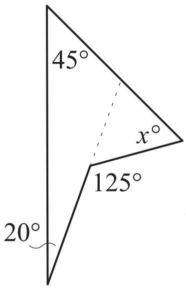
$x^\circ =$

e) Find the value of x°.



$x^\circ =$

f) Find the value of x°.



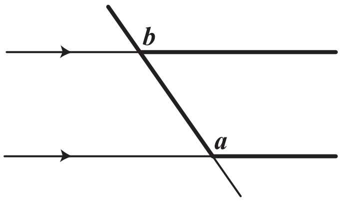
$x^\circ =$

Skill 27.7 Working with pairs of alternate, co-interior and corresponding angles.

Mauve 1 1 2 2 3 3 **4** 4
Lime 1 1 2 2 **3** 3 4 4

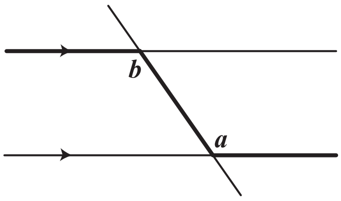
- Consider the classification and properties of the angles formed by intersecting a pair of parallel lines by a transversal. (see Glossary, pages 386, 391 and 394)

CORRESPONDING ANGLES



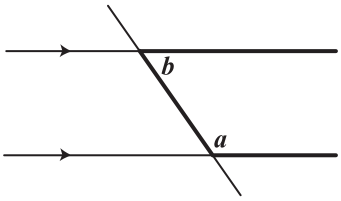
$\angle a = \angle b$

ALTERNATE ANGLES



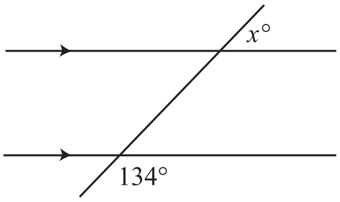
$\angle a = \angle b$

CO-INTERIOR ANGLES

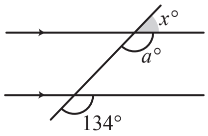


$\angle a + \angle b = 180°$

Q. Find the value of $x°$.



A.



x° and 134° are not in any category above

a° and 134° are corresponding angles

$\Rightarrow a° = 134°$

x° and a° are supplementary angles

$\Rightarrow x° + a° = 180°$

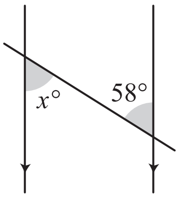
Substitute $a° = 134° \Rightarrow$ *the equation becomes:*

$$x° + 134° = 180°$$

$$x° + 134° - 134° = 180° - 134°$$

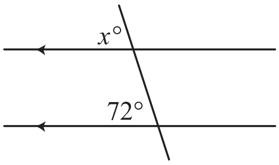
$$x° = \mathbf{46°}$$

a) Find the value of $x°$.



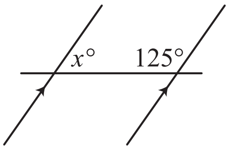
alternate angles $\Rightarrow x° =$ ☐

b) Find the value of $x°$.



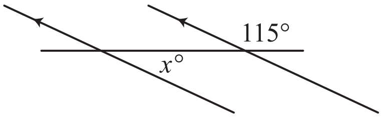
$\Rightarrow x° =$ ☐

c) Find the value of $x°$.



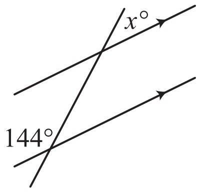
$\Rightarrow x° =$ ☐

d) Find the value of $x°$.



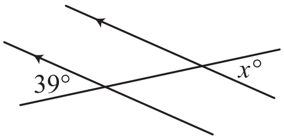
$\Rightarrow x° =$ ☐

e) Find the value of $x°$.



$\Rightarrow x° =$ ☐

f) Find the value of $x°$.

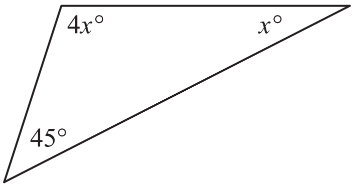


$\Rightarrow x° =$ ☐

Skill 27.8 Finding the value of an angle in a variety of diagrams.

- Use the properties of angles. (see skills 27.1 to 27.7, pages 323 to 329 and Maths Facts, page 460)
- Write an equation involving the unknown angle $x°$.
- Solve the equation for $x°$.

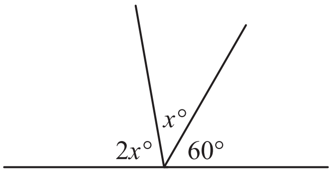
Q. Find the value of $x°$.



A.

$$4x° + x° + 45° = 180°$$
$$5x° + 45° - 45° = 180° - 45°$$
$$5x° = 135°$$
$$5x° \div 5° = 135° \div 5°$$
$$x° = \mathbf{27°}$$

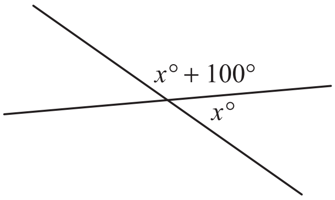
a) Find the value of $x°$.



$$3x° + 60° - 60° = 180° - 60°$$
$$3x° \div 3 = 120° \div 3$$

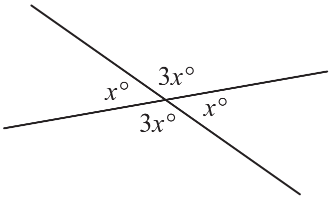
$x° =$

b) Find the value of $x°$.



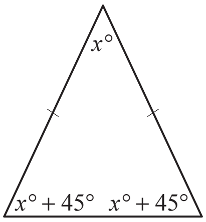
$x° =$

c) Find the value of $x°$.



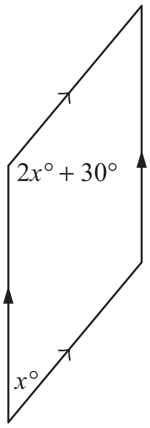
$x° =$

d) Find the value of $x°$.



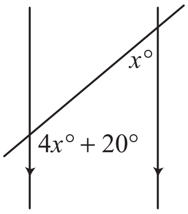
$x° =$

e) Find the value of $x°$.



$x° =$

f) Find the value of $x°$.



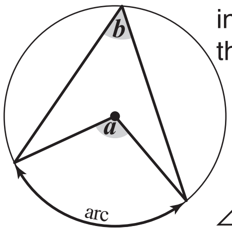
$x° =$

Skill 27.9 Finding the value of an angle in a circle (1).

Mauve 11 22 33 44
Lime 11 22 33 44

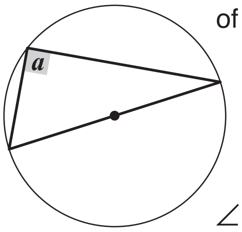
- Use the properties of angles in circles.

Property 1 The angle at the centre of a circle is twice the size of the inscribed angle which intersects the same arc of the circle.



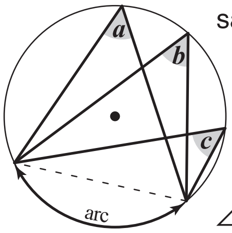
$\angle a = 2 \times \angle b$

Property 2 The angle formed on the circumference from a diameter of a circle is always a right angle.



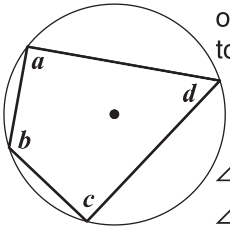
$\angle a = 90°$

Property 3 All angles at the circumference standing on the same arc, in the same segment, are equal.



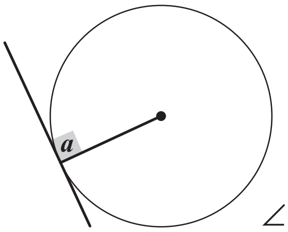
$\angle a = \angle b = \angle c$

Property 4 The opposite angles in a cyclic quadrilateral (all 4 vertices are on the circumference) add up to 180° (are supplementary).



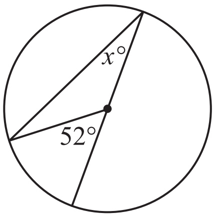
$\angle a + \angle c = 180°$
$\angle b + \angle d = 180°$

Property 5 Any tangent drawn on a circle meets the radius of the circle at right angles.

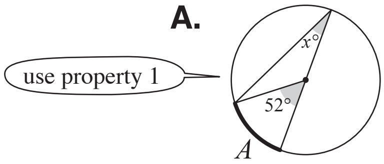


$\angle a = 90°$

Q. Find the value of $x°$.



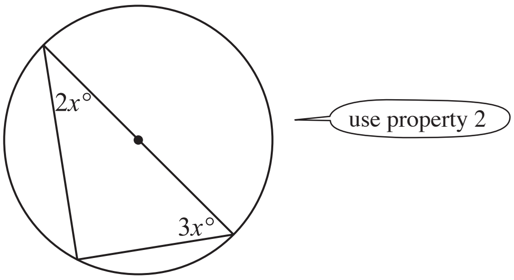
A.



52° is an angle at the centre intercepting arc A
x° is an inscribed angle intercepting arc A
A ⇒ 2x° = 52°
Solve the equation:

$$2x° = 52°$$
$$2x° \div 2 = 52° \div 2$$
$$x° = \mathbf{26°}$$

a) Find the value of $x°$.

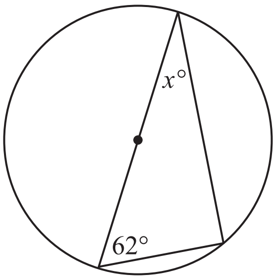


$2x° + 3x° = 90°$

$5x° \div 5 = 90° \div 5$

$x° =$ ☐

b) Find the value of $x°$.

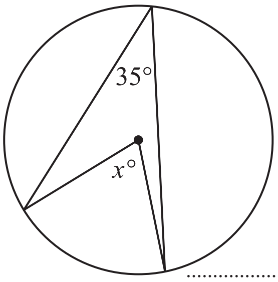


$x° =$ ☐

Skill 27.9 Finding the value of an angle in a circle (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

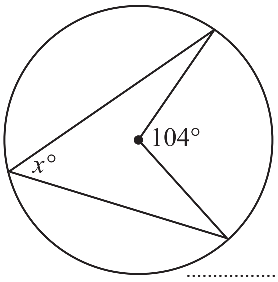
c) Find the value of $x°$.



$x° = 2 \times 35°$

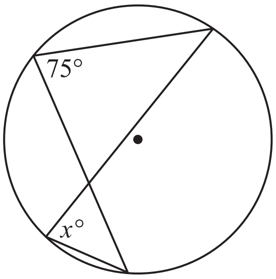
$x° =$

d) Find the value of $x°$.



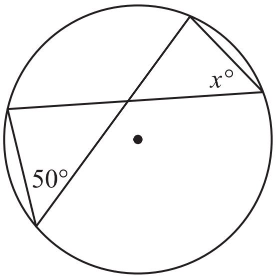
$x° =$

e) Find the value of $x°$.



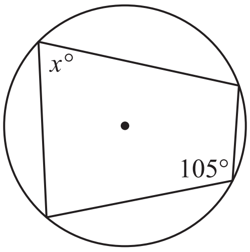
$x° =$

f) Find the value of $x°$.



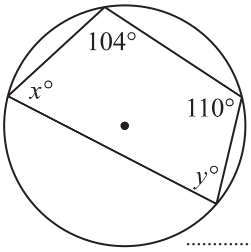
$x° =$

g) Find the value of $x°$.



$x° =$

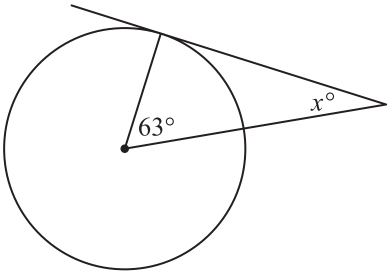
h) Find the values of $x°$ and $y°$.



$x° =$

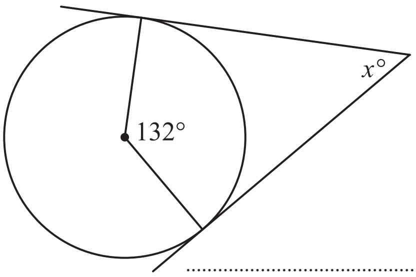
$y° =$

i) Find the value of $x°$.



$x° =$

j) Find the value of $x°$.



$x° =$

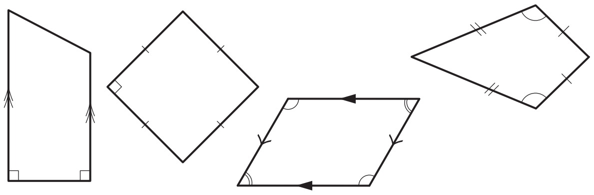
28. [Geometric Reasoning]

Skill 28.1 Recognising polygons, quadrilaterals and triangles.

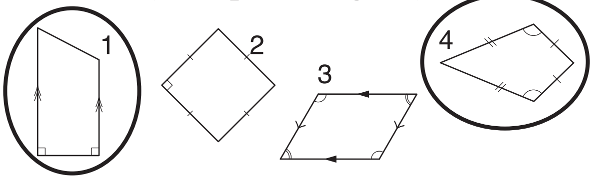
Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Consider the definition of a polygon. (see Glossary, page 423)
- Consider the properties of a parallelogram:
  - both pairs of opposite sides are parallel and equal in length.
- Consider the properties of an isosceles triangle:
  - two sides and two corresponding angles are equal.

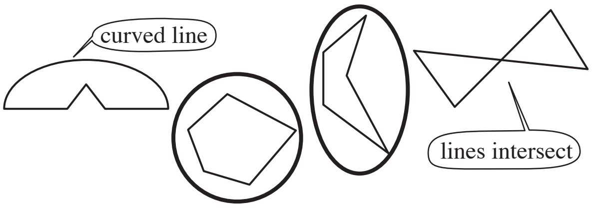
Q. Circle the shapes that are **not** parallelograms.



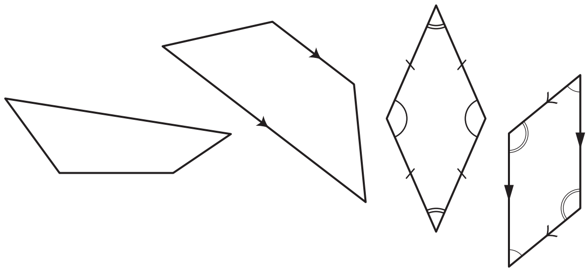
A. *1st shape - has only two opposite sides parallel (**not a parallelogram**).*
2nd shape - has both pairs of opposite sides equal in length (parallelogram).
3rd shape - has both pairs of opposite sides parallel (parallelogram).
*4th shape - doesn't have any parallel sides (**not a parallelogram**).*



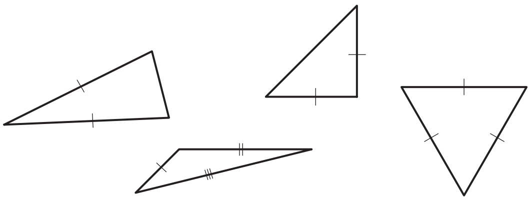
a) Circle the shapes that are polygons.



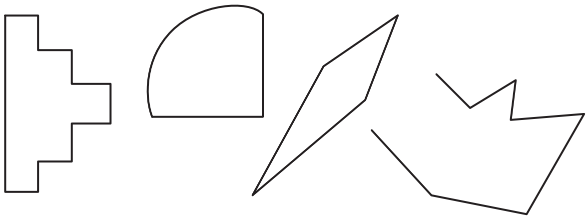
b) Circle the shapes that are **not** parallelograms.



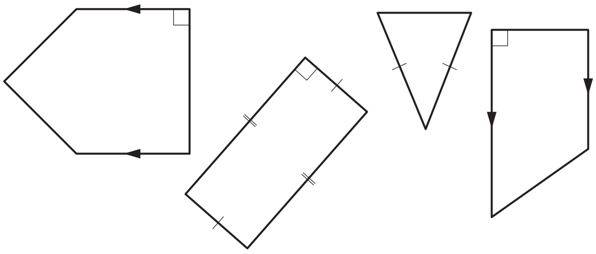
c) Circle the shape that is **not** an isosceles triangle.



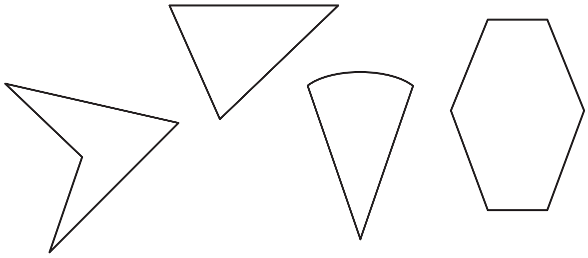
d) Circle the shapes that are **not** polygons.



e) Circle the shape that is a parallelogram.



f) Circle the shape that is **not** a polygon.



Skill 28.2 Classifying triangles.

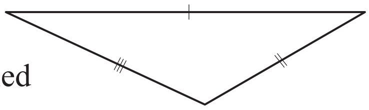
- Look for equal sides or equal angles.
- Look at the types of angles inside the triangle.

Sides and angles	Triangle type
no equal sides/angles	**scalene**
two equal sides/angles	**isosceles**
three equal sides/angles	**equilateral**

Angles	Triangle type
all acute angles	**acute-angled**
one right angle	**right-angled**
one obtuse angle	**obtuse-angled**

Q. Which two options describe this triangle?

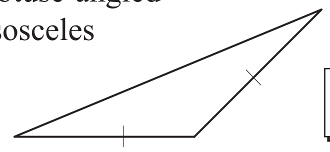
A) scalene
B) equilateral
C) obtuse-angled



A. *A) scalene (no equal sides/angles) ⇒ true*
B) equilateral (all equal sides) ⇒ false
C) obtuse-angled (1 obtuse angle) ⇒ true
*The answer is **A** and **C**.*

a) Which two options describe this triangle?

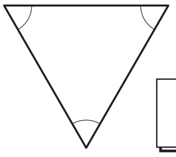
A) right-angled
B) obtuse-angled
C) isosceles



B and C

b) Which two options describe this triangle?

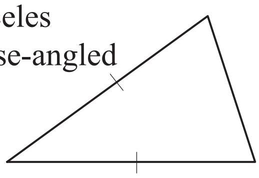
A) equilateral
B) scalene
C) acute-angled



and

c) Which two options describe this triangle?

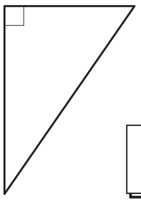
A) acute-angled
B) isosceles
C) obtuse-angled



and

d) Which two options describe this triangle?

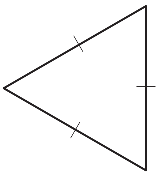
A) acute-angled
B) right-angled
C) scalene



and

e) Which two options describe this triangle?

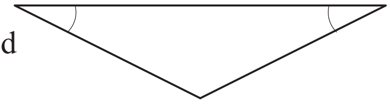
A) acute-angled
B) scalene
C) equilateral



and

f) Which two options describe this triangle?

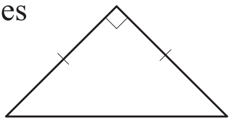
A) isosceles
B) obtuse-angled
C) right-angled



and

g) Which two options describe this triangle?

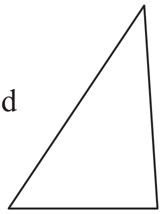
A) acute-angled
B) right-angled
C) isosceles



and

h) Which two options describe this triangle?

A) scalene
B) isosceles
C) acute-angled



and

Skill 28.3 Describing the properties of quadrilaterals.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Consider the properties of squares, rectangles, rhombi, parallelograms, kites and trapeziums. (see Glossary, page 428)

Q. I am a quadrilateral whose diagonals are not equal in length and bisect each other at right angles. What am I?

A) square
B) parallelogram
C) rhombus
D) kite

A. *A) diagonals are equal* ⇒ *A false*
B) diagonals do not bisect each other at right angles ⇒ *B false*
C) diagonals are not equal and bisect each other at right angles ⇒ *C true*
D) diagonals do not bisect each other ⇒ *D false*

The answer is ***C****.*

a) I am a two-dimensional shape with 4 sides. My diagonals are not equal in length and bisect each other but not at right angles. What am I?

A) rhombus
B) parallelogram
C) kite
D) trapezium

B

b) I am a quadrilateral with both pairs of opposite sides parallel and diagonals equal in length. What am I?

A) rhombus
B) trapezium
C) parallelogram
D) rectangle

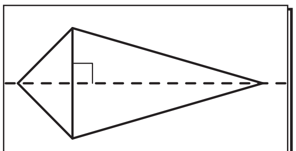
c) I am a two-dimensional shape with 4 sides. Adjacent angles are not equal and I have two axes of symmetry. What am I?

A) trapezium
B) kite
C) rhombus
D) rectangle

d) I am a quadrilateral with both pairs of opposite sides equal in length, but no axis of symmetry. What am I?

A) square
B) trapezium
C) parallelogram
D) rhombus

e) Draw and name the quadrilateral whose diagonals are perpendicular, but has only one axis of symmentry.



kite

f) Draw and name the quadrilateral whose diagonals are equal in length and bisect each other at right angles.

g) Draw and name the quadrilateral whose pairs of opposite angles are equal, but does not have any axis of symmetry.

h) Draw and name the quadrilateral whose diagonals bisect each other at right angles and has two axes of symmetry.

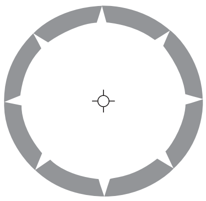
Skill 28.4 Recognising rotational symmetry in 2-dimensional shapes.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

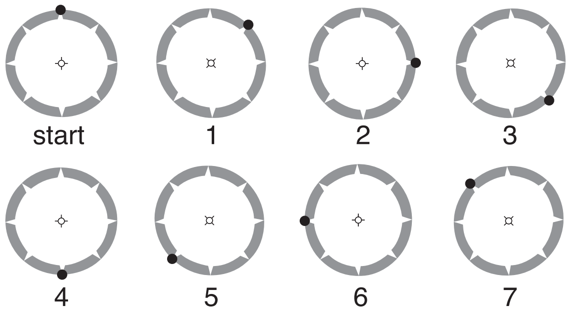
- Try to visualise the shape during a full turn of 360°.
- Count how many times during the full turn the image of the shape exactly covers the original shape.
- This number is called the order of rotational symmetry.

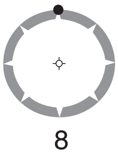
Hints: A shape does not have rotational symmetry if, during a full rotation of 360°, the image of the shape does not exactly cover the original shape.
To count how many times the image of the shape exactly covers the original shape, mark a point on the shape, so you know when the shape has done a complete rotation of 360°.

Q. What is the order of rotational symmetry for this shape?



A.

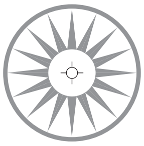


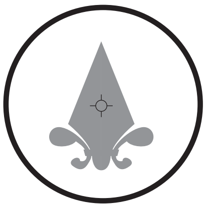


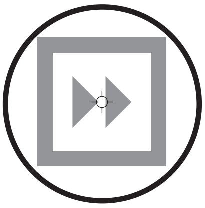
The image of the shape covers the original figure exactly 8 times.

The order of rotational symmetry = ***8***

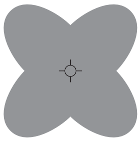
a) Circle the shapes that do not have rotational symmetry.

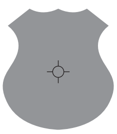


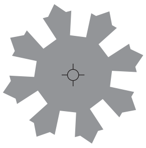




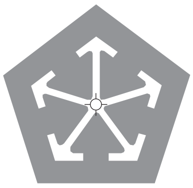
b) Circle the shapes that have rotational symmetry.



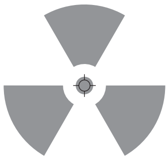




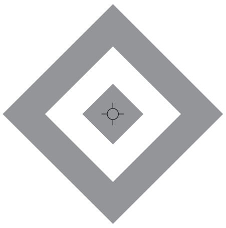
c) This shape has rotational symmetry. How many times during a full turn (360°) does the image of the shape exactly cover the original figure (order of rotational symmetry)?



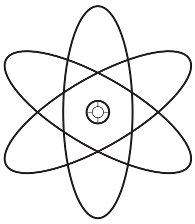
d) This shape has rotational symmetry. How many times during a full turn (360°) does the image of the shape exactly cover the original figure (order of rotational symmetry)?



e) What is the order of rotational symmetry for this shape?

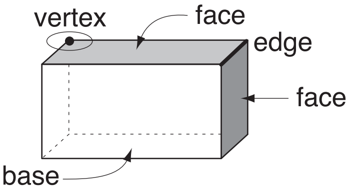


f) What is the order of rotational symmetry for this shape?



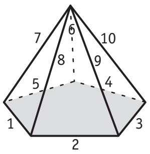
Skill 28.5 Describing the properties of 3-dimensional shapes.

- Count the number of: - faces
  - edges
  - vertices (points/corners)

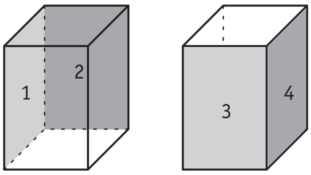


Q. How many edges does a pentagonal pyramid have?

A. *Count the number of edges, or straight lines in the pyramid: five edges in the base and five vertical edges.*
The answer is ***10***



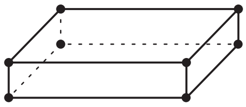
a) Of the 6 faces of a square prism, how many are rectangles?



4

b) Of the 7 faces of a pentagonal prism, how many are rectangles?

c) How many vertices does a rectangular prism have?



d) How many vertices does a triangular prism have?

e) How many edges does a tetrahedron have?

f) How many edges does a rectangular pyramid have?

g) How many faces does a pentagonal pyramid have?

h) How many faces does a triangular prism have?

i) Sketch and name the three-dimensional shape that has 6 faces, all of which are squares.

j) Sketch and name the three-dimensional shape that has 4 faces, all of which are triangles.

k) Sketch and name the three-dimensional shape that has 6 faces, five of which are triangles.

l) Sketch and name the three-dimensional shape that has 6 faces, all of which are rectangles.

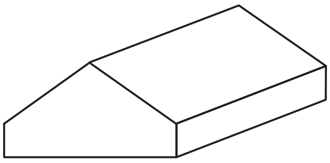
Skill 28.6 Using Euler's formula for polyhedra.

Mauve 1 1 2 2 3 3 4 4
Lime **1** 1 2 2 3 3 4 4

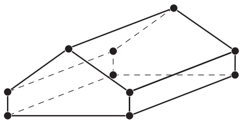
Euler's formula for any polyhedra: **Edges** = **Vertices** + **Faces** – 2 OR **E** = **V** + **F** – 2

Q. Euler's formula, **E** = **V** + **F** – 2 defines the relationship between **E**dges, **V**ertices and **F**aces of any polyhedron. Verify Euler's formula for this solid:

$\boxed{} = \boxed{} + \boxed{} - 2$



A.



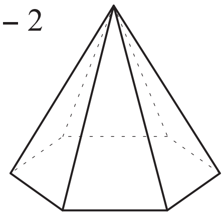
$E = 15$
$V = 10$
$F = 7$
$E = V + F - 2$
$15 = 10 + 7 - 2$
$15 = 15$ *(true)*

Substitute 15, 10, 7 into Euler's formula

$\boxed{15} = \boxed{10} + \boxed{7} - 2$

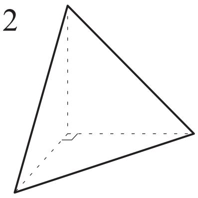
a) Euler's formula, **E** = **V** + **F** – 2 defines the relationship between **E**dges, **V**ertices and **F**aces of any polyhedron. Verify Euler's formula for a hexagonal pyramid:

$\boxed{12} = \boxed{7} + \boxed{7} - 2$



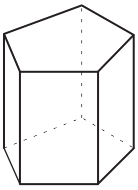
b) Euler's formula, **E** = **V** + **F** – 2 defines the relationship between **E**dges, **V**ertices and **F**aces of any polyhedron. Verify Euler's formula for a triangular pyramid:

$\boxed{} = \boxed{} + \boxed{} - 2$



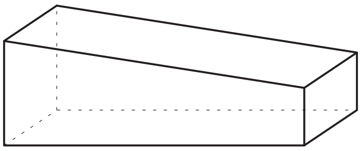
c) Euler's formula, **E** = **V** + **F** – 2 defines the relationship between **E**dges, **V**ertices and **F**aces of any polyhedron. Verify Euler's formula for a pentagonal prism:

$\boxed{} = \boxed{} + \boxed{} - 2$



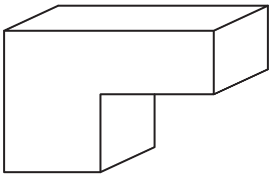
d) Euler's formula, **E** = **V** + **F** – 2 defines the relationship between **E**dges, **V**ertices and **F**aces of any polyhedron. Verify Euler's formula for this prism:

$\boxed{} = \boxed{} + \boxed{} - 2$



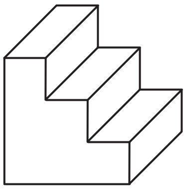
e) Euler's formula, **E** = **V** + **F** – 2 defines the relationship between **E**dges, **V**ertices and **F**aces of any polyhedron. Verify Euler's formula for this prism:

$\boxed{} = \boxed{} + \boxed{} - 2$



f) Euler's formula, **E** = **V** + **F** – 2 defines the relationship between **E**dges, **V**ertices and **F**aces of any polyhedron. Verify Euler's formula for this prism:

$\boxed{} = \boxed{} + \boxed{} - 2$

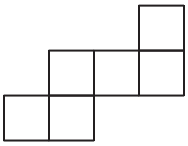


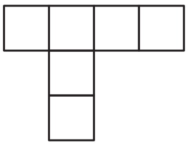
Skill 28.7 Recognising nets of 3-dimensional shapes.

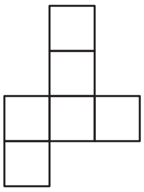
Mauve 1 1 **2** 2 3 3 4 4
Lime **1** 1 2 2 3 3 4 4

- Identify the shapes in the net.
- Imagine the shape folded. OR Make a model by tracing, cutting out and folding the net.

Q. Circle the net that **cannot** be folded to make a model of a cube.

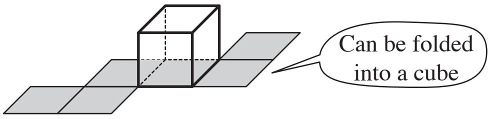




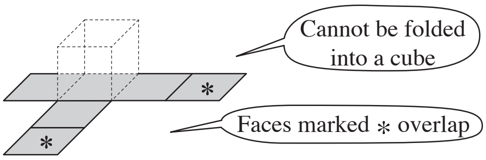


A. *Enlarge, trace and cut out the shape, folding to try to form a cube.*
OR Imagine the shape folded:

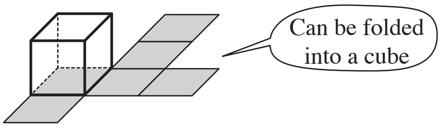
1)



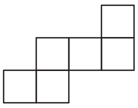
2)



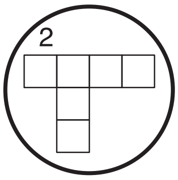
3)



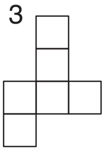
1



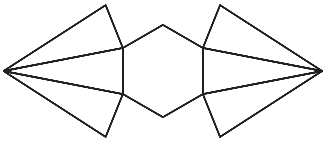
2



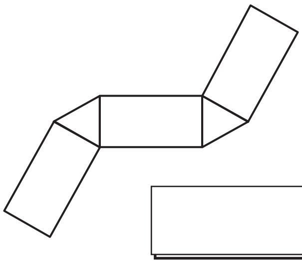
3



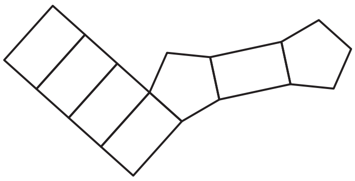
a) What three-dimensional shape can this net be used to make?



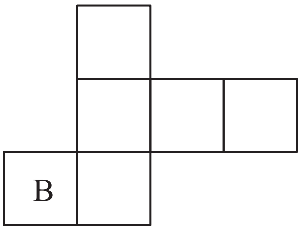
b) What three-dimensional shape can this net be used to make?



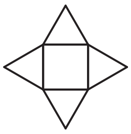
c) What three-dimensional shape can this net be used to make?

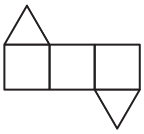


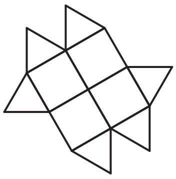
d) On this net of a cube, a face is marked B. Label the opposite face with a T.



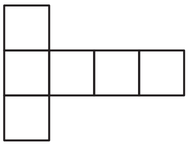
e) Circle the net that **cannot** be folded to make a model of a three-dimensional shape.

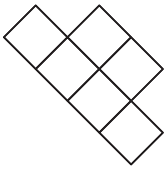


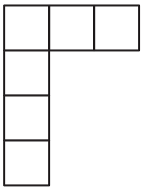




f) Circle the net that **can** be folded to make a model of a cube.







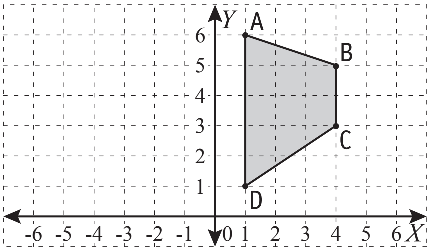
Skill 28.8 Drawing translations, reflections and rotations on a Cartesian plane (1).

To draw a shape translated on a grid
- Move it up (positive, vertically), down (negative, vertically), left (negative, horizontally) or right (positive, horizontally) on the grid, without flipping, turning or changing its size.

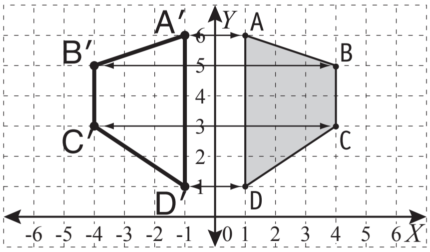
To draw a shape reflected in a given line (mirror line)
- Draw a perpendicular line to the mirror line from each vertex of the shape.
- Extend the perpendicular line beyond the mirror line by the same distance.
- Plot and join the reflected points.

To draw a shape rotated by a given angle about a point (centre of rotation)
- Rotate each vertex by the given angle, in the given direction.
- Plot and join the rotated points.

Q. Draw the reflection of the trapezium ABCD in the line of equation $x = 0$.

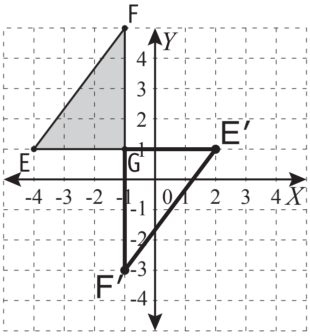


A. *Reflect A, B, C and D in the mirror line $x = 0$:*
A is one unit to the right of line $x = 0$
$\Rightarrow$ *draw A′ one unit to the left of line $x = 0$.*

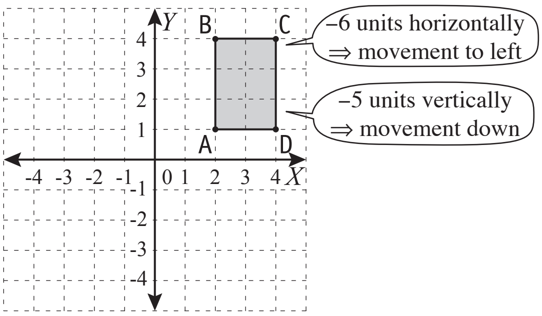


A′B′C′D′ *is the reflection of ABCD in the line $x = 0$*

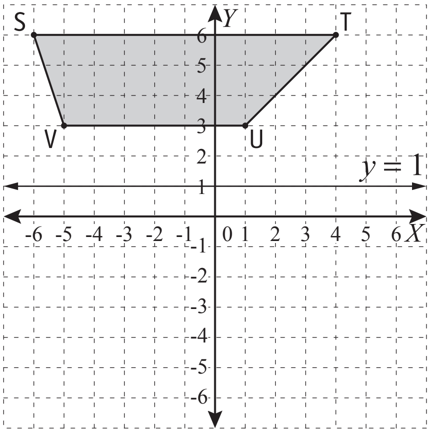
a) Redraw the triangle EFG after rotating it 180° about the point of coordinates (–1,2).



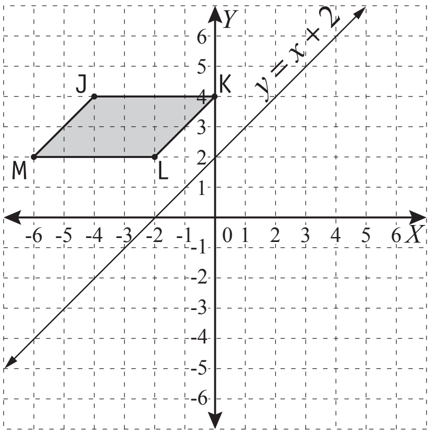
b) Redraw the rectangle ABCD after translating it –6 units horizontally and –5 units vertically.



c) Draw the reflection of the trapezium STUV in the line of equation $y = 1$.



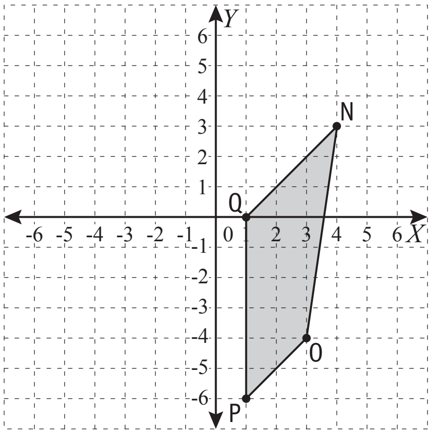
d) Draw the reflection of the parallelogram JKLM in the line of equation $y = x + 2$.



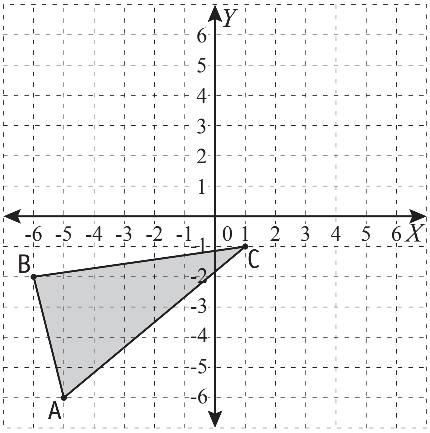
Skill 28.8 Drawing translations, reflections and rotations on a Cartesian plane (2).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

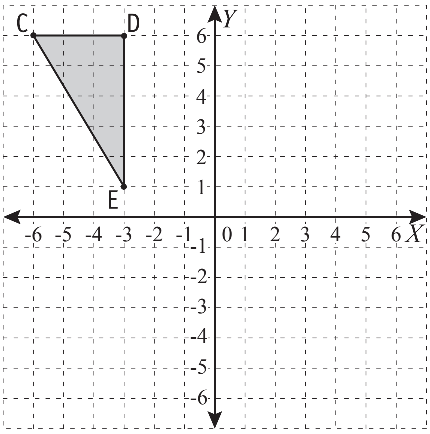
e) Redraw the quadrilateral NOPQ after rotating it 180° about the point of coordinates (1,0).



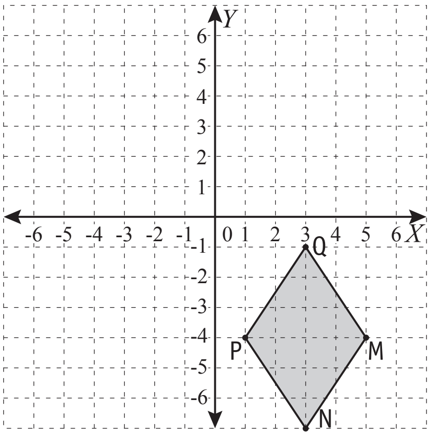
f) Redraw the triangle ABC after reflecting it in the *x*-axis and then translating it 5 units horizontally.



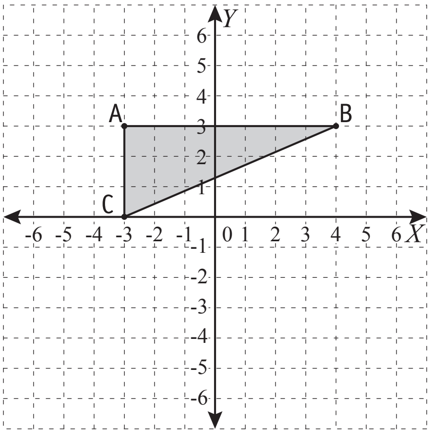
g) Redraw the triangle CDE after rotating it 90° clockwise about the point of coordinates (–3,1) and then translating it 4 units horizontally and –5 units vertically.



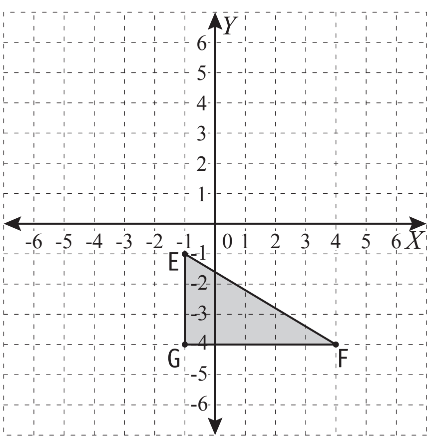
h) Redraw the kite MNPQ after rotating it 180° about the point of coordinates (3,–1) and then reflecting it in the *y*-axis.



i) Redraw the triangle ABC after rotating it 90° anticlockwise about the point of coordinates (–3,0) and then translating it 9 units horizontally and –5 units vertically. Label the transformation A'B'C'. Are triangles ABC and A'B'C' congruent or similar?



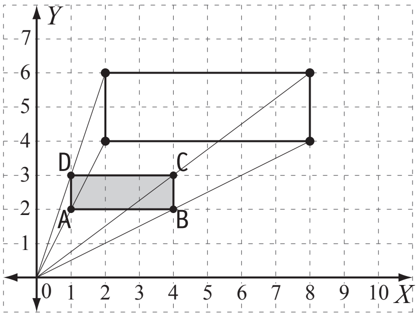
j) Redraw the triangle EFG after rotating it 180° about the point of coordinates (–1,–1) and then reflecting it in the *y*-axis. Label the transformation E'F'G'. Are triangles EFG and E'F'G' congruent or similar?



Skill 28.9 Recognising and drawing enlargements and reductions on a Cartesian plane.

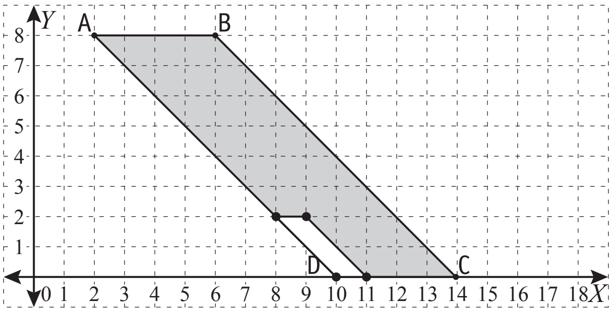
- Use the definitions of enlargement, reduction, factor of enlargement and factor of reduction. (see Glossary, pages 400, 430 and 403 respectively)

Q. Find the scale factor of enlargement applied to rectangle ABCD to produce the larger rectangle.

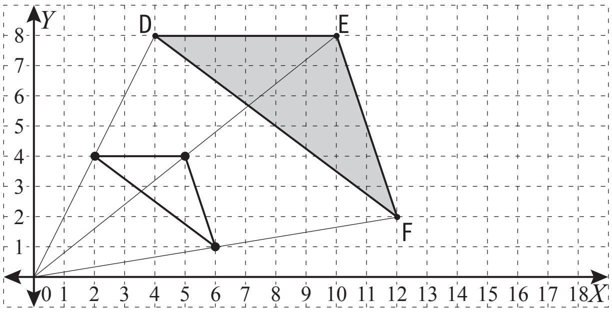


A. *The length and width of the rectangle ABCD have doubled in the enlargement.*
Scale factor of enlargement = ***2***

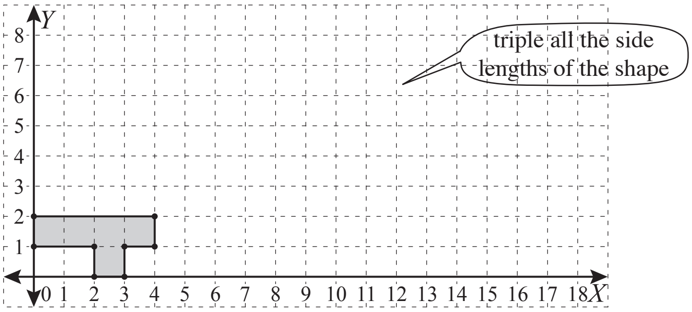
a) Find the scale factor of reduction applied to parallelogram ABCD to produce the smaller parallelogram.



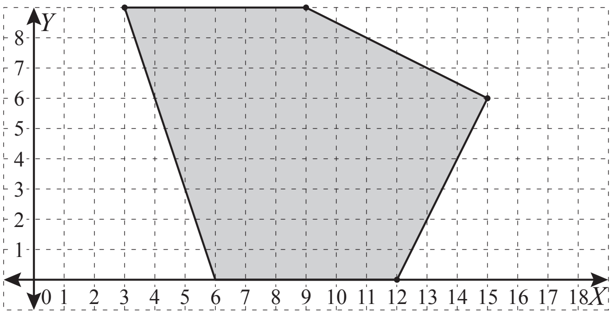
b) Find the scale factor of reduction applied to triangle DEF to produce the smaller triangle.



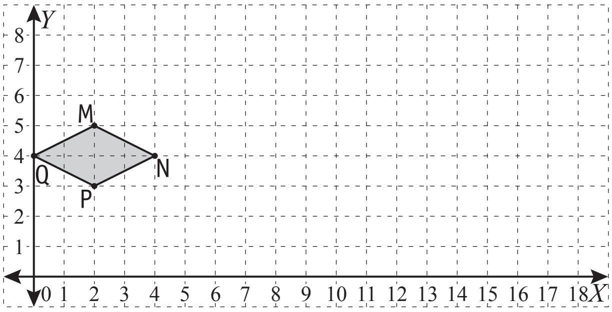
c) Redraw the shape enlarged by a scale factor of 3 about the origin of the axes.



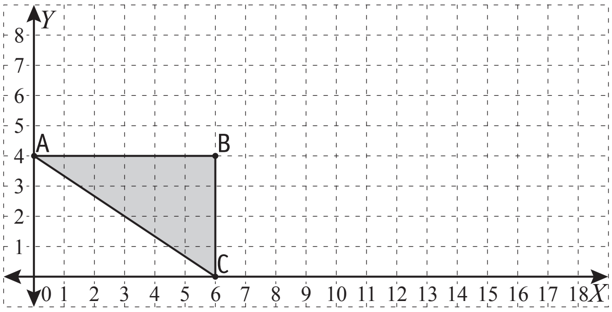
d) Redraw the shape reduced by a scale factor of $\frac{1}{3}$ about the point of coordinates (6,0).



e) Redraw the rhombus MNPQ enlarged by a scale factor of 4 about point Q.



f) Redraw the triangle ABC enlarged by a scale factor of 2 about the origin of the axes. Label the enlargement A'B'C'. Are triangles ABC and A'B'C' congruent or similar?

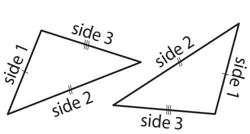


Skill 28.10 Recognising congruence tests for triangles.

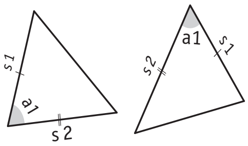
Mauve 1 1 2 2 **3** 3 4 4
Lime 1 1 **2** 2 3 3 4 4

- Use the **tests for congruence** to check if two shapes are congruent (same size and shape).

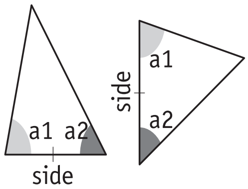
Side-side-side **(SSS)**



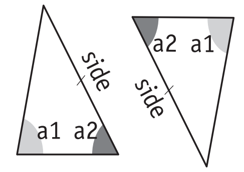
Side-angle-side **(SAS)**



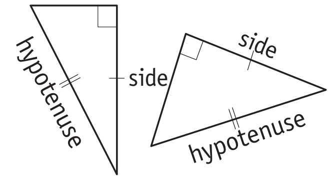
Angle-side-angle **(ASA)**



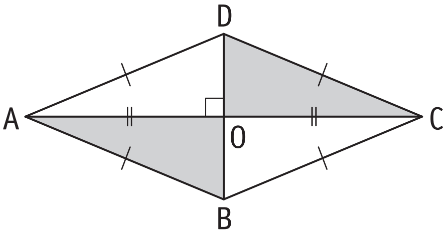
Angle-angle-side **(AAS)**



Right angle-hypotenuse-side **(RHS)**

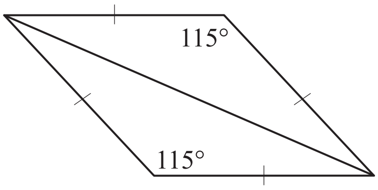


Q. Which congruence test (SSS, SAS, ASA, AAS, RHS) can be applied to show that ΔAOB is congruent to ΔCOD?

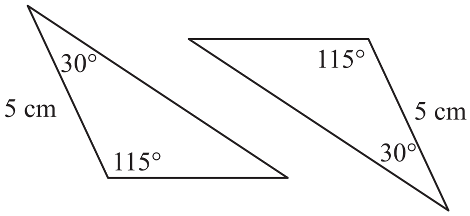


A. *Triangles AOB and COD are right-angled.*
AB = CD (hypotenuse)
AO = OC (side)
⇒ *ΔAOB ≡ ΔCOD*
based on the congruence test
RHS

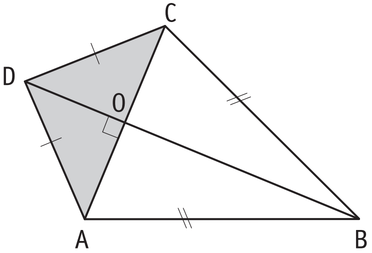
a) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that these triangles are congruent?



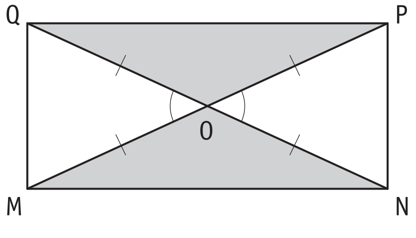
b) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that these triangles are congruent?



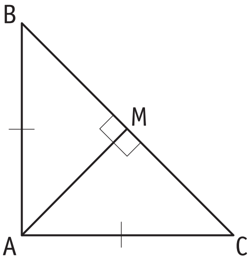
c) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that triangle AOD is congruent to triangle COD?



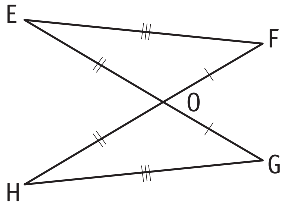
d) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that triangle MON is congruent to triangle POQ?



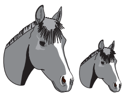
e) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that ΔAMB is congruent to ΔAMC?



f) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that ΔEOF is congruent to ΔGOH?



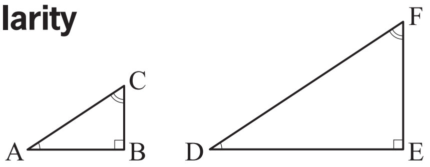
Skill 28.11 Recognising similarity of 2-dimensional shapes.



To find the value of an unknown side:

- Locate two pairs of corresponding sides; one pair should involve the unknown side.
- Set up the proportion (two equal ratios written as an equation).
- Solve the equation to find the value of the unknown.

similarity



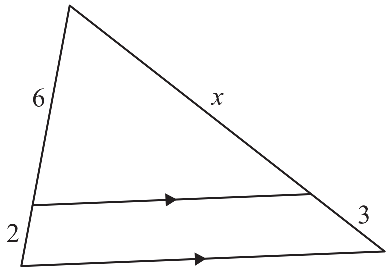
Corresponding angles are equal:

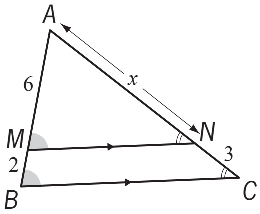
$\angle A = \angle D$ and $\angle B = \angle E$ and $\angle C = \angle F$

Corresponding sides are proportional:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Q. Find the value of x. [All measurements are in cm.]





A. *Triangles AMN and ABC are similar because:*

$\angle MAN = \angle BAC$ *(the same angle)*

$\angle AMN = \angle ABC$ *(corresponding angles)*

$\angle ANM = \angle ACB$ *(corresponding angles)*

The sides are proportional:

$$\frac{AM}{AB} = \frac{AN}{AC} \Rightarrow \frac{6}{8} \times \frac{x}{x+3}$$

Cross multiply

$$6x + 18 = 8x$$

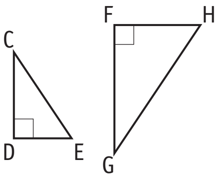
$$6x + 18 - 6x = 8x - 6x$$

Inverse of + 6x is – 6x

$$2x = 18$$

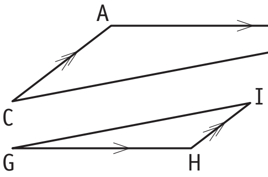
$$x = \mathbf{9}$$

a) Complete the pairs of equal angles for these similar triangles.



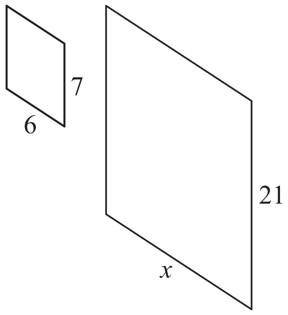
$\angle C =$, $\angle D =$, $\angle E =$

b) Complete the ratios of corresponding sides for these similar triangles.



$$\frac{AB}{\square} = \frac{BC}{\square} = \frac{AC}{\square}$$

c) If these two parallelograms are similar, what is the value of x? [All measurements are in cm.]

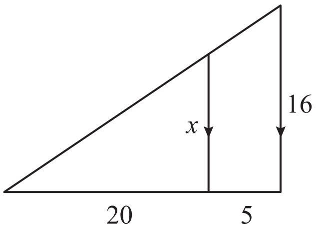


....

....

$x =$

d) Find the value of x. [All measurements are in cm.]



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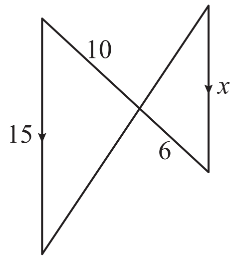
$x =$

e) Find the value of x. [All measurements are in cm.]



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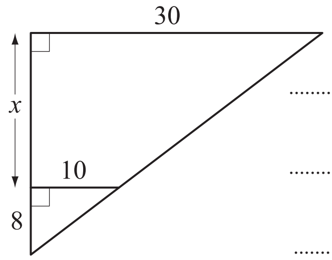
....

$x =$

f) Find the value of x. [All measurements are in cm.]



....

....

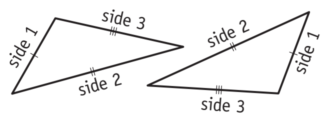
$x =$

Skill 28.12 Identifying equal sides and angles to prove that two triangles are congruent.

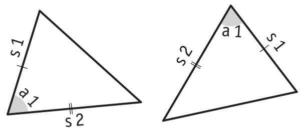
Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 **3** 4 4

- Find the equal sides or angles in the pair of congruent triangles (sides or angles which are in the same position in their respective triangles and are marked the same way).
- Use the **tests for congruence** to check if two shapes are congruent (same size and shape).

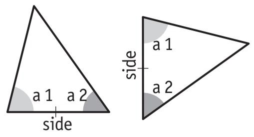
Side-side-side **(SSS)**



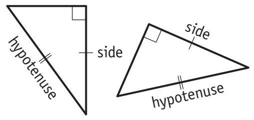
Side-angle-side **(SAS)**



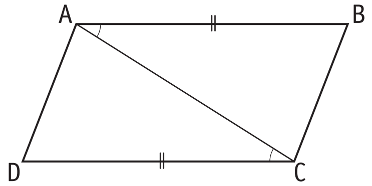
Angle-angle-side **(AAS)**



Right angle-hypotenuse -side **(RHS)**



Q. Write the pairs of equal sides and angles to prove that ΔABC and ΔACD are congruent. Which congruence test did you use?



A. *The equal sides (in a matching position in the 2 triangles) are:*

AB = CD

AC = AC *(a common side)*

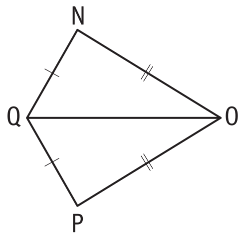
⇒ *The equal angles (in a matching position in the 2 triangles) are:*

∠BAC = ∠ACD

ΔABC ≡ ΔACD, based on the congruence test

SAS

a) Write the pairs of equal sides and angles to prove that ΔNOQ and ΔPOQ are congruent. Which congruence test did you use?



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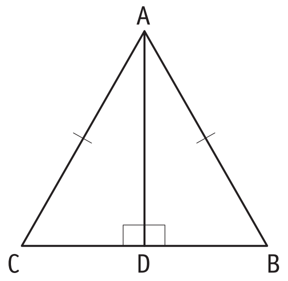
congruence test:

b) Write the pairs of equal sides and angles to prove that ΔACD and ΔABD are congruent. Which congruence test did you use?



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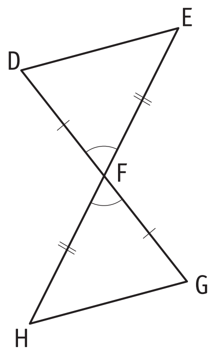
congruence test:

c) Write the pairs of equal sides and angles to prove that ΔDEF and ΔFGH are congruent. Which congruence test did you use?



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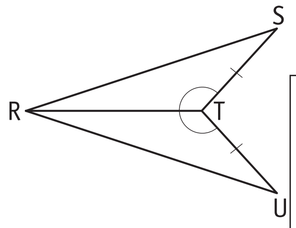
congruence test:

d) Write the pairs of equal sides and angles to prove that ΔRST and ΔRTU are congruent. Which congruence test did you use?



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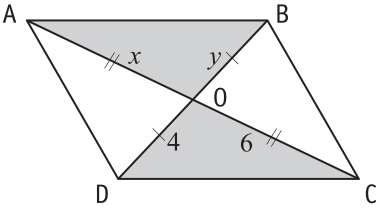
congruence test:

Skill 28.13 Using congruent triangles to find unknown sides and angles.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 **3** 4 4

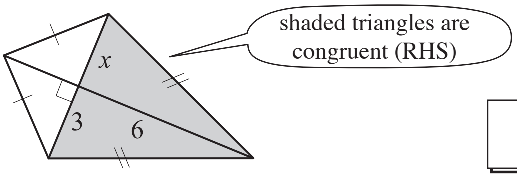
- Find the sides marked the same. They are equal in length.
- Find the angles marked the same. They are equal in size.
- Use the given values of sides and angles to find the unknown sides and angles.

Q. Find the value of x and y in this pair of congruent triangles. [All measurements are in cm.]



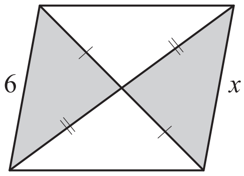
A. *Triangles AOB and COD are congruent.*
$AO = OC = 6 \Rightarrow x = \mathbf{6}$
$BO = OD = 4 \Rightarrow y = \mathbf{4}$

a) Find the value of x given the pair of shaded triangles are congruent. [All measurements are in cm.]

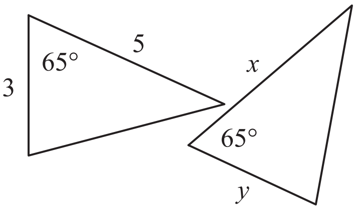


3

b) Find the value of x given the pair of shaded triangles are congruent. [All measurements are in cm.]

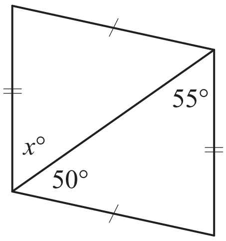


c) Find the value of x and y in this pair of congruent triangles.

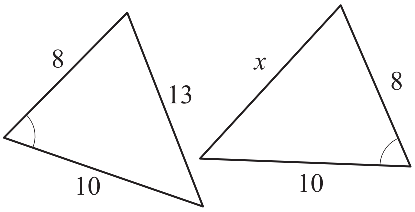


$x =$ $y =$

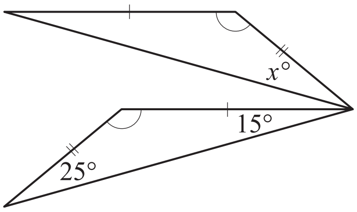
d) Find the value of $x°$ in this pair of congruent triangles.



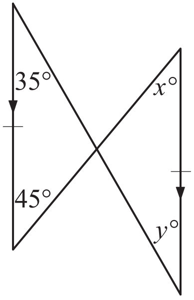
e) Find the value of x in this pair of congruent triangles. [All measurements are in cm.]



f) Find the value of $x°$ in this pair of congruent triangles.

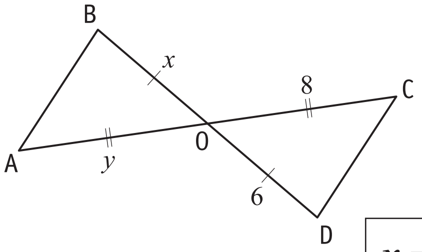


g) Find the value of $x°$ and $y°$ in this pair of congruent triangles.



$x° =$ $y° =$

h) Find the value of x and y in this pair of congruent triangles. [All measurements are in cm.]



$x =$ $y =$

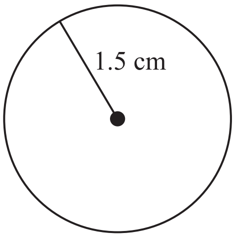
Skill 28.14 Drawing 2-dimensional shapes to scale.

Mauve 11 22 33 44
Lime 11 22 33 **4**4

To redraw a shape to scale:
- Use a ruler and a set square to make the exact dimensions and proportions.
- Estimate the required side length, by comparing it to the other dimensions of the shape.

To find the scale ratio of a model:
- Convert the real-life dimension to the same unit of measurement of the model dimension.
- Divide the real-life dimension by the model dimension, to find the scale factor.
- Write the scale ratio of the model in the form of **1 : scale factor**.

Q. Determine the scale factor used when this circle represents the plan of a lake of diameter 600 m.

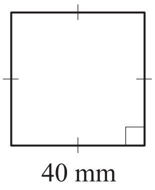


A. *600 m = 600 × 100 cm* (1 m = 100 cm)
= 60 000 cm
scale factor = 60 000 cm ÷ 3 cm
= 20 000

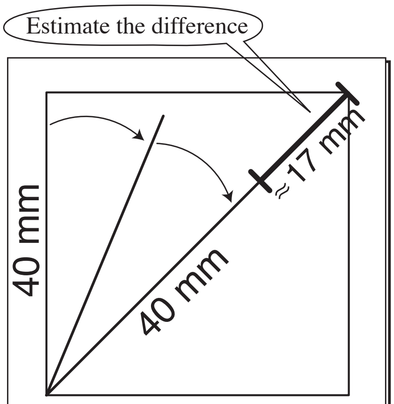
diameter = 2 × radius
= 2 × 1.5
= 3

scale ratio = 1 : ***20 000***

a) Redraw the square to scale and estimate the length of its diagonal.

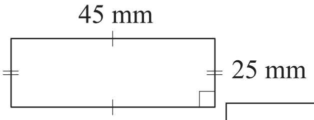


Actual length = 56.57 mm
Accept between 50 mm and 60 mm



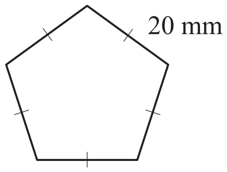
diagonal = mm

b) Redraw the rectangle to scale and estimate the length of its diagonal.



diagonal = mm

c) Determine the scale factor used when this regular pentagon represents the plan of a bike track of perimeter 6 km.



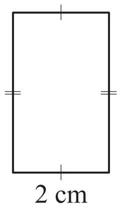
6 km = 6 × 1 000 000 mm

= 6 000 000 mm

scale factor = 6 000 000 mm ÷ 100 mm

= 1 :

d) Determine the scale factor used when this rectangle represents the plan of a football stadium of width 60 m.



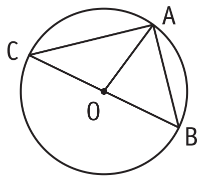
scale factor =

= 1 :

Skill 28.15 Recognising elements of circle geometry.

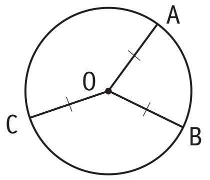
Radius, diameter, chord

- $\overline{OA}$ is a radius
- $\overline{BC}$ is a diameter
- $\overline{AC}$ and $\overline{AB}$ are chords



Radius property

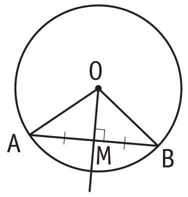
- The radii in a circle are equal in length.



OA = OB = OC

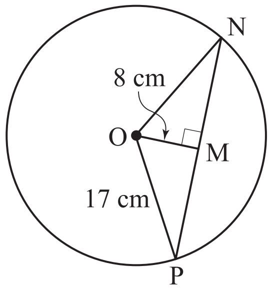
Chord property

- A line through the centre, perpendicular to a chord, bisects the chord.



AM = MB

Q. A chord NP is 8 cm from the centre of a circle of radius 17 cm. Find the length of chord NP. [Hint: Pythagoras' theorem will help.]



A. *Apply Pythagoras' theorem in ΔOMP:*

$OP^2 = OM^2 + MP^2$ find the length MP first

$17^2 = 8^2 + MP^2$

$MP^2 = 289 - 64$

$MP^2 = 225$

$MP = \sqrt{225}$

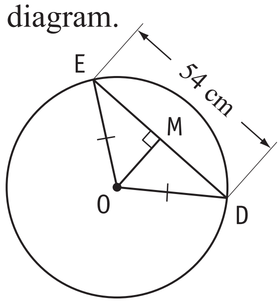
$MP = 15$ MP = MN = 15 cm

$\Rightarrow NP = 2 \times MP$

$= 2 \times 15$

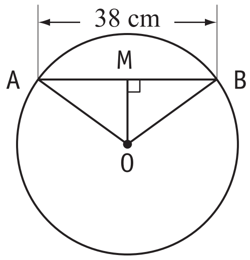
$= \mathbf{30\ cm}$

a) Find the length of the segment DM in this diagram.



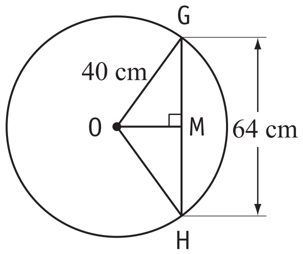
cm

b) Find the length of the segment BM in this diagram.



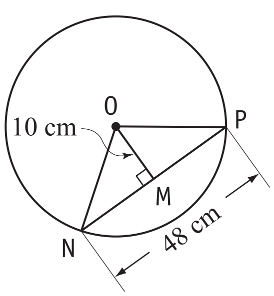
cm

c) Find the length of the segment OM in this diagram. [Hint: Pythagoras' theorem will help.]



OM = = cm

d) Find the length of the radius of the circle in this diagram. [Hint: Pythagoras' theorem will help.]



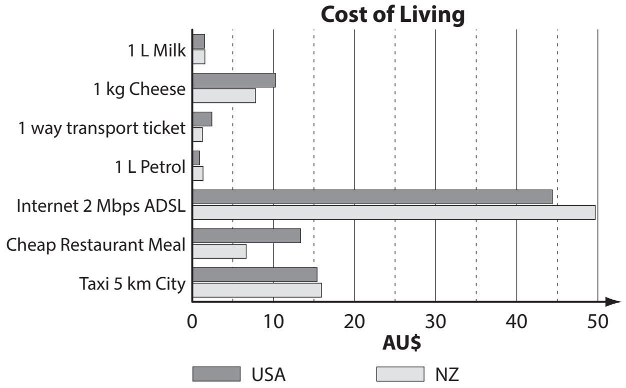
ON = = cm

29. [Statistics]

Skill 29.1 Interpreting data in column or bar graphs (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

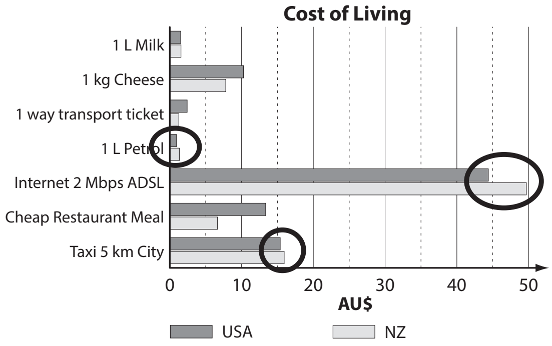
Q. How many of the cost of living items are more expensive in New Zealand than the USA?



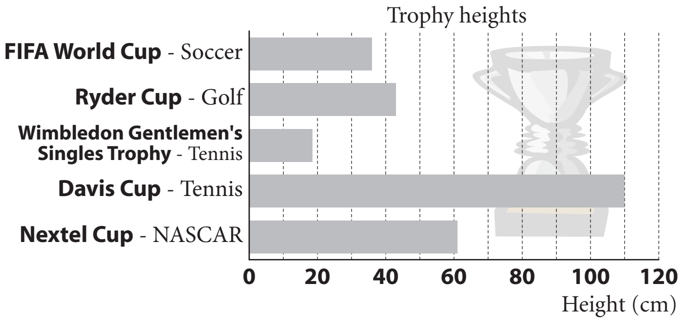
A. *3*

Find the New Zealand bars in the graph.

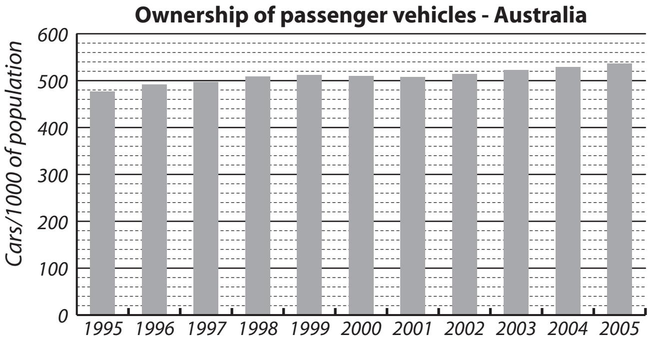
Measure which ones are longer than their USA equivalents.



a) Of the trophies listed below, which sport has the 3rd highest trophy?

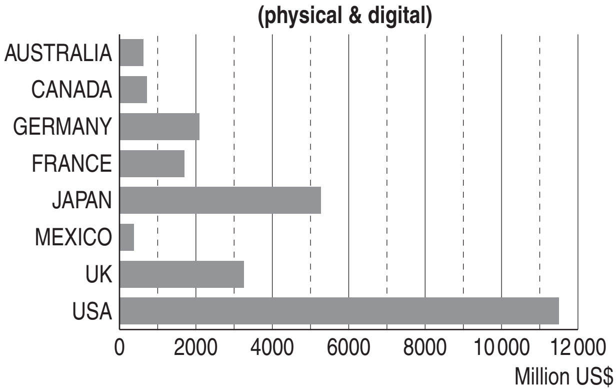


b) In which year did Australian car ownership first exceed 50%?

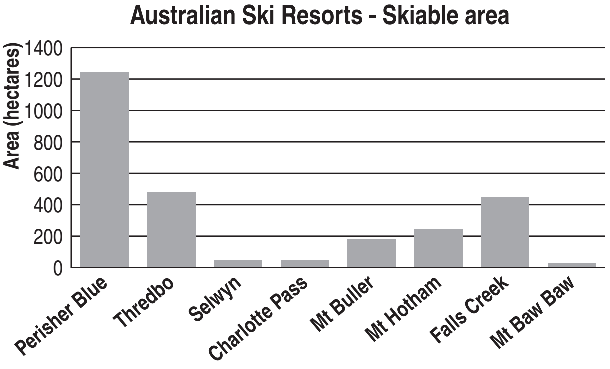


c) How many of the countries listed below had retail music sales in 2007 greater than US$3000 million?

Value Of Total Music Retail Sales 2007 (physical & digital)

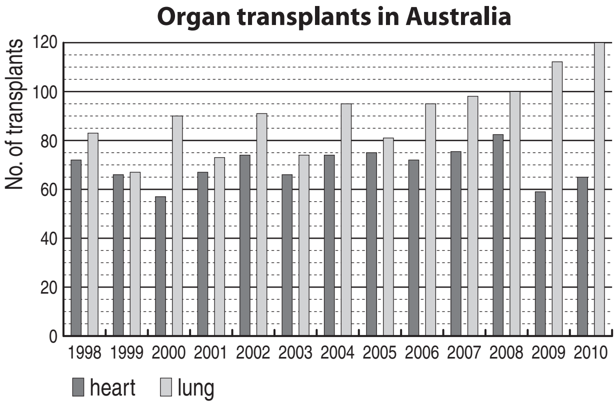


d) Which Australian ski resort listed below has an area closest to 400 hectares?

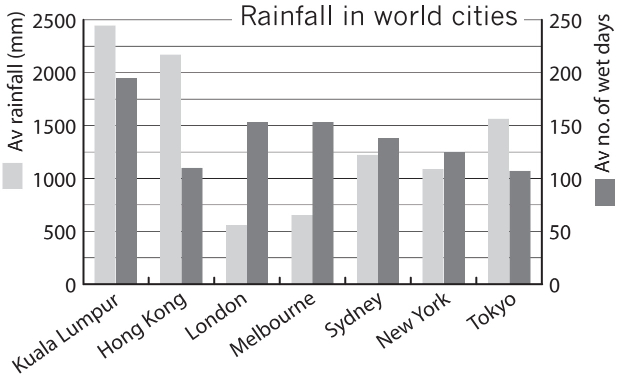


Skill 29.1 Interpreting data in column or bar graphs (2).

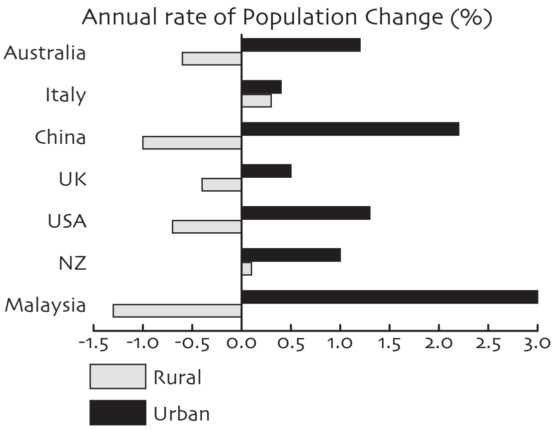
e) In which year were the number of heart and lung transplants most similar?



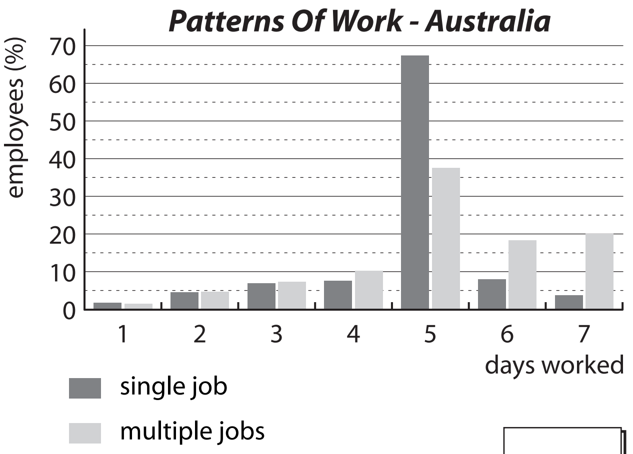
f) Which city has the most wet days compared to the amount of rainfall?



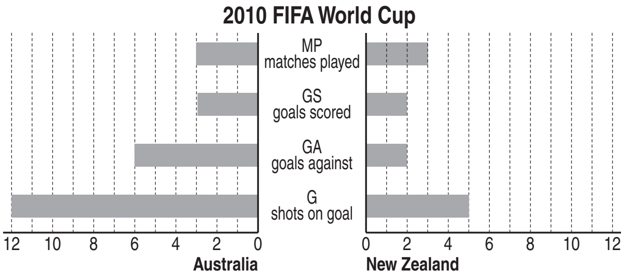
g) Of the countries shown below which have an increasing population in both rural and urban areas?



h) What percentage of multiple job employees work 4 days a week?

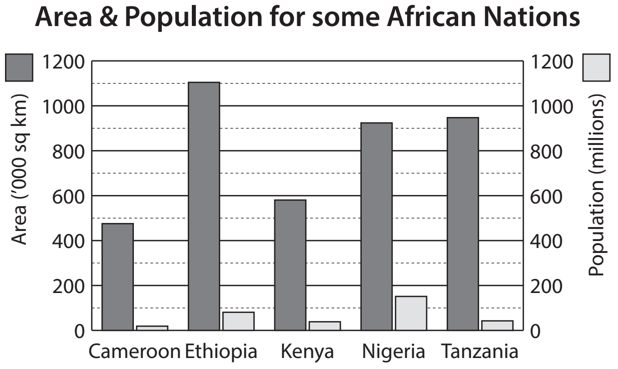


i) What percentage of Australia's shots on goal in the 2010 FIFA world cup were actually converted to goals?



%

j) Which of the African nations shown is the most densely populated? [Hint: people/area]

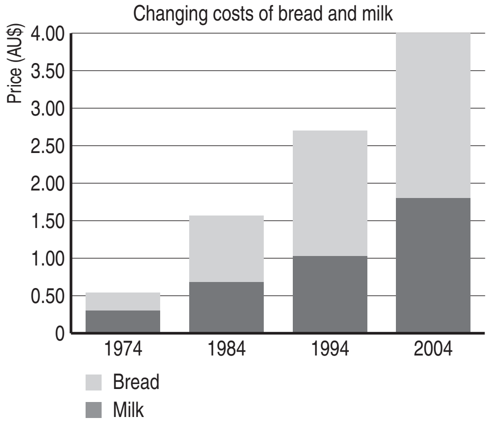


Skill 29.2 Interpreting data in stack graphs.

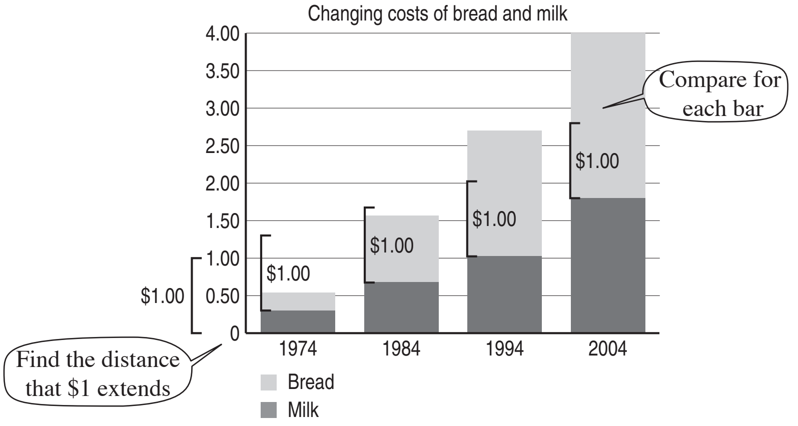
- Read such that each piece of the bar represents a percentage or proportion of the total.
Example:

50%	25%	25%

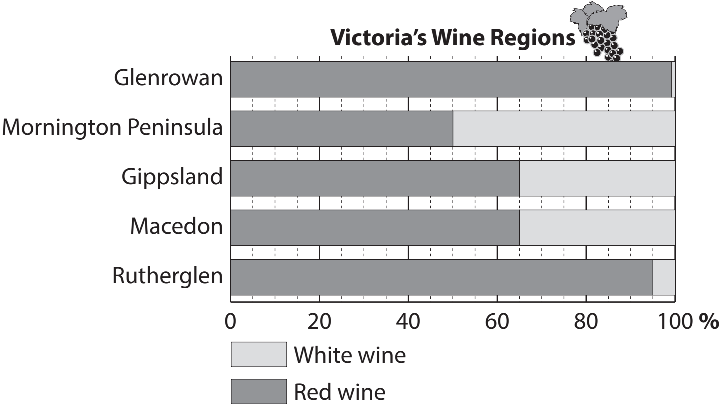
Q. In which of the years shown was the price of bread in Australia closest to $1.00?



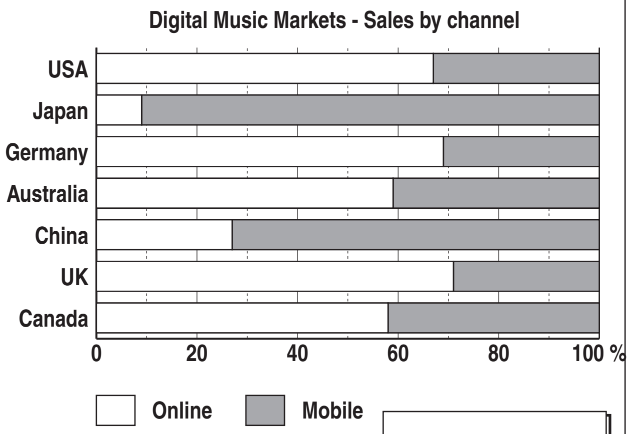
A. ***1984***



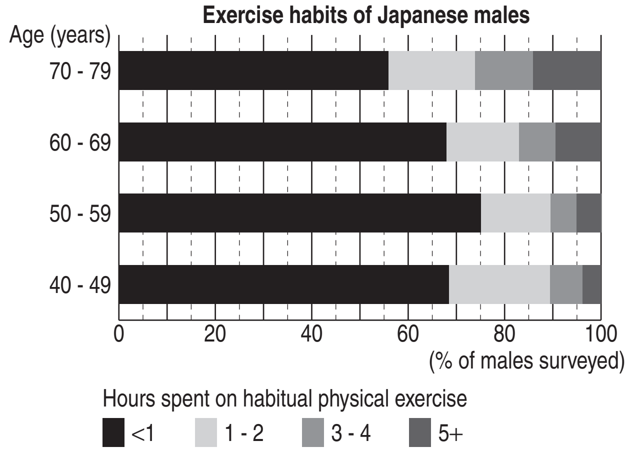
a) In which Victorian wine region does the amount of white wine produced equal 5%?



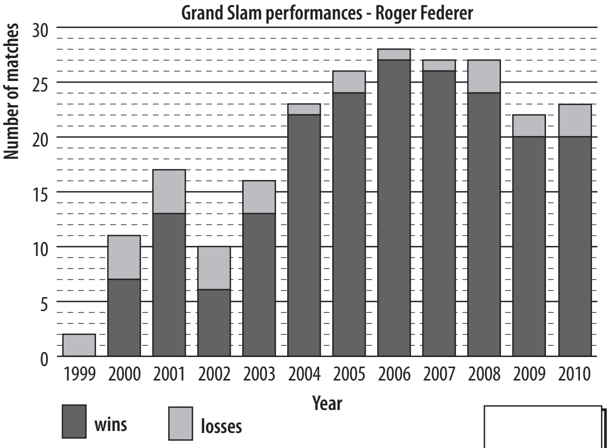
b) In which country do mobile sales comprise approximately 73% of digital sales?



c) In which age bracket did the highest percentage of respondents spend 3 to 4 hours on habitual physical exercise?



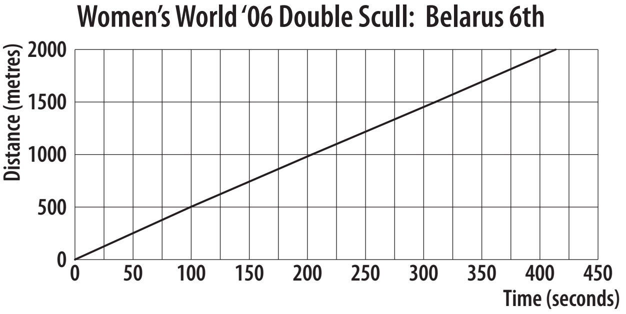
d) Between 2000 and 2010, Roger Federer had played in all 4 grand slam tournaments each year. How many grand slam tournaments has Roger Federer won in that time?
[Hint: The grand slams are knock-out tournaments.]



Skill 29.3 Interpreting data in line graphs (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

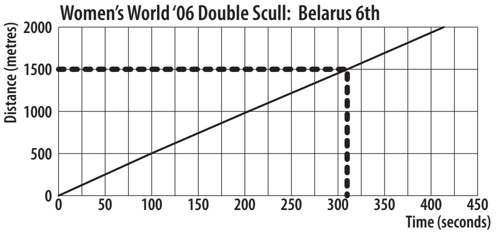
Q. How far into the race was the Belarus team after 5 minutes and 10 seconds?



A. *5 mins + 10 seconds* Convert to seconds

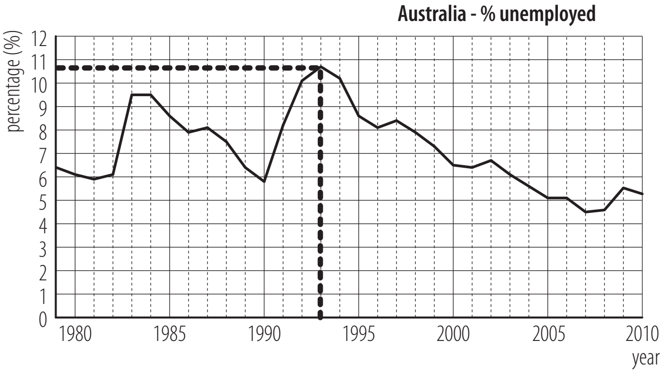
$= 5 \times 60 + 10$

$= 310$

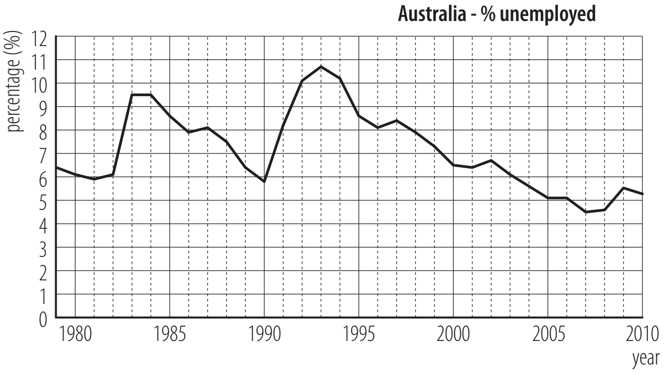


1500 m

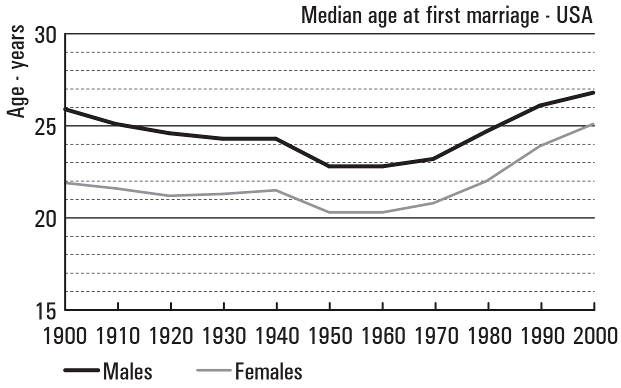
a) In which year between 1979 and 2007 was the highest percentage of Australians unemployed?



b) Using the data below, between which 2 years did the percentage of unemployed Australians increase most?

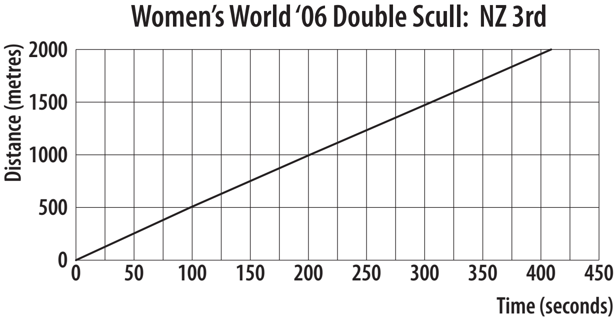


c) In which of the years shown has there been the greatest difference in marrying age of men and women in the USA?



d) At what approximate speed, in m/s, did the New Zealand team row?

A) 0.5 m/s B) 2 m/s C) 5 m/s D) 10 m/s



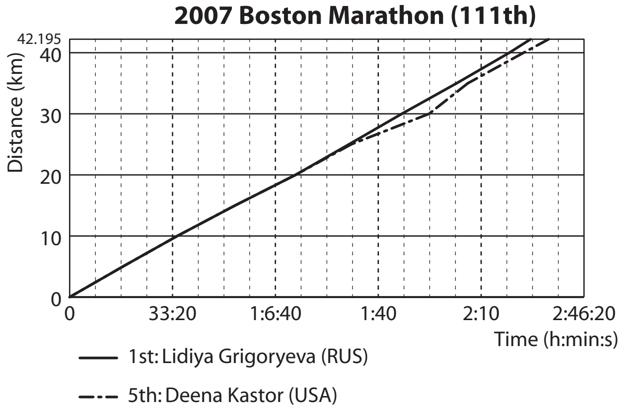
....

....

Skill 29.3 Interpreting data in line graphs (2).

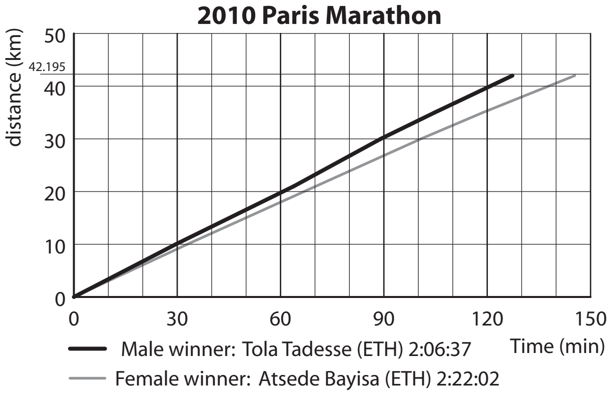
Mauve **1 1** 2 2 3 3 4 4
Lime 1 **1** 2 2 3 3 4 4

e) How far into the marathon was Deena Kastor when she began to recover from stomach cramps?



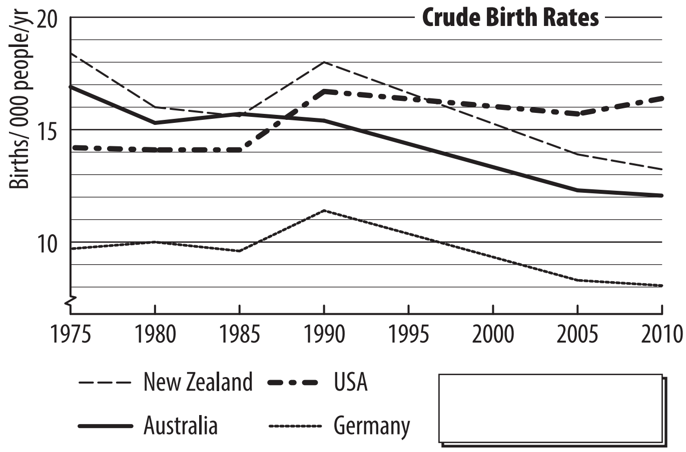
km

f) Approximately how much longer did it take Atsede than Tola to reach the 30 km mark of the 2010 Paris marathon?

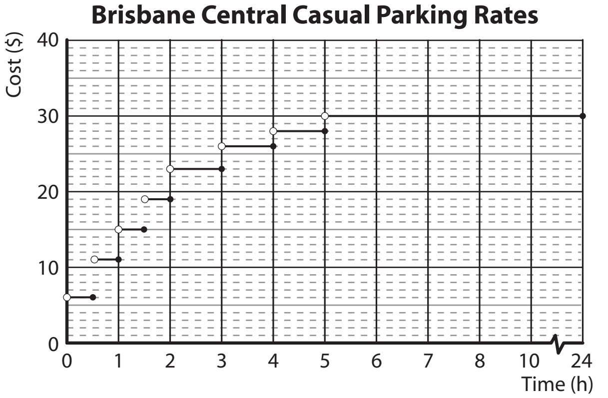


min

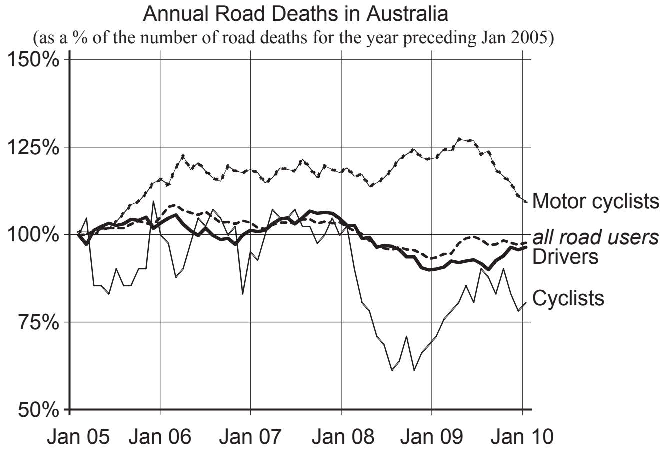
g) During which 5-year interval did Australia's crude birth rate go most against the trend?



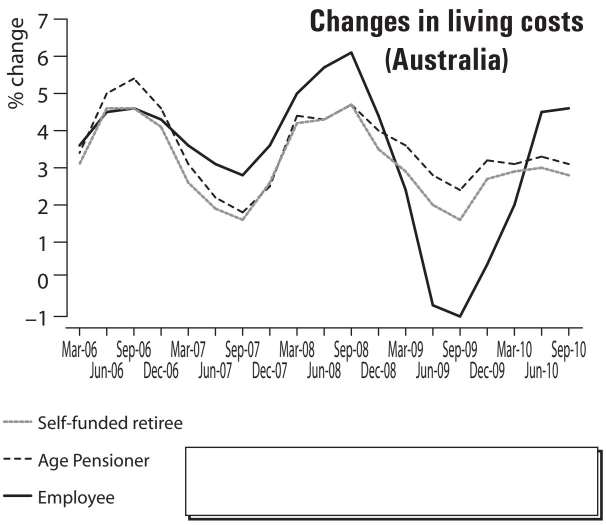
h) What is the cost to park for 3.5 hours?



i) Since 2005, which type of road user has the pecentage change in road deaths most similar to the '*all road users*' average?

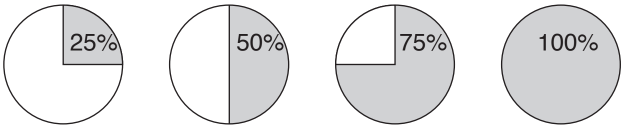


j) Which Australian household type had the greatest % change in living costs for the March 2010 to June 2010 quarter?



Skill 29.4 Interpreting data in pie charts.

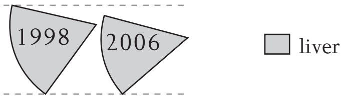
- Pie graphs are circular. Consider each section of the graph as a piece of the pie.
  Hint: Each piece of pie represents a percentage of the total.
  Example:



Q. Which of the years shown had the largest proportion of liver transplants?

Patient transplants by organ

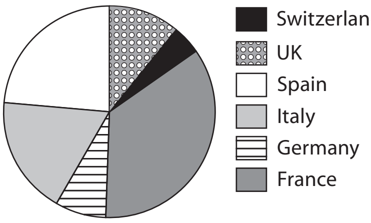
A. Compare the relative sizes of the sectors (pieces of the pie charts) for both years.



1998

a) Of the European countries listed below which had the third highest number of international tourist arrivals?

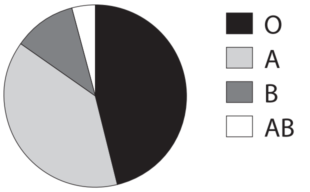
International tourist arrivals



Italy

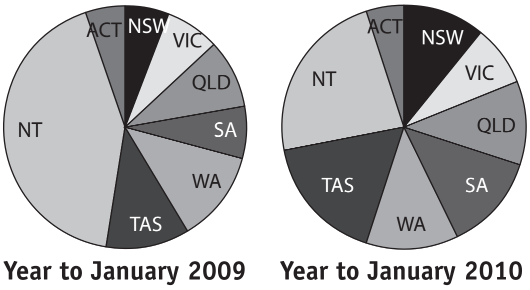
c) Which blood type accounts for closest to 10% of the population?

Who has which Blood Type?



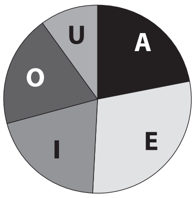
e) Which state or territory saw a reduction in the number of deaths in the year to Jan 2010 compared to the previous year?

Australia's Road deaths



b) In the Concise Oxford Dictionary, the chance of a vowel being a "U" is closest to:

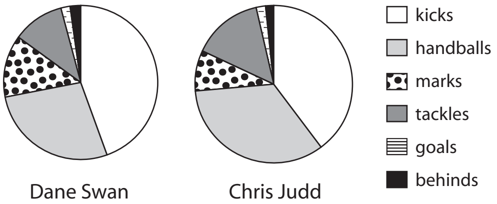
A) 5% B) 10% C) 15% D) 25%



Frequency of vowels -
Concise Oxford Dictionary

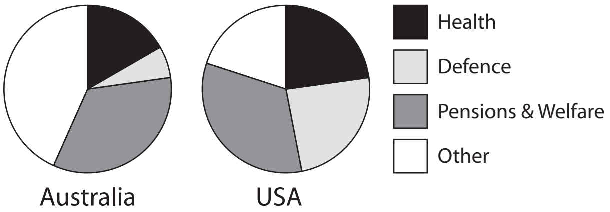
d) Which player shown below had the highest percentage of their match statistics as marks?

2010 AFL match statistics



f) Which country spends approximately 25% of its federal budget on defence?

FEDERAL BUDGETS FOR 2011



Skill 29.5 Calculating the median of sets of data.

Mauve 1 1 **2** 2 3 3 4 4
Lime **1** 1 2 2 3 3 4 4

Median (middle value)

- Write all the values in order.
- Odd numbered set - middle value.
  Even numbered set - average of the 2 middle values.

Set of data (even): 5, 1, 5, 3, 2, 1, 5, 2
Ordered set: 1, 1, 2, 2,| 3, 5, 5, 5

Median $\frac{2+3}{2} = \frac{5}{2} = \mathbf{2.5}$

Q. Calculate the median of this set of data:
3, 3, 4, 2, 3, 2, 4, 6, 1, 9, 8, 8

A. *1, 2, 2, 3, 3, 3,| 4, 4, 6, 8, 8, 9* order values

find middle value

Median: $\frac{3+4}{2} = \frac{7}{2} = \mathbf{3.5}$

a) Calculate the median of this set of data:
3, 4, 8, 5, 2, 4, 3, 6, 7

order values

find middle value

2, 3, 3, 4,| 4,| 5, 6, 7, 8

9 values so 5th value is the middle $\boxed{4}$

b) Calculate the median of this set of data:
1, 3, 4, 4, 5, 2, 6, 1, 7, 9, 4

c) Calculate the median of this set of data:
1.2, 4.1, 3.2, 3, 4.1, 2.3, 2, 3.1, 2

d) Calculate the median of this set of data:
5, 2, 3, 7, 8, 4, 6, 4

e) Calculate the median of this set of data:
12, 12, 11, 10, 11, 13, 12, 15, 12

order values

find middle value

10, 11, 11, 12,| 12,| 12, 12, 13, 15

f) Calculate the median of this set of data:
1, 3, 1, 4, 4, 4, 2, 3, 4, 5, 2, 3

g) Calculate the median of this set of data:
2, 2, 2, 2.5, 3.5, 3.5, 4, 4.5

h) Calculate the median of this set of data:
9, 10, 11, 10, 15, 11

Skill 29.6 Calculating the mode and range of sets of data.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

Mode (most common value)

Range

- Write all the values in order.
- Subtract the lowest value from the highest value.

Hint: A set of data can have more than one mode,
if two or more values repeat the same number of times.

Set of data: 5, 1, 5, 3, 2, 1, 5, 2
Ordered set: 1, 1, 2, 2, 3, 5, 5, 5
Mode 5
Range 5 – 1 = **4**

Q. Calculate the mode and range of this set of data:
1, 2, 3, 3, 4, 5, 2, 6, 8, 5, 3

A. *1, 2, 2, 3, 3, 3, 4, 5, 5, 6, 8*
Mode: ***3***
Range: 8 – 1 = 7

order values

The value 3 is in the set 3 times

difference between highest and lowest

a) Calculate the mode of this set of data:
2, ㉑, ㉑, 15, 16, 15, ㉑

The value 21 is in the set 3 times

21

b) Calculate the mode of this set of data:
3, 2, 2, 4, 5, 6, 7, 4, 5, 2, 5, 3, 4, 2

c) Calculate the mode of this set of data:
18, 21, 20, 18, 22, 18, 20, 21, 22

d) Calculate the mode of this set of data:
102, 99, 98, 100, 101, 98, 102, 98

e) Calculate the range of this set of data:
12, 14, 16, 14, 15, 13

16 – 12 =

f) Calculate the range of this set of data:
19, 22, 23, 15, 12, 16, 13, 15, 24, 14, 17, 18

g) Calculate the mode and range of this set of data:
3, 5, 4, 8, 5, 6, 8, 6, 4, 7, 4, 7, 8, 4

mode = range =

h) Calculate the mode and range of this set of data:
31, 32, 35, 32, 34, 29, 30, 31, 33, 32

mode = range =

i) Calculate the mode and range of this set of data:
2.8, 3.1, 3.5, 3.6, 3.6, 4, 4.2, 4.5, 4.7, 4.9

mode = range =

j) Calculate the mode and range of this set of data:
14, 18, 19, 19, 24, 23, 29, 18, 28, 19

mode = range =

Skill 29.7 Calculating the mean of sets of data.

Mauve 1 1 **2** 2 3 3 4 4
Lime **1** 1 2 2 3 3 4 4

Mean (or average)

- Add all the values in the set.
- Divide the total by the number of values in the set.

Set of data: 5, 1, 5, 3, 2, 1, 5, 2

Mean $1 + 1 + 2 + 2 + 3 + 5 + 5 + 5 = 24$

8 values so $24 \div 8 =$ **3**

Q. Calculate the mean of this set of data:
10, 10, 16, 14, 15

A. *10 + 10 + 16 + 14 + 15 = 65*

65 ÷ 5 5 values in the set, so divide by 5

= ***13***

a) Calculate the mean of this set of data:
6, 22, 21, 14, 18, 15

6 + 22 + 21 + 14 + 18 + 15 = 96

96 ÷ 6 = 16

b) Calculate the mean of this set of data:
1, 3, 3, 4, 7, 9, 15

1 + 3 + 3 + 4 + 7 + 9 + 15 =

c) Calculate the mean of this set of data:
8, 8, 9, 10, 10, 10, 11, 12, 12

d) Calculate the mean of this set of data:
2.1, 2.2, 2.2, 2.5, 2.5, 2.5, 2.7, 3.3

e) Calculate the mean of this set of data:
1, 3, 3, 4, 4, 4, 6, 7, 7, 7, 9

f) Calculate the mean of this set of data:
8, 8, 9, 10, 11, 11, 13

g) Calculate the mean of this set of data:
2, 2, 5, 6, 8, 10, 14, 17

h) Calculate the mean of this set of data:
0, 0, 2, 1.5, 1.8, 2, 2.2, 3, 3.5, 4

i) Calculate the mean of this set of data:
10, 12, 13, 16, 17, 18, 20, 22

j) Calculate the mean of this set of data:
3, 4, 6, 7, 8, 10, 11, 13, 16, 17

Skill 29.8 Calculating the mean, median and mode of sets of data.

Mean (or average)
- Add all the values in the set.
- Divide the total by the number of values in the set.

Median (middle value)
- Write all the values in order.
- Odd numbered set - middle value.
  Even numbered set - average of the 2 middle values.

Mode (most common value)

Set of data: 5, 1, 5, 3, 2, 1, 5, 2

Mean $1 + 1 + 2 + 2 + 3 + 5 + 5 + 5 = 24$
8 values so $24 \div 8 = \mathbf{3}$

Ordered set: $1, 1, 2, 2, \mid 3, 5, 5, 5$

Median $\frac{2+3}{2} = \frac{5}{2} = \mathbf{2.5}$

Mode **5**

Q. Which set of data has the same mean, median and mode?

A) 1, 2, 4, 4, 4, 6, 7
B) 3, 5, 5, 8, 9
C) 1, 2, 2, 2, 4, 4, 6

A. *A)* *Mean* $1 + 2 + 4 + 4 + 4 + 6 + 7 = 28$
$28 \div 7 = \mathbf{4}$
Median $1, 2, 4, \mid 4, \mid 4, 6, 7 \Rightarrow \mathbf{4}$
Mode 1, 2, ④ ④ ④ 6, 7 $\Rightarrow \mathbf{4}$

B) *Mean* $3 + 5 + 5 + 8 + 9 = 30$
$30 \div 5 = \mathbf{6}$
Median $3, 5, \mid 5, \mid 8, 9 \Rightarrow \mathbf{5}$
Mode 3, ⑤ ⑤ 8, 9 $\Rightarrow \mathbf{5}$

C) *Mean* $1 + 2 + 2 + 2 + 4 + 4 + 6 = 21$
$21 \div 7 = \mathbf{3}$
Median $1, 2, 2, \mid 2, \mid 4, 4, 6 \Rightarrow \mathbf{2}$
Mode 1, ② ② ② 4, 4, 6 $\Rightarrow \mathbf{2}$

So ***A)*** *has the same mean, median and mode.*

a) Which set of data has the same mean, median and mode?

A) −2, 0, 0, 1, 2, 2, 2, 3
B) 10, 10, 11, 11, 11, 12, 12

b) Which set of data has the same mean, median and mode?

A) 1, 2, 3, 3, 3, 4, 5
B) 5, 5, 6, 7, 9, 10

c) Which set of data has the same mean, median and mode?

A) 8, 8, 9, 10, 11
B) −1, −1, 1, 1, 1, 3, 3
C) 2, 3, 3, 4, 5, 7

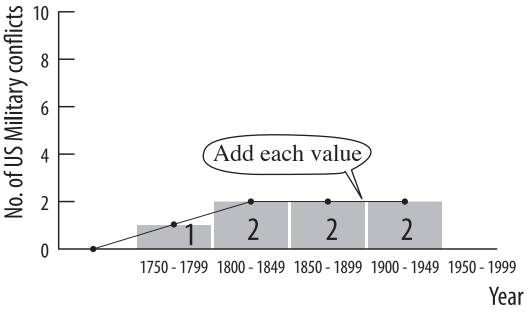
d) Which set of data has the same mean, median and mode?

A) 29, 30, 30, 32, 34
B) 6, 6, 7, 9, 9, 9, 10
C) −2, 2, 3, 3, 3, 5, 7

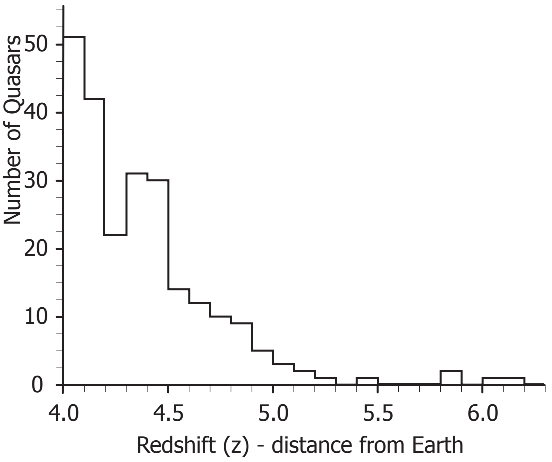
Skill 29.9 Interpreting histograms.

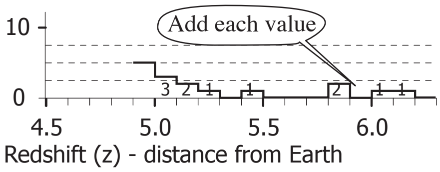
Q. Using this histogram, how many times did the USA engage in military conflict prior to 1950?

A. *1 + 2 + 2 + 2*
= *7*



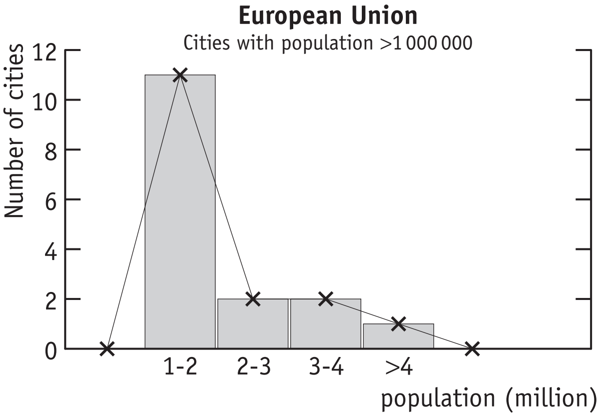
a) How many quasars have been discovered with a redshift of 5 or greater?





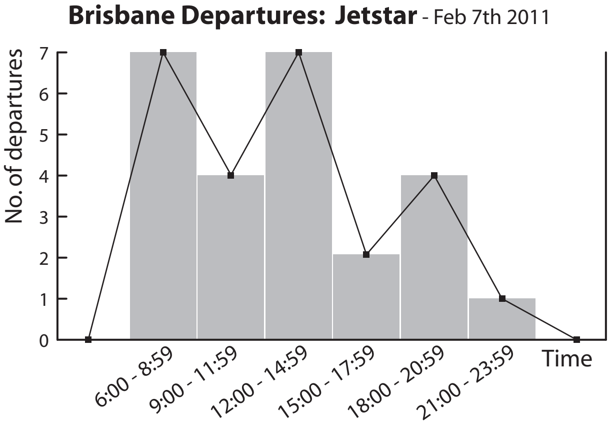
3 + ...

b) How many cities in the European Union have a population greater than 2 million people?



....

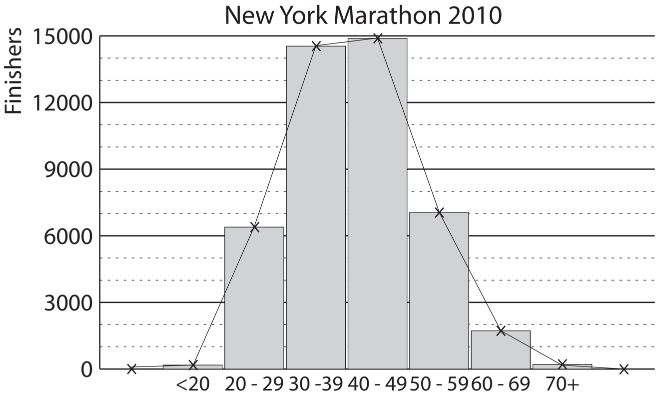
c) On February 7th, 2011 how many departures did Jetstar have out of Brisbane after 6:00 pm?



....

d) The best approximation for the number of finishers in the 2010 New York Marathon is:

A) 30 000 B) 45 000 C) 60 000



....

Skill 29.10 Interpreting stem-and-leaf plots (1).

To complete a stem-and-leaf plot from a given set of data:

- Write the values from the data set - each unit digit is a leaf beside its corresponding tens (or hundreds) digit, which is a stem.

Hint:

tens value	units value	
STEM	LEAF	
0	2	= 2
1	5 7	= 15 and 17

hundreds & tens values	units value	
STEM	LEAF	
23	7	= 237

To calculate values from a stem-and-leaf plot:

Mode (most common value)

- Find the leaf digit that repeats most.
- Read the number resulting from the corresponding stem and leaf.

Median (middle value)

- Count the number of leaves.

If an odd number of leaves:

- Count from the top left leaf until you reach the middle leaf.
- This digit is the unit and must be put with the corresponding stem.

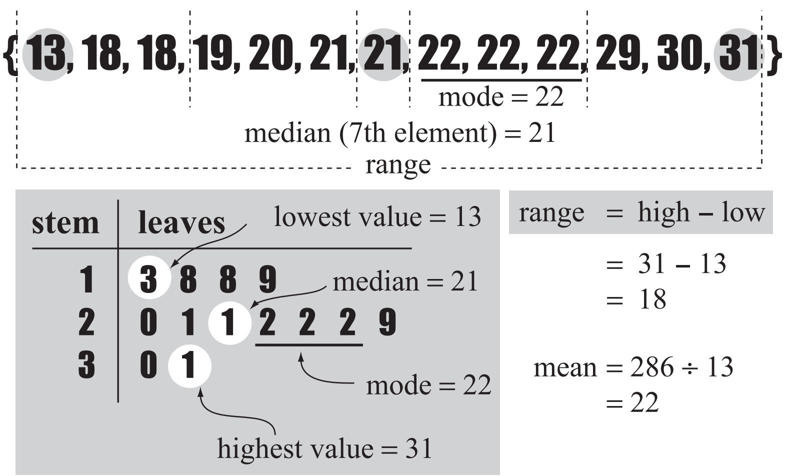
If an even number of leaves:

- Count from the top left leaf until you reach the two middle leaves.
- Read the digits with their corresponding stems.
- Find the average of the 2 middle numbers.

Range

- Subtract the lowest number (top left leaf) from the highest number (bottom right leaf).

Data set of 13 elements:



range = high − low
= 31 − 13
= 18

mean = 286 ÷ 13
= 22

Q. This back-to-back stemplot shows Richmond and North Melbourne scores during the 2010 AFL home and away season. Find the difference between the medians of the two sets of data.

North Melbourne		Richmond
9	3	
	4	5 8
7 5	5	3 3 4 6 9
3 3 0	6	4 7
8 2	7	3 7 8
4 2 2	8	0 6 9
7 1 0	9	4 5
4 3	10	0 0 5
9 6 3 0	11	2
9 3	12	6

middle leaves

middle leaves

A. *22 scores for each team ⇒*
median = average of 11th and 12th scores

North Melbourne:

$$median = \frac{84+90}{2} = \frac{174}{2} = 87$$

Richmond:

$$median = \frac{77+78}{2} = \frac{155}{2} = 77.5$$

difference = 87 − 77.5 = **9.5**

a) Complete the stem-and-leaf plot for this data:
3, 12, 16, 17, 20, 21, 32, 35, 35, 37, 39, 43, 48

stem	leaves
0	3
1	2 6 7
2	0 1
3	___ ___ ___ ___ ___
4	___ ___

b) Complete the stem-and-leaf plot for this data:
202, 204, 207, 210, 223, 223, 226, 228, 229, 230, 231, 232, 236

stem	leaves
20	___ ___ ___
21	___
22	___ ___ ___ ___ ___
23	___ ___ ___ ___

Skill 29.10 Interpreting stem-and-leaf plots (2).

c) The stem plot shows a set of scores obtained by a year 9 Maths class. Find the median and range of the data.

stem	leaves
2	0 2 3 6
3	2 3 5 6 7
4	6 8 9
5	4 7
6	2
7	3 8 9
8	5

19 scores

46 - middle score

nine leaves
+ middle leaf
+ nine leaves

median =

range = 85 – 20 =

median = range =

d) The stem-and-leaf plot shows a set of IQ scores obtained by 30 year 10 students. Find the median and mode of the data.

STEM	LEAF
8	7
9	0 2 3 4 4 5 7 8 9 9
10	1 2 2 2 5 7 9 9
11	4 5 7 9
12	1 1 2 4 6 8
13	
14	1

median =

mode =

median = mode =

e) Complete the back-to-back stem-and-leaf plot for the following two sets of data representing the ages of the teachers at the local high school. Find which set has the greater median.

Male: 27, 28, 33, 36, 39, 40, 47, 47, 48, 49, 50 52, 55

Female: 22, 23, 26, 27, 29, 29, 34, 36, 38, 38, 41 43, 44, 44, 45, 48, 49, 50, 56, 59, 61

male		female
_ _	2	_ _ _ _ _ _
_ _ _	3	_ _ _ _
_ _ _ _ _	4	_ _ _ _ _ _ _
_ _ _	5	_ _ _
	6	_

f) This back-to-back stemplot shows the numbers of sausages and soft drinks sold at the school fundraisers in one year. Find the difference between the medians of the two sets of data.

sausages		soft drinks
	14	3 4
4 3 2	15	2 4 8 8
2 1 1	16	0 2 3
6	17	7
9 9 7 6 5	18	0 2

g) The back-to-back stemplot below shows the English and Maths scores of a year 10 class. Find the difference between the medians of the two sets of scores.

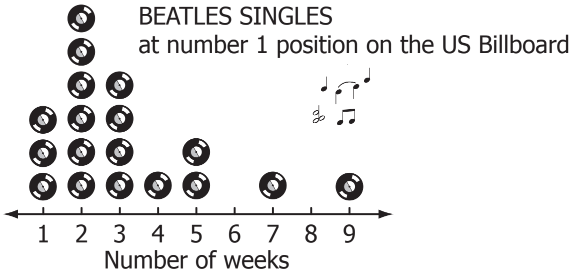
MATHS		ENGLISH
2 1	4	0 5 8
8 8 7 6 5 3 0	5	1 3 0 7
9 9 8 5 5 5 3 2 1	6	4 6 9 9
6 6 5 0	7	2 2 5 5 7 9 9 9
2	8	0 4 6
8	9	4 5

h) This back-to-back stemplot shows Collingwood and Fremantle scores during the 2010 AFL home and away season. Find the difference between the medians of the two sets of data.

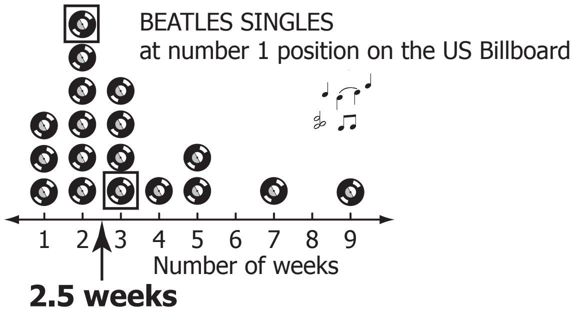
Collingwood		Fremantle
	3	9
1	4	
4 0	5	
	6	7 9
6	7	6
6 0	8	1 2 2 7
6 5	9	3 3 6 7
7 5 5 0	10	0
3	11	1 2 3 4 5 8
9 7 3 0	12	5
5 3	13	9
	14	
7 5	15	
2	16	0

Skill 29.11 Interpreting dot plots.

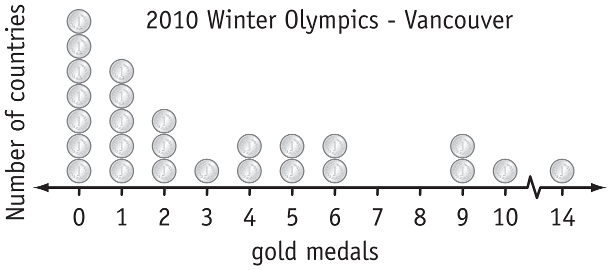
Q. What is the median number of weeks in which The Beatles songs were at number one on the US Billboard charts?



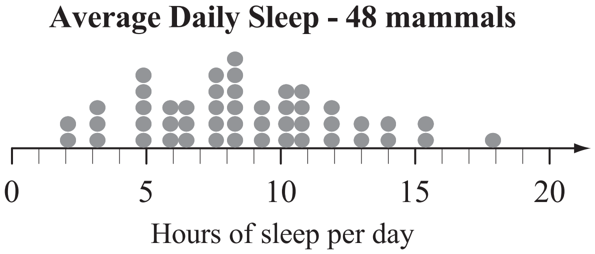
A. *2.5* There were 18 songs, so the median number of weeks belongs between the 9th and 10th songs.



a) How many countries won more than 7 medals at the 2010 Winter Olympics?



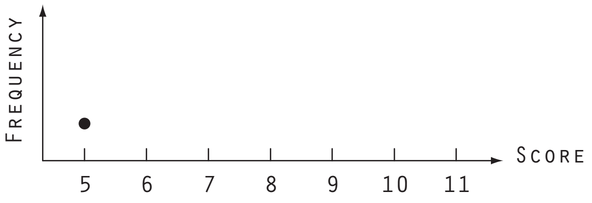
b) Estimate to the nearest hour, the most common number of hours of sleep required by a mammal.



h

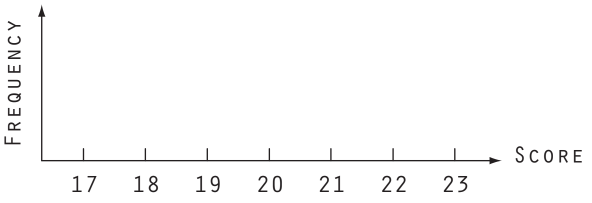
c) Complete the dot plot and find the median of the following data:

~~5~~, 9, 7, 6, 9, 8, 8, 8, 7, 6, 10, 11

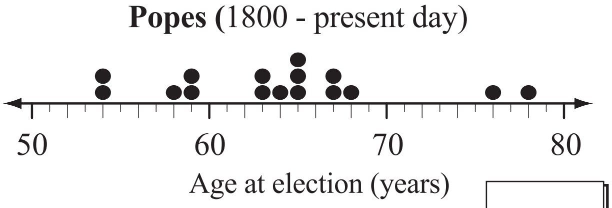


d) Complete the dot plot and find the median of the following data:

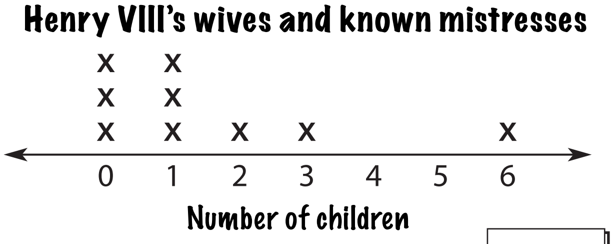
21, 18, 21, 23, 22, 19, 17, 22, 20, 17, 19, 21



e) This dot plot shows the age of a Pope at his election to office. What is the median number of the distribution?



f) This dot plot shows the number children Henry VIII had with his 6 wives and 3 mistresses. What is the median number of the distribution?



Skill 29.12 Interpreting frequency tables.

Mode (most common value)

- Find the highest number in the frequency row.

Median (middle value)

- Add all the frequencies to know how many scores are in total.

If an odd numbered total:

- Add from the left on the frequency line till you reach the middle score.

If an even numbered total:

- Add from the left on the frequency line till you reach the two middle scores.
- Find the average of the 2 middle scores.

Mean (or average)

- Multiply each score by its frequency.
- Add the results.
- Divide by the total number of frequencies (scores).

Range

- Subtract the lowest score from the highest score.

Q. How many scores are there of 6 or less in the following distribution?

Score	5	6	7	8	9
Frequency	3	2	4	3	1

How many times the score occurs

A. *There are 2 lots of 6 and 3 lots of 5 scored.*

$2 + 3$

$= \mathbf{5}$

a) How many scores are there of more than 13 in the following distribution?

Score	10	11	12	13	14
Frequency	4	8	3	6	5

$14 > 13$

b) How many scores are there of 8 or more in the following distribution?

Score	4	6	8	10	12
Frequency	3	7	1	4	5

c) Find the median and range of the following distribution.

Score	15	16	17	18	19
Frequency	3	5	2	7	3

20 scores

$$median = \frac{17+18}{2} = \frac{35}{2} = 17.5$$

$$range = 19 - 15 = 4$$

median = range =

d) Find the mean and mode of the following distribution.

Score	2	3	4	5	6
Frequency	4	2	1	1	2

10 scores

$$mean = \frac{2\times 4+3\times 2+4\times 1+5\times 1+6\times 2}{10} = \frac{35}{10}$$

mode =

mean = mode =

e) Find the median and range of the following distribution.

Score	0	1	2	3	4
Frequency	5	11	6	14	5

median =

range =

median = range =

f) Find the mean and mode of the following distribution.

Score	0	1	2	3	4
Frequency	4	6	5	3	2

mean =

mode =

mean = mode =

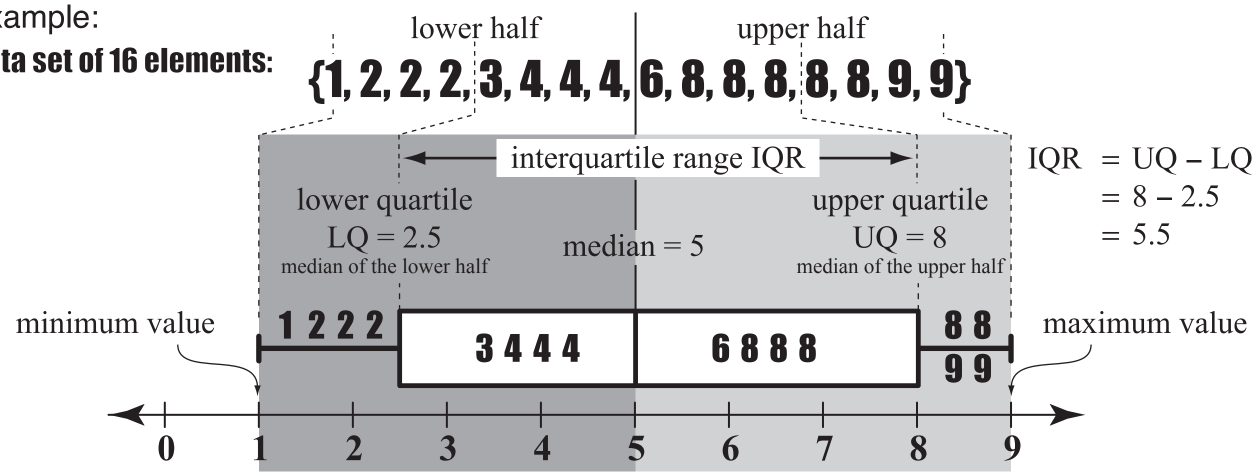
Skill 29.13 Calculating the median, range, upper quartile (UQ), lower quartile (LQ) and interquartile range (IQR) for box-and-whisker plots.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 3 **3** 4 4

- Find the median or middle value of the set of data.
- Divide the data into an upper half and a lower half.
- Find the median of the upper values of the set of data, or the upper quartile (UQ).
- Find the median of the lower values of the set of data, or the lower quartile (LQ).
- Find the interquartile range (IQR) of the set of data by subtracting the LQ from the UQ.

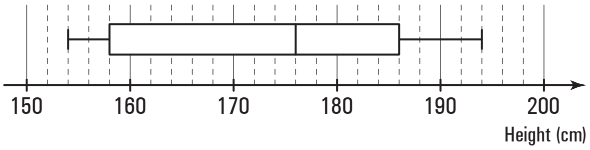
Example:

Data set of 16 elements: **{1, 2, 2, 2, 3, 4, 4, 4, 6, 8, 8, 8, 8, 8, 9, 9}**



IQR = UQ – LQ
= 8 – 2.5
= 5.5

Q. For this box-and-whisker plot, find the median and upper quartile (UQ).

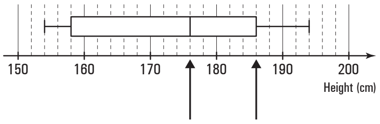


A. *median = middle value*

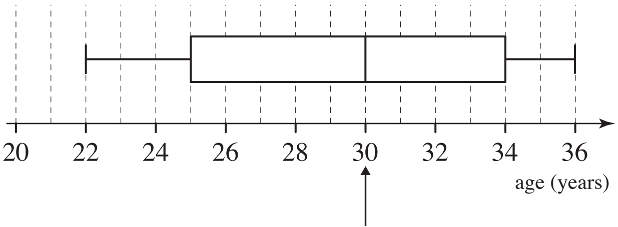
median = ***176***

UQ is the median of the upper scores ⇒

UQ = ***186***



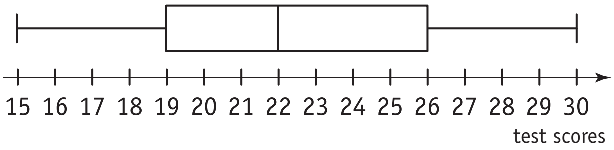
a) For this box-and-whisker plot, find the median and lower quartile (LQ).



median = 30

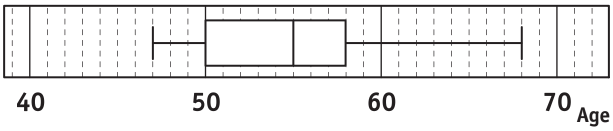
median = 30 LQ =

b) For this box-and-whisker plot, find the lower quartile (LQ) and upper quartile (UQ).



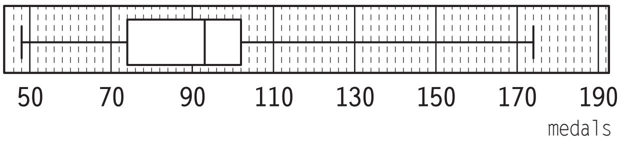
LQ = UQ =

c) What is the median and upper quartile (UQ) of the set of ages of the 19th century American presidents when they were first elected?



median = UQ =

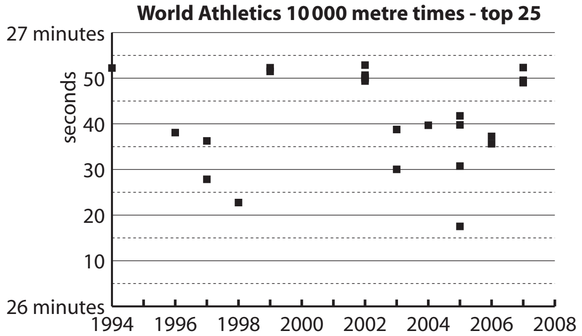
d) What is the median and interquartile range (IQR) of the number of medals won by the USA at each of the Olympics between 1908 and 2004, as shown on this boxplot?



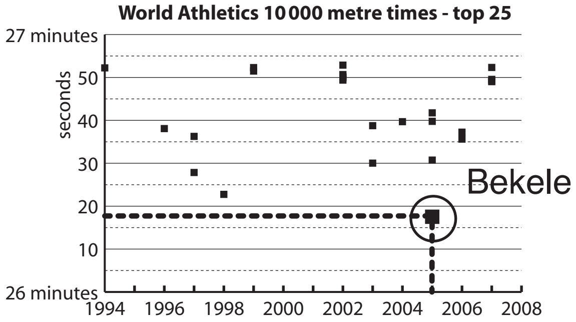
median = IQR =

Skill 29.14 Interpreting scatter plots.

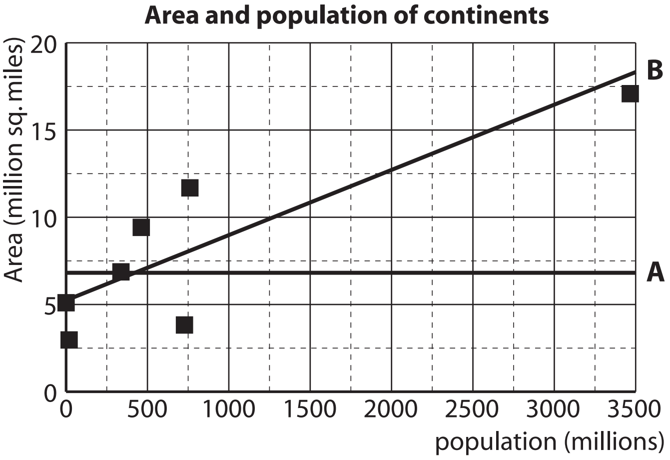
Q. In what year did Bekele (Ethiopia) set the world record time of 26:17.53 for the 10 000 m event?



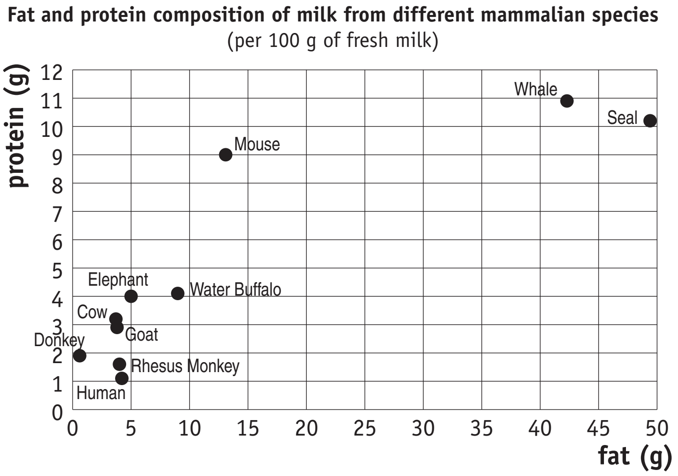
A. *2005*



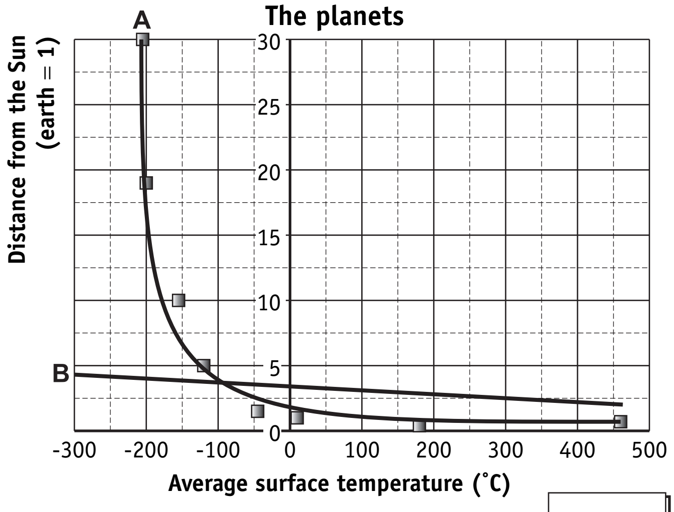
a) Select the most appropriate trend line for this scatter plot. [Hint: The sums of the distances from the points above and below the line, to the line, are approximately equal.]



b) How many of the mammals tested have less than 5 g of fat and less than 2 g of protein in any 100 g of their milk?

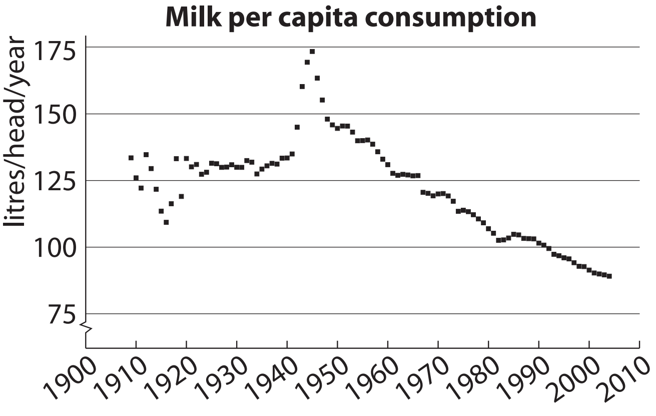


c) Select the most appropriate trend line for this scatter plot. [Hint: The sums of the distances from the points above and below the line, to the line, are approximately equal.]



d) The difference in milk consumption between 1960 and 1990 is:

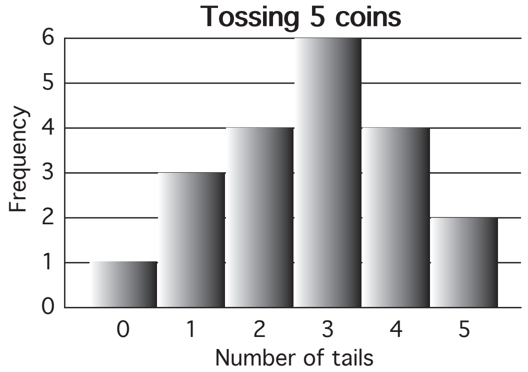
A) < 25 litres/head/year
B) = 25 litres/head/year
C) > 25 litres/head/year



Skill 29.15 Interpreting frequency histograms.

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 **2** 3 3 4 4

Q. Find the range, median and mean of the distribution.



A. *The data for the number of tails is:*

$$\underbrace{0, 1, 1, 1, 2, 2, 2, 2, 3, 3,}_{\textit{first 10 terms}} \Big| \underbrace{3, 3, 3, 3, 4, 4, 4, 4, 5, 5}_{\textit{last 10 terms}}$$

range = last term – first term = 5 – 0 = ***5***

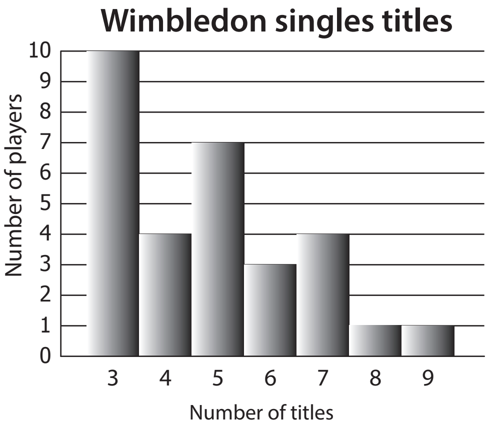
median = average of 10th and 11th term = ***3***

mean = sum of all numbers ÷ number of terms

= 65 ÷ 20

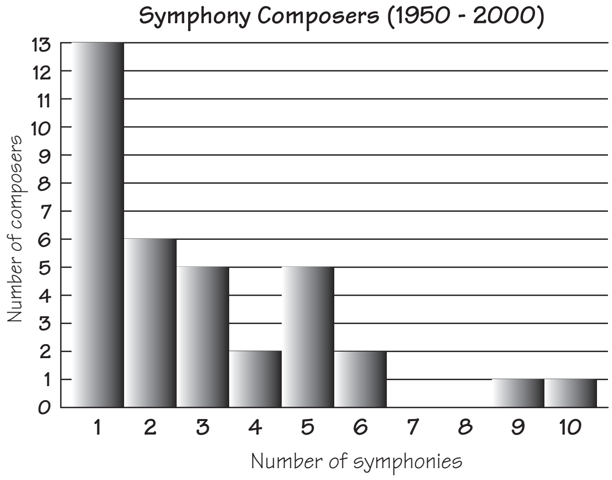
= ***3.25***

a) Find the range and median of the distribution.



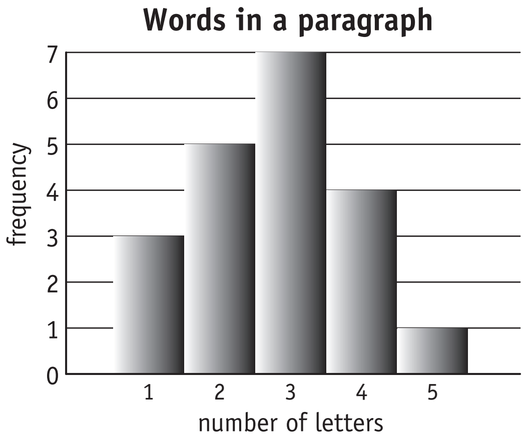
range = median =

b) Find the range and median of the distribution.



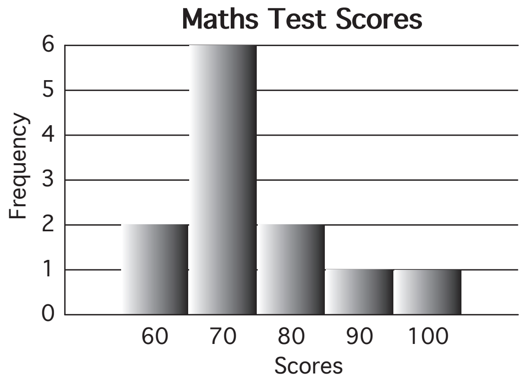
range = median =

c) Find the range and mean of the distribution.



range = mean =

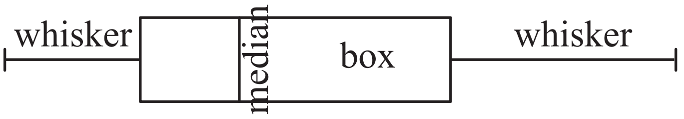
d) Find the median and mean of the distribution.
[Round the mean to the nearest whole number.]



median = mean =

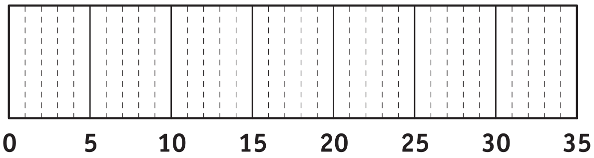
Skill 29.16 Drawing box-and-whisker plots.

- Order the given data set.
- Find the median, lowest and greatest values, lower quartile and upper quartile.
- Mark the maximum and minimum values with whiskers.
- Mark the median of all values with a vertical line.
- Mark the upper quartile and lower quartile with the box edges as shown below.



Q. Draw a box-and-whisker plot for this set of data:

7, 8, 13, 15, 20, 22, 24, 27, 30, 32



A. 7, 8, 13, 15, 20, 22, 24, 27, 30, 32

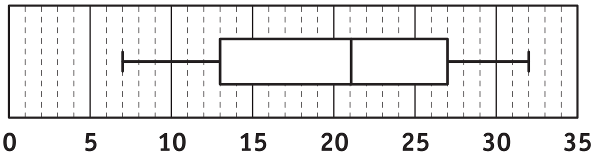
maximum value = 32

minimum value = 7

median = $\frac{22+20}{2}$ = 21

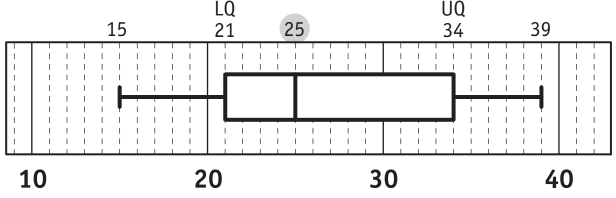
upper quartile = 27 median of upper half

lower quartile = 13 median of lower half



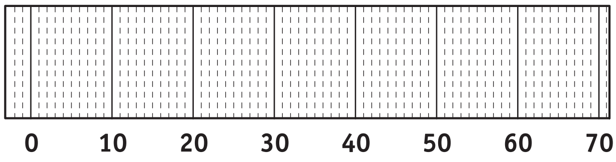
a) Draw a box-and-whisker plot for this set of data:

15, 21, 21, 23, 25, 27, 32, 36, 39



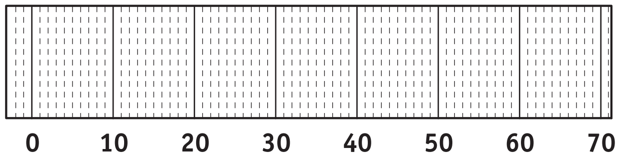
b) Draw a box-and-whisker plot for this set of data:

34, 47, 11, 15, 57, 24, 20, 11, 19, 50, 28, 37

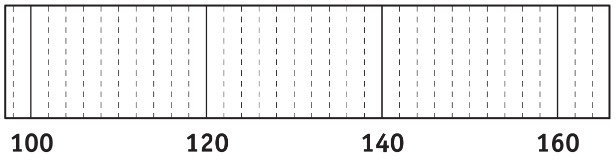


c) Draw a box-and-whisker plot for this set of data:

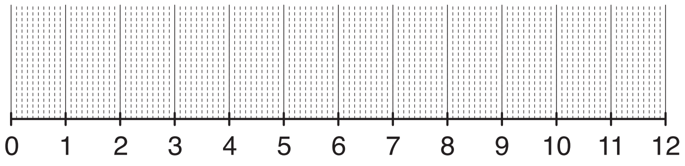
51, 17, 25, 39, 7, 49, 62, 41, 20, 6, 43, 13



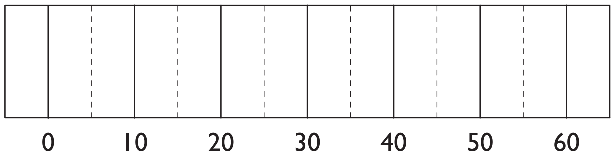
d) Draw a box-and-whisker plot for the set of data whose lowest value is 104, greatest value is 158, median is 136, lower quartile is 116 and upper quartile is 142.



e) Draw a box-and-whisker plot for the set of data whose lowest value is 0, greatest value is 11.8, median is 4.1, lower quartile is 1.2 and upper quartile is 8.5



f) Draw a box-and-whisker plot for the set of data whose lowest value is 15, greatest value is 60, median is 35, lower quartile is 25 and upper quartile is 40.



Skill 29.17 Calculating the median, upper quartile (UQ), lower quartile (LQ) and interquartile range (IQR) for frequency tables and stem-and-leaf plots.

- Find the median or middle value of the set of data.
- Divide the data into an upper half and a lower half.
- Find the median of the upper values of the set of data, or the upper quartile (UQ).
- Find the median of the lower values of the set of data, or the lower quartile (LQ).
- Find the interquartile range (IQR) of the set of data by subtracting the LQ from the UQ.

(see skill 29.13, page 346)

Q. Calculate the median and upper quartile (UQ) for the data displayed in this frequency table.

Athens, 2004 - bronze medal winning countries

Number of medals	1	2	3	4	5	6	7	8	9	10
Frequency	15	13	7	6	2	2	1	0	4	0

scores 1 to 15 are all 1

scores 16 to 28 are all 2

A. *50 scores altogether ⇒*
median = average of 25th and 26th scores
25th and 26th scores are 2 } ⇒
median = ***2***
UQ is the median of the 25 upper scores ⇒
UQ = 13th score counting from the top score:
9, 9, 9, 9, 7, 6, 6, 5, 5, 4, 4, 4, 4 ⇒
UQ = ***4***

13th score from the top

a) For this stem-and-leaf plot, find the median and the lower quartile (LQ).

stem	leaves
1	3 8 8 9
2	0 1 1 2 2 2 9
3	0 1

13 scores ⇒ median = 7th term

$LQ = \frac{18+19}{2} = \frac{37}{2} =$

median = 21 LQ = 18.5

b) Calculate the median and lower quartile (LQ) for the data displayed in this frequency table.

Athens, 2004 - silver medal winning countries

Number of medals	1	2	3	4	5	6	7	8	9	10
Frequency	15	11	6	5	2	4	1	0	4	0

median = LQ =

c) Calculate the median and upper quartile (UQ) for the data displayed in this frequency table.

Adam Scott - PGA, 2006 **rounds 72 or less**

Score	63	64	65	66	67	68	69	70	71	72
Frequency	1	1	4	4	5	5	11	7	11	9

median = UQ =

d) Calculate the median and lower quartile (LQ) for the data displayed in this frequency table.

Score	5	6	7	8	9	10	11	12	13	14
Frequency	2	3	1	4	0	10	5	0	12	3

median = LQ =

e) Find the interquartile range (IQR) for the set of data shown in this stem-and-leaf plot.

STEM	LEAF
1	1 1 2 3 4 4
1	7 8 9
2	0 2 2 4
2	5 5 6 8 9
3	0 1 2 2 3 4
3	6 6 7 8 9
4	1 1 2 3 3 4

IQR =

f) This stem-and-leaf plot shows the ages of the 19th century American presidents when they were first elected. Find the interquartile range (IQR) for the set of data.

STEM	LEAF
4	7 8 9
5	0 0 0 1 2 2 4
5	5 5 6 7 8 8 8 9
6	2
6	5 6 8

IQR =

30. [Probability]

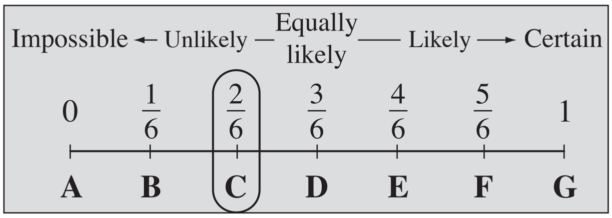
Skill 30.1 Describing the probability of an event using probability scales (1).

Mauve 1 1 2 2 3 3 4 4
Lime 1 1 2 2 3 3 4 4

- Find the number of favourable outcomes for the event.
- Find the total number of possible outcomes.
- Divide the number of favourable outcomes by the number of possible outcomes:

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$

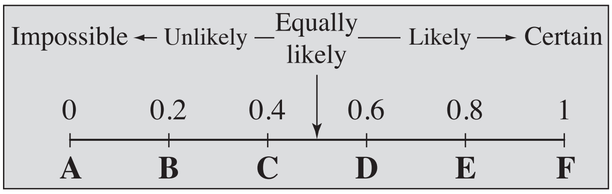
Example:			
	Experiment	throwing a standard die	
	Event	throwing a number greater than 4	
	Possible outcomes (PO)	6	(throwing a 1, 2, 3, 4, 5 or a 6)
	Favourable outcomes (FO)	2	(throwing a 5 or a 6)
	Probability (Pr)	2 out of 6 = $\frac{2}{6}$	(FO out of PO)



Hints: Probability ranges from 0 to 1.
The closer the probability is to 1, the more likely the event is to happen.
The closer the probability is to 0, the more unlikely the event is to happen.

Q. Which letter A to F best describes the probability of this event?

'A class captain will be elected from 5 candidates.'



A. *PO* = 5

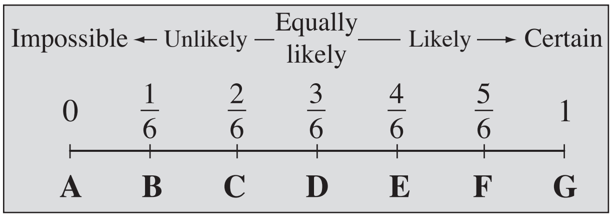
FO = 1

$$Pr\ (event) = \frac{FO}{PO} = \frac{1}{5} = 0.2$$

The answer is ***B***.

a) Which letter A to G best describes the probability of this event?

'A standard die is rolled and a number less than 5 turns up.'

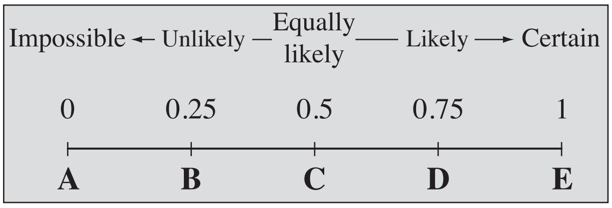


PO = 6 *FO* = 4

$\frac{FO}{PO} = \frac{4}{6}$ ⇒

b) Which letter A to E best describes the probability of this event?

'A tossed coin lands heads.'

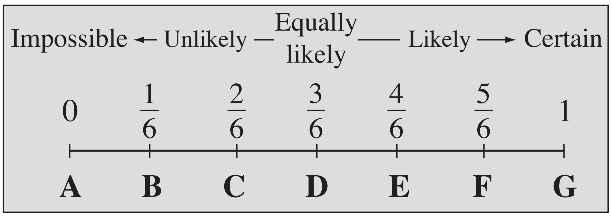


PO = *FO* =

$\frac{FO}{PO} =$ ⇒

Skill 30.1 Describing the probability of an event using probability scales (2).

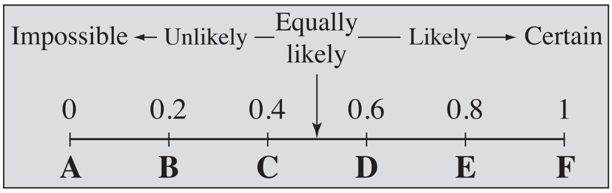
c) Which letter A to G best describes the probability of this event?
'A standard die is rolled and a number greater than 6 turns up.'



$PO =$ $FO =$

$\frac{FO}{PO} =$ ⇒ ☐

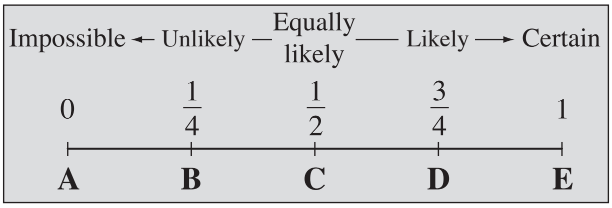
d) Which letter A to F best describes the probability of this event?
'Mathematics is taught at our school.'



$PO =$ $FO =$

$\frac{FO}{PO} =$ ⇒ ☐

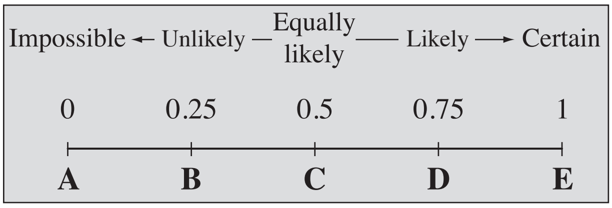
e) Which letter A to E best describes the probability of this event?
'The next baby born in a family will be a boy.'



$PO =$ $FO =$

$\frac{FO}{PO} =$ ⇒ ☐

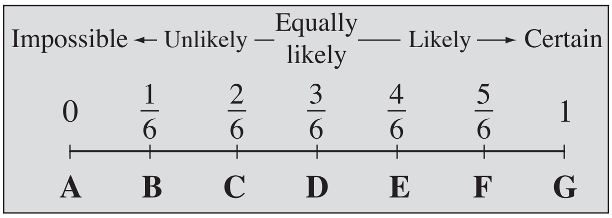
f) Which letter A to E best describes the probability of this event?
'There is a holiday on 1 January.'



$PO =$ $FO =$

$\frac{FO}{PO} =$ ⇒ ☐

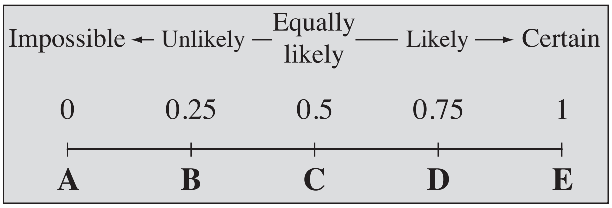
g) Which letter A to G best describes the probability of this event?
'A standard die is rolled and a number greater than 1 turns up.'



$PO =$ $FO =$

$\frac{FO}{PO} =$ ⇒ ☐

h) Which letter A to E best describes the probability of this event?
'A spinner numbered 1 to 8 is spun and a multiple of 3 is obtained.'



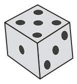
$PO =$ $FO =$

$\frac{FO}{PO} =$ ⇒ ☐

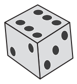
Skill 30.2 Calculating the probability of a simple event (1).

- Find the total number of possible outcomes.
- Find the number of favourable outcomes for the event.
- Divide the number of favourable outcomes by the number of possible outcomes:

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$



OR



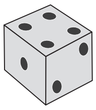
Example:	Experiment	throwing a standard die	
	Event	throwing a number greater than 4	
	Possible outcomes (PO)	6	(throwing a 1, 2, 3, 4, 5 or a 6)
	Favourable outcomes (FO)	2	(throwing a 5 or a 6)
	Probability (Pr)	2 out of 6 = $\frac{2}{6}$	(FO out of PO)

Q. A number from 10 to 17 is chosen at random. What is the probability that the number chosen is a multiple of 3? [Give your answer as a decimal.]

A. *PO* = 8 (10, 11, 12, 13, 14, 15, 16, 17)
FO = 2 (12 and 15)

$$Pr(\text{multiple of } 3) = \frac{FO}{PO} = \frac{2^{\div 2}}{8_{\div 2}} = \frac{1}{4} = \mathbf{0.25}$$

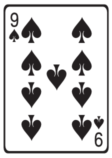
a) A standard die is rolled. What is the probability of rolling a multiple of 2?



PO = 6 *FO* = 3 (2, 4, 6)

$$Pr(\text{multiple of } 2) = \frac{FO}{PO} = \frac{3^{\div 3}}{6_{\div 3}} =$$

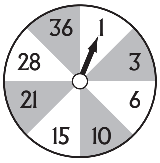
b) A 52-card deck of playing cards is shuffled and one card is dealt from the top of the deck. What is the probability that it is a nine?



PO = *FO* =

$$Pr(\text{nine}) = \frac{FO}{PO} = \qquad =$$

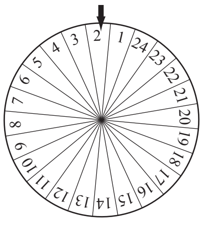
c) This spinner is spun once. What is the probability of spinning a multiple of 3?



PO = *FO* =

Pr(multiple of 3) = = =

d) This spinner is spun once. What is the probability of spinning a prime number?

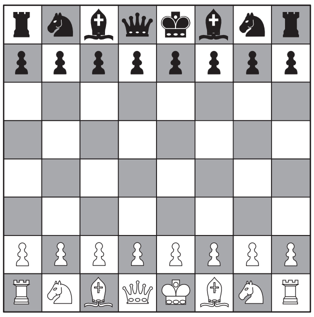


PO = *FO* =

Pr(prime number) = = =

Skill 30.2 Calculating the probability of a simple event (2).

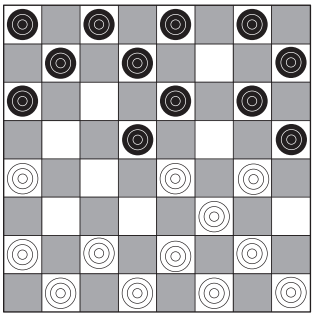
e) A fly lands onto one square of this chess board. What is the probability that the fly lands on a square containing a knight (♘ or ♞)?



PO = *FO* =

Pr(knight) = = =

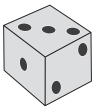
f) A fly lands onto one square of this draught board. What is the probability that the fly lands on an empty white square?



PO = *FO* =

Pr(empty white) = = =

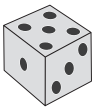
g) A standard die is rolled. What is the probability of rolling a number greater than 2?



PO = *FO* =

Pr(> 2) = = =

h) A standard die is rolled. What is the probability of rolling a prime number?



PO = *FO* =

Pr(prime number) = = =

i) Based on these speed figures, find the probability that a driver, chosen at random, was travelling at more than 110 km/h.

Speed (km/h)	Frequency
81 - 90	9
91 - 100	12
101 - 110	11
111 - 120	8

PO = *FO* =

Pr(> 110) = = =

j) Based on these conference figures, find the probability that a participant, chosen at random, comes from Queensland (QLD).

STATE	PARTICIPANTS
VIC	35
NSW	28
QLD	15
SA	15
WA	12

PO = *FO* =

Pr(Qld) = = =

k) Amanda has six 50-cent coins, nine 20-cent and nine 10-cent coins in her purse. Find the probability that the first coin she takes out of her purse will be a 10-cent coin.

PO = *FO* =

Pr(10-cent coin) = = =

l) In a lotto draw, balls numbered 1 to 45 are mixed together. What is the probability that the first number drawn is greater than 33?

PO = *FO* =

Pr(> 35) = = =

Skill 30.3 Recognising the probability of complementary events.

- Identify the complementary events.
- Find or calculate the probability of the event.

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$

- Subtract this value from 1 in order to find the probability of the complementary event:

$$\text{Pr(complementary event)} = 1 - \text{PR(event)}$$

Hints: The complement of the event “the aeroplane will be on time” is “the aeroplane will not be on time”.
Winning - not winning, voting “yes” - voting ”no” are examples of complementary events.

Q. If the probability of winning a prize is $\frac{3}{200}$, what is the probability of not winning a prize?

A. *Event = winning a prize*
Complementary event = not winning a prize
Pr(winning a prize) $= \frac{3}{200}$
Pr(not winning a prize) $= 1 - \frac{3}{200}$
$= \frac{200}{200} - \frac{3}{200} = \mathbf{\frac{197}{200}}$

a) If the probability of raining on New Year’s Eve is $\frac{7}{20}$, what is the probability of not raining?

Pr(raining) $= \frac{7}{20}$

Pr(not raining) $= 1 - \frac{7}{20} =$

b) If the probability of Scott winning his next tennis match is $\frac{3}{5}$, what is the probability of him not winning?

Pr(winning) =

Pr(not winning) = 1 − =

c) The probability that Nina votes “yes” in the referendum is 0.37. What is the probability that she votes “no”?

Pr(yes) =

Pr(no) = 1 − =

d) The probability that a couple will have a child with blue eyes is $\frac{1}{4}$. What is the probability that the child will not have blue eyes?

Pr(blue) =

Pr(not blue) = 1 − =

e) There are 10 purple, 24 yellow and 16 green discs. Find, as a decimal, the probability that a disc drawn at random from the barrel is not yellow.

FO = 24, PO = 50

Pr(yellow) $= \frac{24^{\div 2}}{50_{\div 2}} = \frac{12}{25}$

Pr(not yellow) $= 1 - \frac{12}{25} = \frac{25}{25} - \frac{12}{25} = \frac{13}{25}$

$= \frac{13^{\times 4}}{25_{\times 4}} = \frac{52}{100} = \boxed{0.52}$

f) A bag contains 15 green, 10 red and 20 blue marbles. Find the probability that a marble drawn at random from the bag will not be blue.

Pr(blue) =

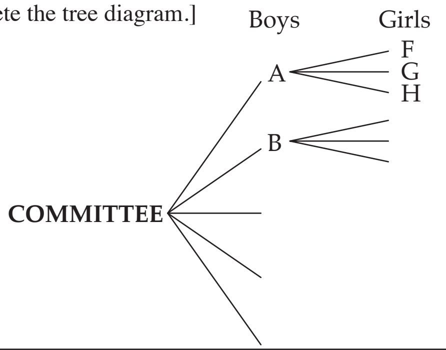
Pr(not blue) = 1 − =

Skill 30.4 Finding the possible outcomes (sample spaces) of an event by completing tree diagrams.

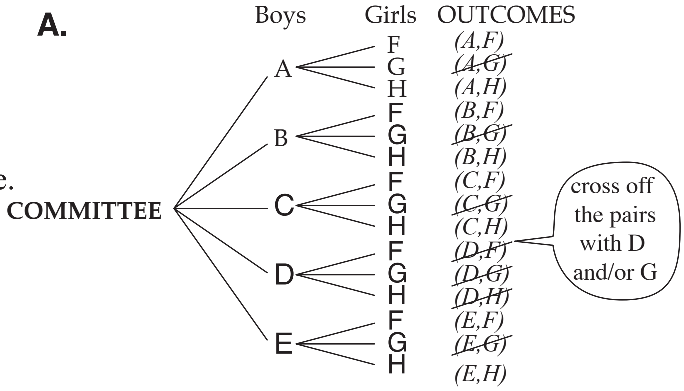
- Complete the tree diagram to show all the possible outcomes (sample space).
- Count the number of favourable outcomes.
- Divide the number of favourable outcomes by the number of possible outcomes.

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{FO}{PO}$$

Q. A committee consisting of one boy and one girl is to be selected from 5 boys (Aaron, Brad, Chris, Dean and Ethan) and 3 girls (Fiona, Gemma and Hannah). Find the probability that Dean and Gemma are not on the committee.
[Complete the tree diagram.]



A.

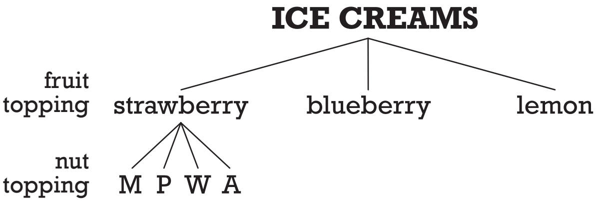


$PO = 5 \times 3 = 15$

$FO = 15 - 7 = 8$

$\Pr(\text{no Dean, no Gemma}) = \frac{FO}{PO} = \boldsymbol{\frac{8}{15}}$

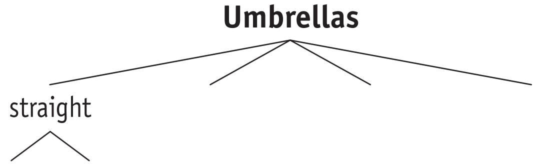
a) The ice cream at the cinema is sold with one fruit (strawberry, blueberry and lemon) and one nut topping (macadamia, pecans, walnuts, almonds). What is the probability of choosing a lemon topping? [Complete the tree diagram.]



$PO = 12$ $FO = 4$

$\Pr(\text{lemon}) = \frac{4 \div 4}{12 \div 4} =$ ☐

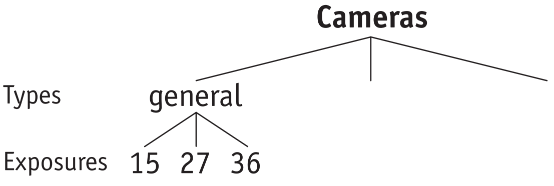
b) Umbrellas are sold in four styles, straight, golf, 2-folded and 3-folded. They can be manual (M) and automatic (A). What is the probability of receiving an automatic umbrella for your birthday? [Complete the tree diagram.]



$PO =$ $FO =$

$\Pr(\text{automatic}) =$ $=$ ☐

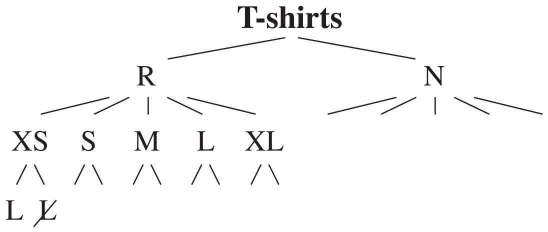
c) A store is selling general, outdoor and waterproof disposable cameras. Each type comes with 15, 27 and 36 exposures. What is the probability of choosing an outdoor disposable camera? [Complete the tree diagram.]



$PO =$ $FO =$

$\Pr(\text{outdoor}) =$ $=$ ☐

d) The T-shirts displayed for sale come in two colours (red, navy) and in 5 sizes (XS, S, M, L, XL). The T-shirts can be purchased with or without a logo. What is the probability of choosing a navy T-shirt without a logo?



$PO =$ $FO =$

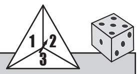
$\Pr(\text{navy, no logo}) =$ $=$ ☐

Skill 30.5 Calculating the probability of multiple events by using tree diagrams or two-way tables to represent the sample spaces (1).

- Complete the tree diagram or two-way table to show all the possible outcomes (sample space).
- Count the number of favourable outcomes.
- Divide the number of favourable outcomes by the number of possible outcomes.

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$

Q. A die is tossed and a spinner labelled 1, 2 and 3 is spun. What is the probability of obtaining one even number and one odd number when the die is tossed and the spinner is spun once? [Complete the table.]



Possible outcomes		Die					
		1	2	3	4	5	6
Spinner	1	(1,1)	(1,2)				
	2	(2,1)	(2,2)				
	3						

A.

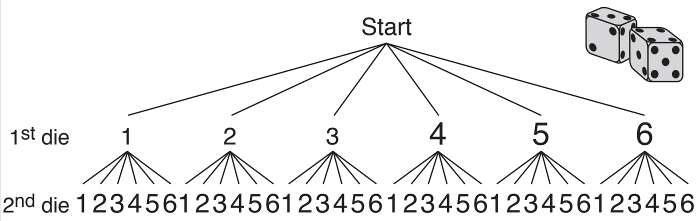
Possible outcomes		Die					
		1	2	3	4	5	6
Spinner	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)

$PO = 6 \times 3 = 18$

$FO = 9$

$$Pr(\text{one odd, one even}) = \frac{FO}{PO} = \frac{9^{\div 9}}{18_{\div 9}} = \frac{1}{2}$$

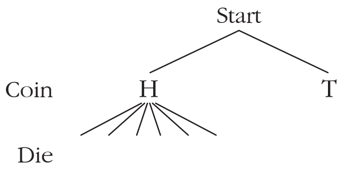
a) A pair of standard dice are rolled. Find the probability of rolling a total of 7. [Complete the tree diagram.]



$PO = 36$ $FO = 6$

$$Pr(\text{total of } 7) = \frac{FO}{PO} = \frac{6^{\div 6}}{36_{\div 6}} = \boxed{\frac{1}{6}}$$

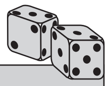
b) A coin is tossed and a die is rolled. Find the probability of tossing heads and rolling a number less than 5. [Complete the tree diagram.]



$PO =$ $FO =$

$Pr(H \text{ and } < 5) = \quad = \quad = \boxed{}$

c) A pair of standard dice are rolled. What is the probability of rolling any pair of identical numbers? [Complete the table.]

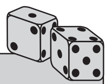


Possible outcomes		Die 1					
		1	2	3	4	5	6
Die 2	1	(1,1)	(1,2)				
	2	(2,1)					
	3	(3,1)					
	4						
	5						
	6						

$PO =$ $FO =$

$Pr(\text{identical no.}) = \quad = \quad = \boxed{}$

d) A pair of standard dice are rolled. Find the probability of rolling no prime numbers in the pair. [Complete the table.]



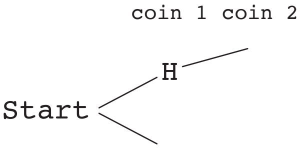
Possible outcomes		Die 1					
		1	2	3	4	5	6
Die 2	1	(1,1)	(1,2)				
	2	(2,1)					
	3	(3,1)					
	4						
	5						
	6						

$PO =$ $FO =$

$Pr(\text{no primes}) = \quad = \quad = \boxed{}$

Skill 30.5 Calculating the probability of multiple events by using tree diagrams or two-way tables to represent the sample spaces (2).

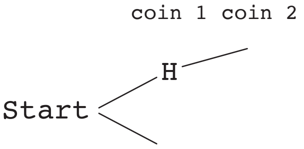
e) Two coins are tossed at the same time. Find the probability of tossing at least one head.
[Complete the tree diagram.]



PO = *FO* =

Pr(at least 1 H) = = =

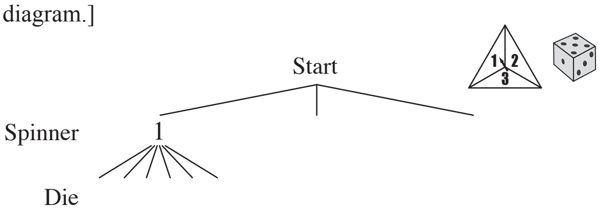
f) Two coins are tossed at the same time. Find the probability of tossing two tails.
[Complete the tree diagram.]



PO = *FO* =

Pr(2 tails) = = =

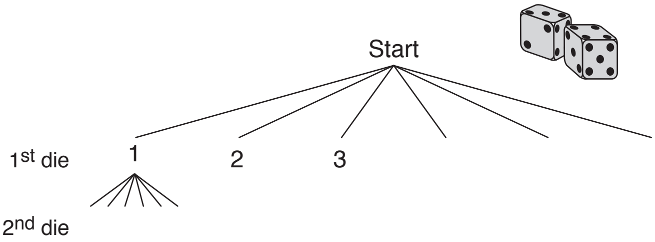
g) A die is tossed and a spinner labelled 1, 2 and 3 is spun. What is the probability of obtaining two odd numbers when the die is tossed and the spinner is spun once? [Complete the tree diagram.]



PO = *FO* =

Pr(2 odd no.) = = =

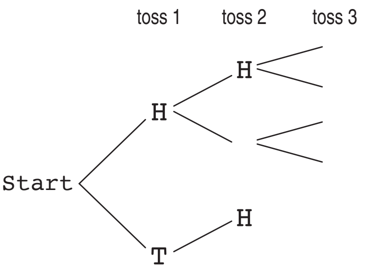
h) A pair of standard dice are rolled. Find the probability of rolling two even numbers in the pair. [Complete the tree diagram.]



PO = *FO* =

Pr(2 even no.) = = =

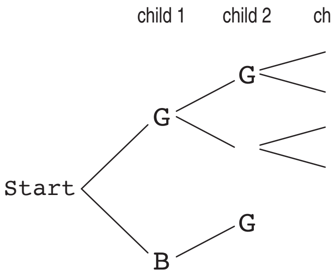
i) A coin is tossed three times. What is the probability of tossing at least one tail?
[Complete the tree diagram.]



PO = *FO* =

Pr(at least 1 T) = = =

j) Chris and Klaus wish to start a family. They plan to have 3 children. If the chance of having a girl or a boy is equal, find the probability that they have 3 children of the same sex.



PO = *FO* =

Pr(3 of same sex) = = =

Skill 30.6 Calculating the probability of mutually exclusive events by using the Addition Law of Probability.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 **3** 3 4 4

- Find the probability of each separate event.

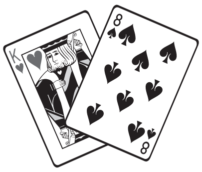
$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$

- Add the probabilities of each event in order to find the probability of either event occurring.

Addition Law of Probability: Pr(A or B) = Pr(A) + Pr(B)

Hint: Mutually exclusive events cannot occur at the same time.

Q. A 52-card deck of playing cards is shuffled and one card is dealt from the top of the deck. What is the probability that it is an eight or a court card?



A. $Pr(8) = \frac{4}{52}$ 8 in all four suites

$Pr(court) = \frac{12}{52}$ J, Q, K in all four suites

$Pr(8 \text{ or } court) = Pr(8) + Pr(court)$

$$= \frac{4}{52} + \frac{12}{52} = \frac{16^{\div 4}}{52_{\div 4}} = \boldsymbol{\frac{4}{13}}$$

a) Based on these car dealership figures, find the probability that a buyer, chosen at random purchased a Toyota or a Holden.

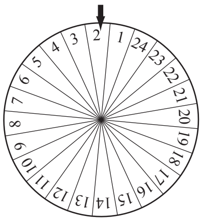
CAR MODEL	SALES
Ford	90
Holden	140
Mitsubishi	55
Toyota	180
Nissan	15

$Pr(T) = \frac{180}{480}$ $Pr(H) = \frac{140}{480}$

$Pr(T \text{ or } H) = Pr(T) + Pr(H) =$

$$= \frac{180}{480} + \frac{140}{480} = \frac{320^{\div 160}}{480_{\div 160}} =$$

b) This spinner is spun once. What is the probability of spinning a multiple of 3 or a multiple of 10?



$Pr(M3) = \frac{8}{24}$ $Pr(M10) =$

$Pr(M3 \text{ or } M10) = \quad + \quad =$

$=$ $=$

c) A bowl contains 40 marbles, numbered 1 to 40. A marble is drawn from the bowl. Find the probability of drawing a multiple of 6 or a multiple of 7.

$Pr(M6) =$ $Pr(M7) =$

$Pr(M6 \text{ or } M7) =$

$=$ $=$

d) A jar contains 23 black, 18 red and 31 white jelly beans. Find the probability that a jelly bean drawn at random from the jar will be white or black.

$Pr(white) =$ $Pr(black) =$

$Pr(white \text{ or } black) =$

$=$ $=$

Skill 30.7 Calculating the probability of non-exclusive events.

Mauve 1 1 2 2 3 3 4 **4**
Lime 1 1 2 2 **3** 3 4 4

- Find the probability of each separate event.

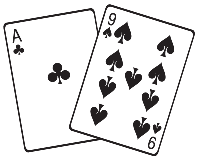
$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$

- Add the probabilities of each event and then subtract the probability of both events occuring at the same time, in order to find the probability of either event occuring.

$$\text{Pr(A or B)} = \text{Pr(A)} + \text{Pr(B)} - \text{Pr(A and B)}$$

Hint: Non-exclusive events can occur at the same time.

Q. A 52-card deck of playing cards is shuffled and one card is dealt from the top of the deck. What is the probability that it is an odd number or a club?



A. $Pr(odd\ number) = \frac{20}{52}$ 1, 3, 5, 7, 9 in all four suites

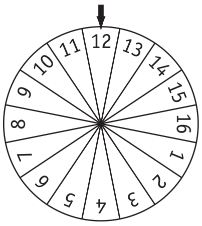
$Pr(club) = \frac{13}{52}$ A quarter of the deck are clubs

$Pr(odd\ and\ club) = \frac{5}{52}$

$Pr(O\ or\ C) = Pr(O) + Pr(C) - Pr(O\ and\ C)$

$= \frac{20}{52} + \frac{13}{52} - \frac{5}{52} = \frac{28^{\div 4}}{52_{\div 4}} = \frac{7}{13}$

a) This spinner is spun once. What is the probability of spinning a multiple of 2 or a multiple of 3?



$Pr(M2) = \frac{8}{16}$ ....

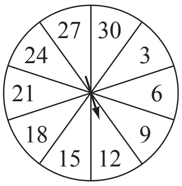
$Pr(M3) = \frac{5}{16}$ ....

$Pr(M2\ and\ M3) = \frac{2}{16}$ ....

$Pr(M2\ or\ M3) = \frac{8}{16} + \frac{5}{16} - \frac{2}{16}$ ....

$= \frac{13}{16} - \frac{2}{16}$ .... $=$ ☐

b) This spinner is spun once. What is the probability of spinning an even number or a number greater than 15?



$Pr(even) =$ ....

$Pr(> 15) =$ ....

$Pr(even\ and > 15) =$ ....

$Pr(even\ or > 15) = \quad + \quad -$ ....

$=$ .... $=$ ☐

c) A 52-card deck of playing cards is shuffled and one card is dealt from the top of the deck. What is the probability that it is a red court card or a heart?

$Pr(red\ court) =$ ....

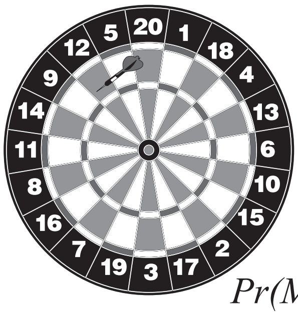
$Pr(heart) =$ ....

$Pr(red\ court\ and\ heart) =$ ....

$Pr(red\ court\ or\ heart) = \quad + \quad -$ ....

$=$ .... $=$ ☐

d) A dart is thrown and hits the board. What is the probability that the dart lands inside a multiple of 3 or a multiple of 4?



$Pr(M3) =$ ....

$Pr(M4) =$ ....

$Pr(M3\ and\ M4) =$ ....

$Pr(M3\ or\ M4) = \quad + \quad -$ ....

$=$ .... $=$ ☐

Skill 30.8 Finding the number of expected successful events.

- Determine the probability of the event.
- Use the formula:

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$

$$\text{Expected number of successful events} = \text{total number of events} \times \text{probability of the event}$$

Q. A roulette wheel has 37 compartments, numbered 0 to 36. The 0 is green and the remaining numbers are divided evenly between red and black. The wheel is spun 740 times. How many times would you expect the marble to land on a red number?

A. *Pr(red)* $= \frac{18}{37}$

Expected successful outcomes $= \overset{20}{\cancel{740}} \times \frac{18}{\underset{1}{\cancel{37}}} =$

$= 20 \times 18$

$= \mathbf{360}$

a) In a class of 28 students, how many would you expect to celebrate their birthday on a Saturday next year?

Pr(Saturday) $= \frac{1}{7}$

Expected birthdays on Sat =

$= 28 \times \frac{1}{7}$ = ☐

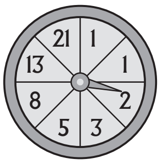
b) A coin is tossed 32 times. How many times would you expect heads to turn up?

Pr(heads) =

Expected heads up =

= = ☐

c) This spinner is spun 100 times. How many times would you expect an even number to be spun?

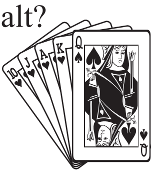


Pr(even) =

Expected even numbers =

= = ☐

d) A 52-card deck of playing cards is shuffled and one card is dealt from the top of the deck. If you repeat this 65 times, how many times would you expect a queen to be dealt?



Pr(Q) =

Expected Queen =

= = ☐

e) The unemployment rate in Australia at the end of March 2007 was approximately 4.5%. Based on this figure, how many people were expected to be unemployed in a population sample of 5000 people?

Pr(unemployed) =

Expected unemployed =

= = ☐

f) A school survey found that 5 out of 9 students voted "yes" in favour of the canteen menu. There are 630 students in the school. How many students would you expect to have voted "yes"?

Pr(yes) =

Expected yes votes =

= = ☐

Skill 30.9 Calculating the probability of independent events by using the Multiplication Law of Probability.

Mauve 11 22 33 44
Lime 11 22 33 44

- Find the probability of each event.

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$

- Multiply the probabilities of each event in order to find the probability of both events occurring.

Multiplication Law of Probability: Pr(A and B) = Pr(A) × Pr(B)

Hint: The probability of independent events is not dependent on the outcome of any other event.

Q. A bag contains 4 red marbles and 6 green marbles. You select a marble at random, replace it, and draw another marble. Find the probability that both marbles are green.

A. $Pr(G) = \frac{6}{10}$

$$\begin{aligned} Pr(G \text{ and } G) &= Pr(G) \times Pr(G) \\ &= \frac{6}{10} \times \frac{6}{10} \\ &= \frac{36}{100}\begin{matrix}\div 4\\ \div 4\end{matrix} \\ &= \frac{9}{25} \quad \text{or } \mathbf{0.36} \end{aligned}$$

6 green

10 possible

a) The probability of Joseph's tennis team winning is 0.6 and each result is independent of other wins or losses. What is the probability that Joseph's team wins three times running?

Pr(W and W and W) = Pr (W) × Pr (W) × Pr (W)

= 0.6 × 0.6 × 0.6 = 0.216

b) What is the probability of a couple having first a boy and then a girl?

Pr(B) = *Pr(G) =*

Pr(B and G) = Pr (B) × Pr (G)

= =

c) In a school, 25% of the students ride bikes to school and 40% have fair hair. One student is selected at random. What is the probability that the student has fair hair but does not ride a bike?

= =

d) The Orange-bellied Parrot has a 7 out of 10 fledgling survival rate. From a brood of two chicks, what is the probability that the first will live and the second will die?

= =

e) You can take 1 duck and 1 chicken to the Sydney show. They will be chosen randomly from the 7 ducks and 9 chickens at your farm. What is the probability that the youngest of each animal will be chosen?

= =

f) The average error rate resulting in patient harm in Australian hospitals is 12%. Two patients are chosen at random. What is the probability that they will both be harmed by hospital error?

= =

Skill 30.10 Completing a probability tree diagram (1).

- From the start, fill in each branch of the tree with the value of the probability of that event.

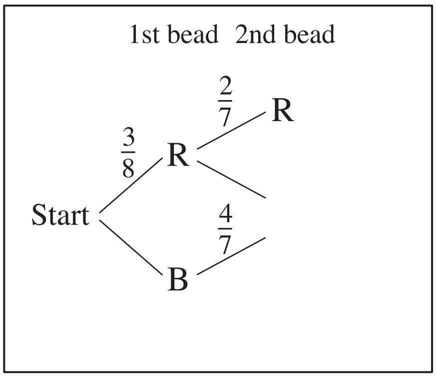
$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$

Hint: The values on all the branches starting from the same point always add to 1.

- List all the possible outcomes.
- To find the probability of any outcome, multiply values horizontally, from two or more consecutive branches.
- Where there is more than one outcome to be considered, add all the probabilities.

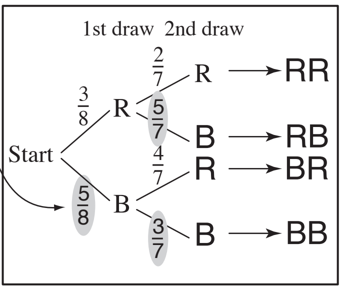
Hint: Remember in a probability tree diagram, always add vertically and multiply horizontally.

Q. Two beads are taken out in succession, without replacing, from a bag containing three red and five blue beads. What is the probability of drawing one red and one blue bead?
[Complete the probability tree diagram.]



A. *Drawing the 1st bead, out of 8:*

$$Pr(blue) = 1 - \frac{3}{8}$$
$$= \frac{5}{8}$$



If 1st bead is red, drawing the 2nd, out of 7:

$$Pr(blue) = 1 - \frac{2}{7} = \frac{5}{7}$$

If 1st bead is blue, drawing the 2nd, out of 7:

$$Pr(blue) = 1 - \frac{4}{7} = \frac{3}{7}$$

Possible outcomes are: RR, RB, BR, BB

multiply horizontal consecutive branches

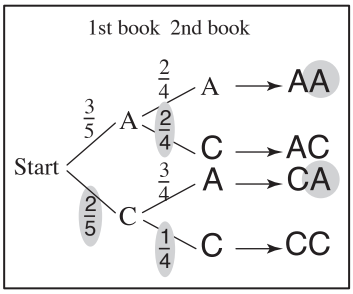
$$Pr(RB) = \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$$

$$Pr(BR) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$$

Addition Law of probability

$$Pr(RB \text{ or } BR) = \frac{15}{56} + \frac{20}{56} = \frac{35^{\div 7}}{56_{\div 7}} = \mathbf{\frac{5}{8}}$$

a) A box contains 3 atlases and 2 comic books. A book is picked from the box and not replaced. What is the probability that a second selection will be an atlas? [Complete the probability tree diagram.]

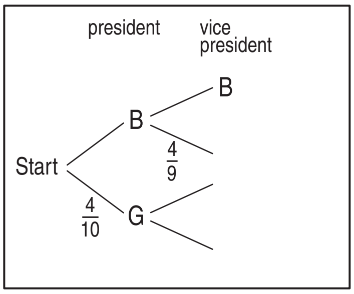


$$Pr(AA) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$$

$$Pr(CA) = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$$

$$Pr(AA \text{ or } CA) = \frac{6}{20} + \frac{6}{20} = \frac{12^{\div 4}}{20_{\div 4}} = \boxed{}$$

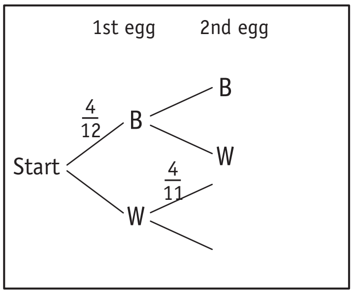
b) A president and a vice president are to be selected from 6 boys and 4 girls. What is the probability that both people selected are boys?
[Complete the probability tree diagram.]



$$Pr(BB) = \quad \times \quad = \quad = \quad \boxed{}$$

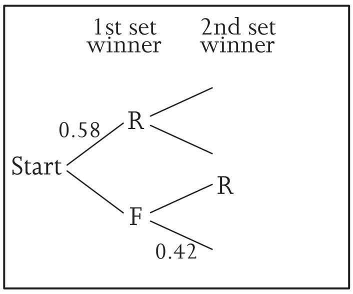
Skill 30.10 Completing a probability tree diagram (2).

c) An egg carton has 4 brown eggs and 8 white eggs. Two eggs are picked up one after the other. What is the probability of picking 2 white eggs? [Complete the probability tree diagram.]



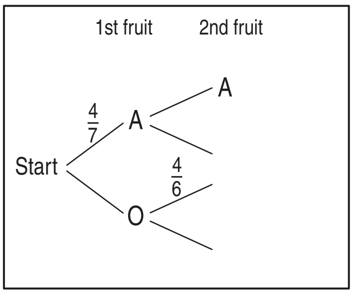
Pr(WW) = × = =

d) Ron and Fred are going to play 2 sets of tennis. Ron's chance of winning a set is 0.58. What is the probability of Ron winning both sets? [Complete the tree diagram. Give the answer correct to two decimal places.]



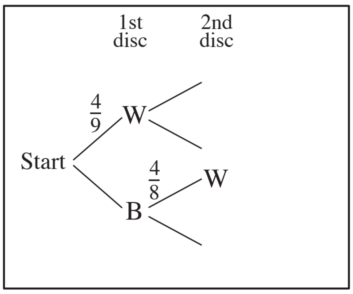
Pr(RR) = × = ≈

e) A bowl contains 4 apples and 3 oranges. Two pieces of fruit are picked one after another. What is the probability of picking two oranges? [Complete the probability tree diagram.]



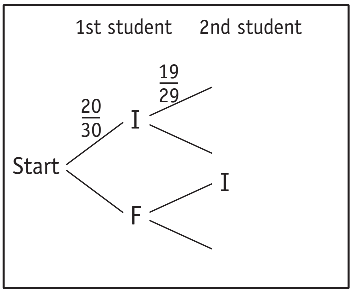
Pr(OO) = × = =

f) Two discs are taken out, without replacing, from a barrel containing 4 white and 5 black discs. What is the probability of choosing two black discs? [Complete the probability tree diagram.]



Pr(BB) = × = =

g) Of the thirty students in Jobe's class, 20 study Italian and 10 study French. If two students are selected at random, what is the probability of one studying Italian and one studying French? [Complete the probability tree diagram.]

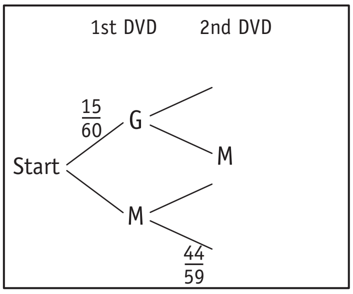


Pr(IF) = =

Pr(FI) = =

Pr(IF or FI) = + =

h) A shelf contains 15 DVD's with games and 45 DVD's with movies. A DVD is picked from the shelf and not replaced. What is the probability that a second DVD selected from the shelf will be a movie? [Complete the tree diagram.]



Pr(GM) = =

Pr(MM) = =

Pr(GM or MM) = + =

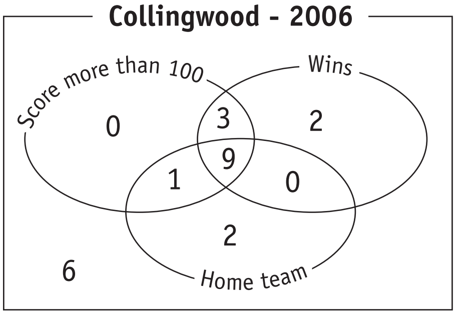
Skill 30.11 Calculating the probability of an event represented by Venn diagrams.

Mauve 11 22 33 44
Lime 11 22 33 44

- Count the total number of possible outcomes.
- Shade the areas inside the Venn diagram that fit the description for favourable outcomes.
- Use the formula for the probability of an event.

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\text{FO}}{\text{PO}}$$

Q. If one 2006 Collingwood match is selected at random, what is the probability that the selected match was lost with a score of 100 points or less?

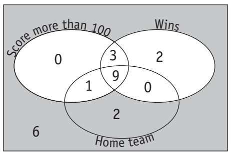


A. *PO = total games played* — possible outcomes

= 0 + 3 + 2 + 1 + 9 + 0 + 2 + 6

= 23

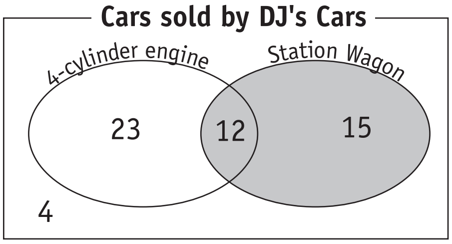
Losses with scores of 100 or less are shown by the shaded area: (white areas are excluded)



FO = 8

$$Pr(event) = \frac{FO}{PO} = \boldsymbol{\frac{8}{23}}$$

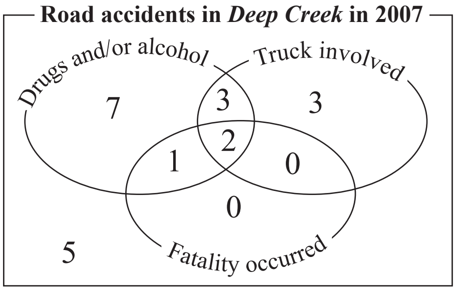
a) If one car is selected at random, what is the probability that it is a station wagon?



PO = 54 *FO = 27*

$Pr(event) = \frac{FO}{PO} =$ $=$

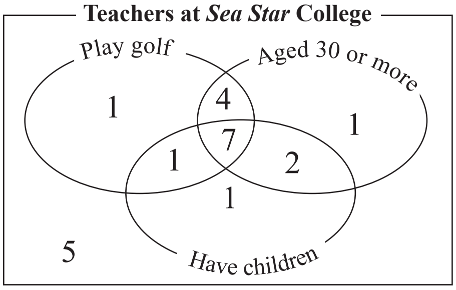
b) If one accident is selected at random for investigation, what is the probability that the selected accident was fatal?



PO = *FO =*

$Pr(event) = \frac{FO}{PO} =$ $=$

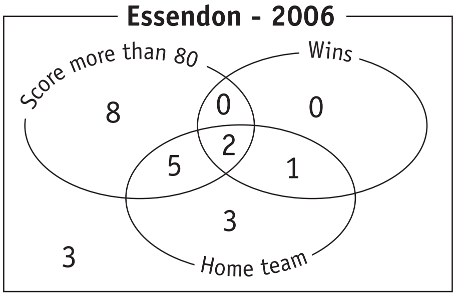
c) If one teacher is selected at random, what is the probability that she or he is 31 years of age, plays golf and has a child?



PO = *FO =*

Pr(event) = $=$

d) If one match is selected at random, what is the probability that the selected match was lost as an away team?



PO = *FO =*

Pr(event) = $=$

Skill 30.12 Calculating the probability of an event represented by two-way tables.

- Fill in the empty spaces of the two-way table by checking that each row and column adds to its respective total.
- Find the total number of possible outcomes, which is the number in the bottom right corner.
- Highlight the areas inside the two-way table that fit the description for favourable outcomes.
- Use the formula for the probability of an event:

$$Pr(event) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{FO}{PO}$$

Q. If one passenger is selected at random, find the probability that he or she flies economy class. [Complete the two-way table.]

	Local	International	**Total**
First class	500	300	
Business class	1800		2700
Economy class	4300		
Total		3400	

A.

	Local	International	**Total**
First class	500	start 300	
Business class	1800		2700
Economy class	4300		
Total	6600	3400	10 000

finish

Follow arrows for order of filling

PO = total = 10 000 possible outcomes

Passengers flying economy are shown by the shaded area:

	Local	International	**Total**
First class	500	300	800
Business class	1800	900	2700
Economy class	4300	2200	6500
Total	6600	3400	10 000

FO = 6500

$$Pr(event) = \frac{FO}{PO} = \frac{6500^{\div 500}}{10\,000_{\div 500}} = \mathbf{\frac{13}{20}}$$

a) If one person is selected at random, find the probability that it is a women who voted 'Yes' at the referendum. [Complete the two-way table.]

	Women	Men	Total
Yes	245	180	425
No		120	
Total	580	300	880

PO = 880 *FO = 245*

$Pr(event) = \frac{FO}{PO} =$ .... $=$ ☐

b) If one car is selected at random for sale, find the probability that it is a manual, 3-door car. [Complete the two-way table.]

	5-door	3-door	**Total**
Manual	28		
Automatic			90
Total	65	70	

PO = *FO =*

$Pr(event) = \frac{FO}{PO}$ .... $=$ ☐

c) If one student is selected at random from the school, find the probability that the person is not blonde. [Complete the two-way table.]

	Boys	Girls	**Total**
Red hair	16	9	
Blonde hair	105		
Brown hair	75		160
Total		304	

PO = *FO =*

Pr(event) = ... $=$ .... $=$ ☐

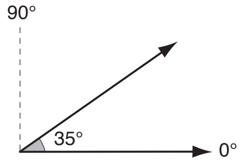
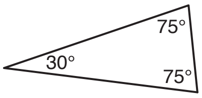
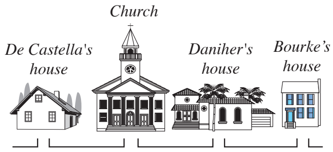
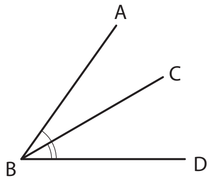
d) If one card is dealt from the top of a standard deck of playing cards, find the probability that it is a red court card. [Complete the two-way table.]

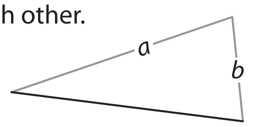
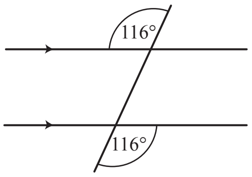
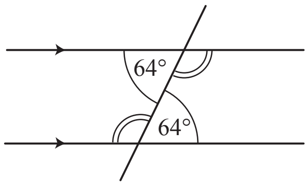
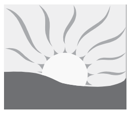
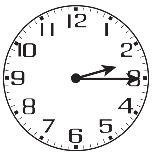
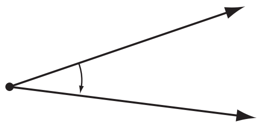
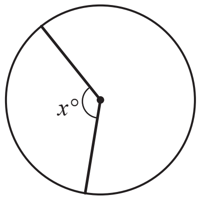
	Black	Red	**Total**
Ace			
Number			
Court card			
Total			52

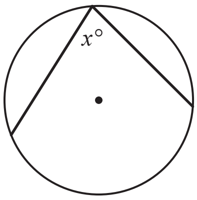
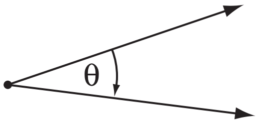
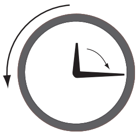
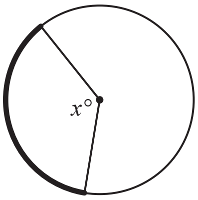
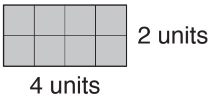
PO = *FO =*

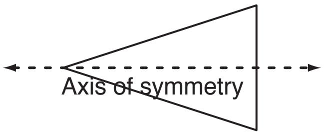
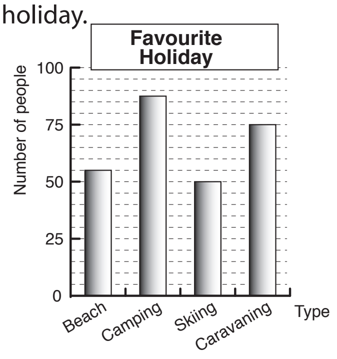
Pr(event) = ... $=$ ☐

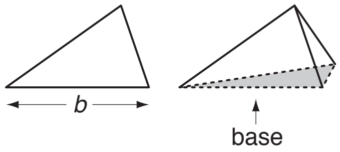
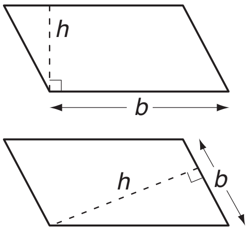
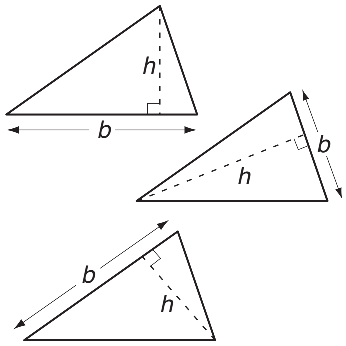
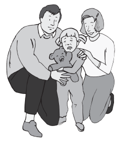
GLOSSARY

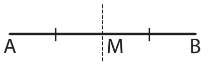
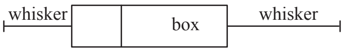
TERMS	DEFINITIONS	EXAMPLES
accuracy	• How close the result of a measuring comes to the true value.	3.14 is a fairly accurate estimation of π.
acute angle	• An *angle* measuring less than 90°.	
acute-angled triangle	• A *triangle* in which all *angles* measure less than 90°.	
add (+)	• To join together.	If you add 1 black cow and 2 white cows, there are $1 + 2 = 3$ cows all together. 
addition	• The *operation* of finding the total or sum of two or more numbers to make one number. • The result is called the *sum* or *total*.	Adding 15 and 6 we reach a total (sum) of 21. $15 + 6 = 21$
addition law of probability	• A method for finding the *likelihood* that either or both of two *events* occur.	Addition rule for mutually exclusive events (either not both): P(A or B) = P(A) + P(B) Addition rule for non exclusive events: P(A or B) = P(A) + P(B) − P(A and B)
adjacent	• Immediately next to.	The Daniher's live adjacent to the Bourke's. 
adjacent angles	• Two angles that have a common side and a common vertex but have no interior points.	∠ABC is adjacent to ∠CBD. 

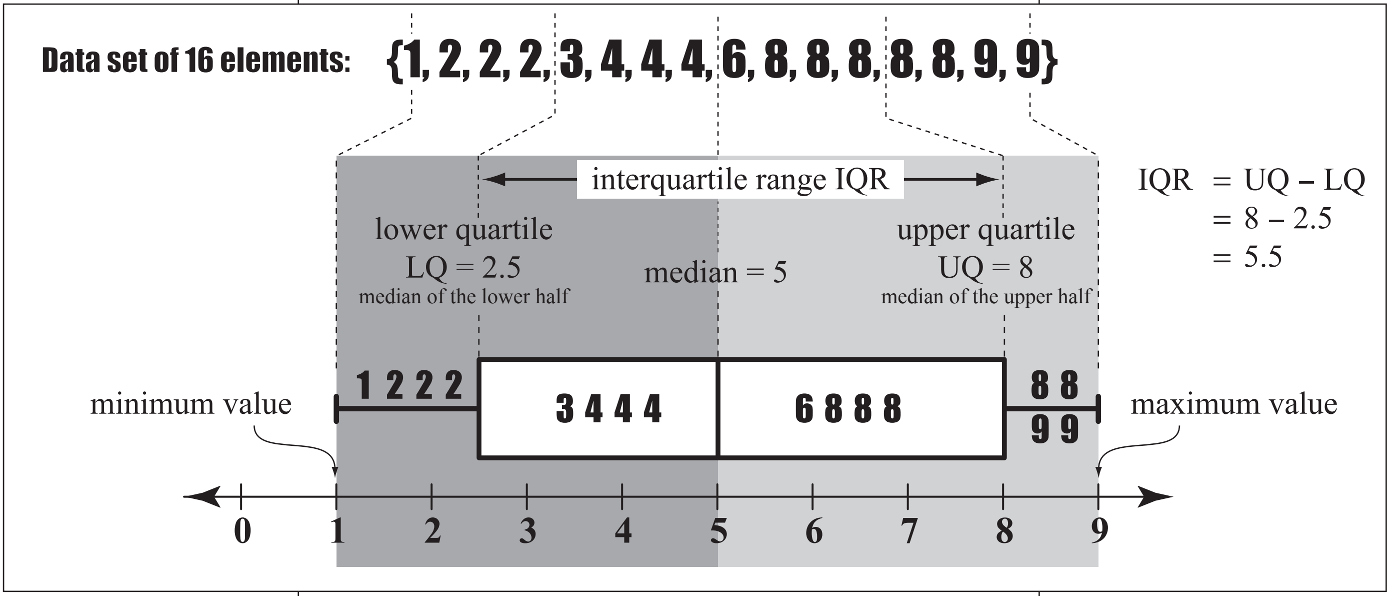
adjacent sides	• *Sides* immediately next to each other in a shape.	Sides *a* and *b* are adjacent to each other. 
algebra	• A branch of Mathematics where numbers are represented by letters or symbols, called *pronumerals* or *variables*.	$x + x = 6$, so x equals 3 ♣ ÷ 3 = 12, so ♣ equals 36
alternate exterior angles	• Angles in the *opposite*, outside corners when a *transversal* crosses a set of *parallel lines*. • Alternate exterior angles are equal.	
alternate interior angles	• Angles in the *opposite*, inside corners when a *transversal* crosses a set of *parallel lines*. • Alternate interior angles are equal.	One set of alternate interior angles measure 64°, the others are marked and measure 116°. 
altitude	• *Height* above sea level.	The plane flies at an altitude of 10 000 metres.
am (**a**nte **m**eridiem)	• The *time* from midnight to midday (morning).	
analogue clock	• A clock or watch that has rotating hands and shows 12 *hour time*.	
angle	• The amount of turning between two straight *lines* that are fixed at a *point*. • An angle is measured in *degrees*.	
angle at the centre of a circle	• An *angle* with the corner in the *centre* of a *circle*.	

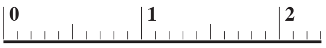
angle at the circumference of a circle	• An *angle* with the corner on the *circumference* of a *circle*.	
angle θ (theta)	• 'θ' (theta) is a letter of the Greek alphabet often used to label an *angle* in *trigonometry*.	
annual	• Happening *once* a *year*.	
annual interest rate	• The *percentage* of the *principal* you earn or pay each year. The principal is an amount of money that is deposited or borrowed.	If you deposit $100 into a savings account that pays 4% interest rate per year, then you earn $4 in interest that year.
anticlockwise	• Moving in the *opposite direction* to the hands on a clock.	
approximate	• Very close to the actual size. • To *estimate* by *rounding off*.	If you have $24.85 in your wallet, you can say you have approximately $25.00.
arc of a circle	• A section of the circumference of a *circle*. It can be measured by the size of the *angle* at its centre or by the length of the arc itself.	
area	• The amount of surface covered by a *2D shape*. • Area is measured in *square units*, e.g. square centimetres (cm^2) or square metres (m^2).	The area of a rectangle is calculated by multiplying length (*l*) by width (*w*): $A = lw$ $= 4 \times 2$ $= 8$ Area = 8 square units 
ascending order	• Arranged from smallest to largest. • Becoming larger, greater or higher.	3, 5 and 7 are in ascending order.

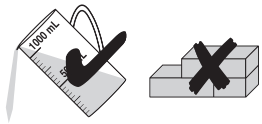
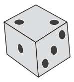
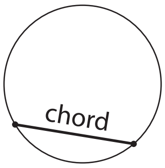
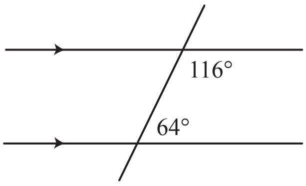
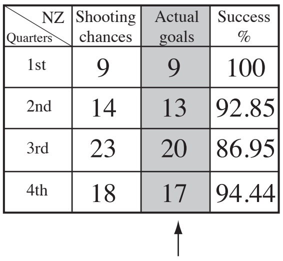
<table>
<tr>
<td>associative property
(of addition and multiplication)</td>
<td>• Rule: When <i>adding</i> or <i>multiplying</i>, no matter how the numbers are grouped, the answers will always be the same.</td>
<td>$a + (b + c) = (a + b) + c$ “+”
$1 + (3 + 4) = (1 + 3) + 4$
$8 = 8$
$a \times (b \times c) = (a \times b) \times c$ “×”
$1 \times (3 \times 4) = (1 \times 3) \times 4$
$12 = 12$</td>
</tr>
<tr>
<td>average</td>
<td>• Or <i>mean</i>, is the total of all scores divided by how many scores there are.
• The number that could be used to replace every number in a set of numbers without changing the total <i>sum</i> for the set.</td>
<td>The average of 5, 7 and 9 is 7.
$5 + 7 + 9 = 21$ and
$21 \div 3 = \mathbf{7}$
So $7 + 7 + 7 = 21$</td>
</tr>
<tr>
<td>average speed</td>
<td>• See <i>speed</i>.</td>
<td></td>
</tr>
<tr>
<td>axis of symmetry</td>
<td>• (pl. axes) See <i>line of symmetry</i>.</td>
<td>
 </td>
</tr>
<tr>
<td>back-to-back stem-and-leaf plot</td>
<td>• A diagram displaying <i>data</i> by <i>place value</i>.
• See <i>stem-and-leaf plot</i></td>
<td>Data A: 5, 18, 18, 19, 19, 21
Data B: 5, 17, 17, 18, 20, 21, 30,
<table>
<tr><th>B</th><th>Stem</th><th>A</th></tr>
<tr><td>5</td><td>0</td><td>5</td></tr>
<tr><td>8 7 7</td><td>1</td><td>8 8 9 9</td></tr>
<tr><td>1 0</td><td>2</td><td>1</td></tr>
<tr><td>0</td><td>3</td><td></td></tr>
</table></td>
</tr>
<tr>
<td>backwards</td>
<td>• Away from your front.
• In reverse of the usual way.</td>
<td></td>
</tr>
<tr>
<td>balance (money)</td>
<td>• The amount of money remaining in a bank account after all transactions have been completed.</td>
<td>The bank account held $32. After $12 was withdrawn the balance of the account was $20.</td>
</tr>
<tr>
<td>bar graph</td>
<td>• A graph using <i>columns</i> to show quantities or numbers so they can be easily compared.</td>
<td>Camping is the favourite holiday.

 </td>
</tr>
</table>

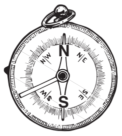
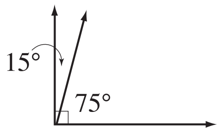
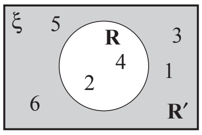
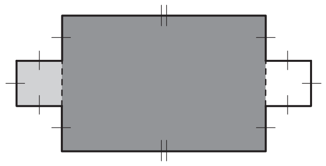
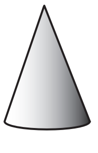
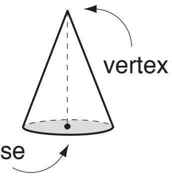
base	• A *line* or surface on which a figure stands.	
base income (money)	• The gross income, for a specified period of *time*, that does not include bonus or overtime income.	
base of a parallelogram	• The base (b) of a *parallelogram* is the *length* of any of its *sides*.	
base of a triangle	• The base (b) of a *triangle* is the *length* of any of its *sides*.	
basic numeral	• Numbers written in their *simplest form*. • *Whole numbers* and *decimal* numbers are basic numerals. • *Fractions*, *percentages*, *powers*, *square roots*, etc. are not basic numerals because an *operation* can be performed to *simplify* them.	9 and 1.8 are basic numerals. $\frac{5}{8}$, 35%, 7^4 and $\sqrt{12}$ are not basic numerals.
between	• At a place bounded by two or more places.	The child is between her parents. 
bi	• (or **di**) Prefix meaning two.	A bicycle has 2 wheels. 
binomial	• A *polynomial* with two *terms*.	$a+3b, 3gh-2g, x^2+3x$ are all binomials.
binomial factors	• *Binomials* written as a *product*. Some *quadratic trinomials* are the product of two binomial factors.	binomial factor trinomial $(x+2)\,(x+1)=x^2+3x+2$ binomial factor

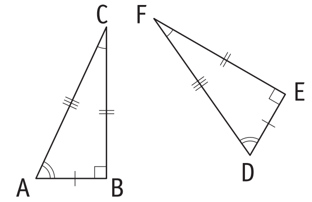
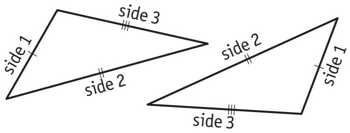
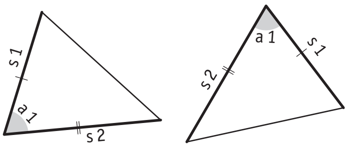
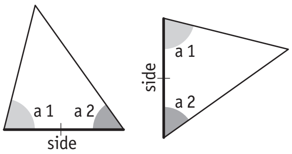
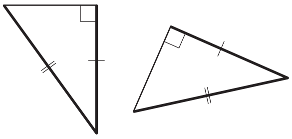
bisect	• To split into two *equal* parts.	$\overline{AM} \equiv \overline{MB}$ 
BODMAS	• The order of operations rule - ***B**rackets, **O**rders (Powers and square roots), **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction.*	See *Order of operations*
box-and-whisker plot	• A *graph* that shows the distribution of a *set* of *data*. • It displays the *median*, *upper quartile*, *lower quartile*, *maximum* and *minimum* values.	

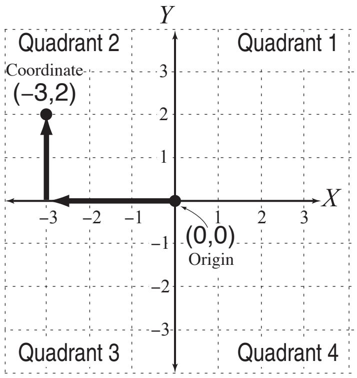
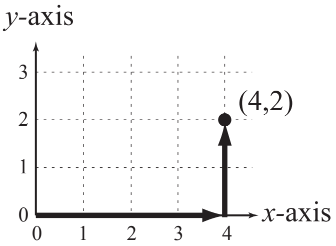
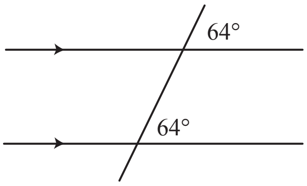
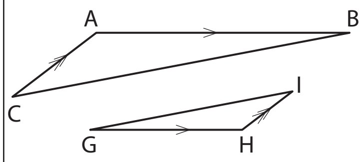
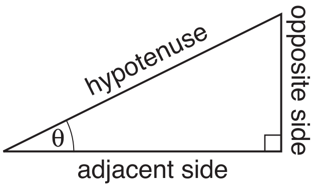


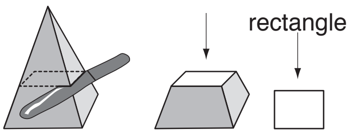
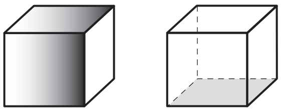
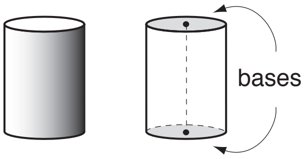
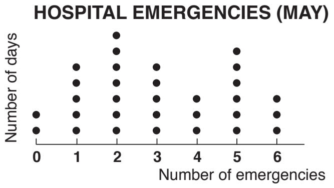
brackets ( )	• A *pair* of symbols used to enclose a mathematical *expression*.	$(12 - 4) \div 2 = 4$ Brackets group 12 take away 4.
calculate	• To work something out.	$3 + 5 + 6 = 14$ 
calendar	• A *time* chart that tells us what *day*, *week*, *month* and *year* it is.	
calibration	• A mark on a *scale*.	
cancel	• To strike out an *equal term* on each side of an *equation*.	$x - 3 = 6$ cancel −3 by adding 3 to both sides of the equation $x \cancel{-3} \cancel{+3} = 6 + 3$ $-3 + 3 = 0$ $x = 9$

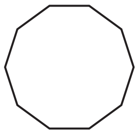
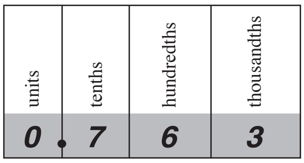
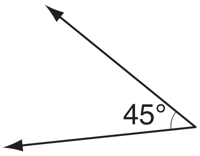
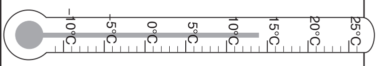
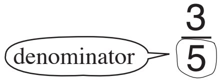
<table>
<tr><td>capacity</td><td>• Or <i>volume</i>, is the measure of the amount of liquid a container can hold.</td><td>A jug has capacity because it can hold liquid, a brick does not.
 </td></tr>
<tr><td>Cartesian plane</td><td>• See <i>coordinate plane</i>.</td><td></td></tr>
<tr><td>chance</td><td>• The likelihood that a particular result or <i>outcome</i> will occur.</td><td>The chance of rolling a 2 with a standard die is 1 in 6.
</td></tr>
<tr><td>chord</td><td>• A <i>line segment</i> on the <i>interior</i> of a <i>circle</i>. A chord has both end points on the <i>circumference</i> of the circle.</td><td> </td></tr>
<tr><td>closest</td><td>• Nearest to.</td><td>The son is closest to the mother.
</td></tr>
<tr><td>co-interior angles</td><td>• Angles in the <i>adjacent</i> corners when a <i>transversal</i> crosses a set of <i>parallel lines</i>.
• Co-interior angles add to 180°.</td><td> </td></tr>
<tr><td>coefficient</td><td>• The number which multiplies a <i>variable</i>.</td><td>3 is the coefficient of $3x$
6 is the coefficient of $6y^4$</td></tr>
<tr><td>column</td><td>• A <i>vertical</i> line of <i>data</i> in a table.</td><td>Netball: Aust v NZ

<table>
<tr><th>NZ / Quarters</th><th>Shooting chances</th><th>Actual goals</th><th>Success %</th></tr>
<tr><td>1st</td><td>9</td><td>9</td><td>100</td></tr>
<tr><td>2nd</td><td>14</td><td>13</td><td>92.85</td></tr>
<tr><td>3rd</td><td>23</td><td>20</td><td>86.95</td></tr>
<tr><td>4th</td><td>18</td><td>17</td><td>94.44</td></tr>
</table></td></tr>
<tr><td>combinations</td><td>• A selection of objects from a collection. Order is irrelevant.</td><td>A class committee is a combination of 2 boys and 2 girls chosen from a total of 12 boys and 15 girls.</td></tr>
<tr><td>commission
(money)</td><td>• The pay or <i>percentage</i> paid to an agent.</td><td>5% of the sale price of $100 000 goes to the agent in commission. The agent gets $5000.</td></tr>
</table>

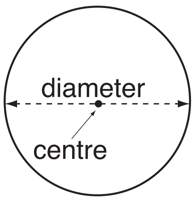
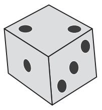
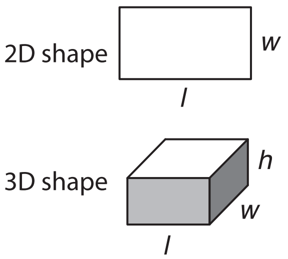
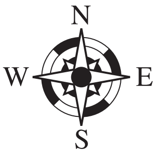
common factor	• A *whole number* that is a *factor* of two or more non-zero whole numbers.	The common factors of 18 and 24 are 1, 2, 3 and 6.
common multiple	• A *whole number* that is a *multiple* of two or more non-zero *whole numbers*.	The common multiples of 5 and 6 are 30, 60, 90, ...
commutative property (of addition and multiplication)	• Rule: When *adding* or *multiplying*, no matter how the numbers are ordered, the answers will always be the same.	$a + b = b + a$ $1 + 3 = 3 + 1$ $4 = 4$ “+” $a \times b = b \times a$ $3 \times 4 = 3 \times 4$ $12 = 12$ “×”
compass	• An instrument that shows *direction*.	 
complement of an angle	• An *angle* that, when added to the first angle, makes a *right angle* (or 90° in total).	75° is the complement of 15°, because 75° + 15° = 90° 
complementary event (′)	• The opposite of an event. The *set* of all outcomes that are not included in the *event*.	The event is to roll a die and R = {2, 4} is the result. The complement of event R is R′. R′ = {1, 3, 5, 6} 
composite shapes	• A combination of two or more *2D* shapes into one figure.	 The above diagram is the composite of 3 rectangular shapes.
compound interest	• Interest calculated, not only on the original *principal*, but also on the interest that has been added at the end of each pay period.	Interest calculated grows the principal amount which in turn grows the interest calculated.
cone	• A *solid* with one circular base and one *vertex*.	 

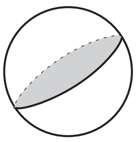
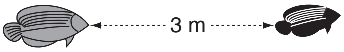
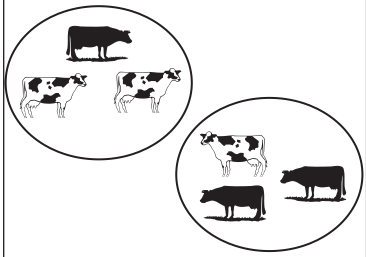
congruent shapes	• Have exactly the same size and shape.	Triangles ABC and DEF are congruent.  **Sides** Corresponding sides are congruent: $\overline{AB} \equiv \overline{DE}$, $\overline{BC} \equiv \overline{EF}$, $\overline{AC} \equiv \overline{DF}$ **Angles** Corresponding angles are congruent: $\angle A \equiv \angle D$, $\angle B \equiv \angle E$, $\angle C \equiv \angle F$
congruence tests for triangles	• Methods used to determine if *triangles* are the same shape and size.	
1. Side-side-side (**SSS**)	If two triangles have: three pairs of sides the same length then the triangles are congruent.	
2. Side-angle-side (**SAS**)	If two triangles have: two pairs of sides the same length and the angle formed by the two sides is the same then the triangles are congruent.	
3. Angle-side-angle (**ASA**)	If two triangles have: two pairs of angles the same and the pair of sides which are in between the two angles the same length then the triangles are congruent.	
4. Right angle-hypotenuse-side (**RHS**)	If two right-angled triangles have: their hypotenuses and a pair of sides the same length then the triangles are congruent.	
consecutive numbers	• Numbers that follow each other.	4 and 5 are consecutive numbers. 
constant term	• A *term* that has a fixed value and does not contain a *variable*.	Opposite to a *variable*. In $y = x + 5$ 5 is constant x and y are variables. The speed of light in a vacuum (c) is a constant. $c = 299\,792\,458$ m/s
conversion factor	• The amount that you *multiply* or *divide* a number by to change it to a different *unit of measurement*.	1 m = 100 cm The conversion factor for changing metres to centimetres is 100.

convert	• Change from a *unit* to another.	90 km/h can be converted to a 25 m/s.
coordinate plane	• A *plane* divided into four *quadrants* by a *horizontal line* called the *x-axis* and a *vertical line* called the *y-axis*.	
coordinates	• An *ordered pair* of numbers that locate a *point* on a *coordinate plane*. • The *first* number tells you how far from the *origin* to move along the *x-axis*. The *second* tells you how far from the origin to move along the *y-axis*. • They are written in *brackets* with a comma in between.	(4,2) are the coordinates of a point located 4 units to the right and 2 units upward from the origin (0,0). 
corresponding angles	• Angles in the matching corners when a *transversal* crosses a set of *parallel lines*. • Corresponding angles are *equal*.	
corresponding sides	• Two *sides* that are situated the same way in different geometric shapes.	$\overline{AC}$ corresponds to $\overline{HI}$ 
cosine	• A *trigonometric* function. • In a *right-angled triangle*, the cosine of an *acute angle* is the *ratio* of the length of the side *adjacent* to the angle, to the length of the *hypotenuse*.	$\cos\theta = \dfrac{\textbf{A}\text{djacent}}{\textbf{H}\text{ypotenuse}}$ 
counting number	• Any of the *whole numbers* from zero onwards.	0, 1, 2, 3, 4, 5...... are counting numbers.

cross multiply	• To simplify a *proportion*, written as two *equal fractions* OR • To *multiply* each *numerator* by the *denominator* of the fraction across from it.	$a:b = c:d$ $\frac{a}{b} \times \frac{c}{d}$ $a \times d = b \times c$ $ad = bc$
cross-section	• The *shape* of the *face* that results when an object is cut through.	
cross simplify	• To *divide* the *diagonal* numbers (when *multiplying* two *fractions*) by the same number to reduce their value before multiplying.	$\frac{3}{4} \times \frac{8}{9} = \frac{\overset{\div 3}{3}}{\underset{\div 4}{4}} \times \frac{\overset{\div 4}{8}}{\underset{\div 3}{9}} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$
cube	• A *solid* with six identical *square* faces.	
cubed	• A number cubed is the third *power* of the number.	5 cubed $= 5^3 = 5 \times 5 \times 5 = 125$
cubic unit	• A unit of *volume* expressed in cubic form.	The volume of a solid is measured in the appropriate cubic units, e.g. cm^3 or m^3.
cylinder	• A *solid* with two *parallel* circular *bases* of the same size.	
data	• Collection of information that can include facts, numbers or measurements.	
day	• A *unit* of *time* equal to 24 *hours*.	A day starts and ends at midnight. 
daylight saving time	• Use of fictitious time in the summer months that prolongs light in the evening hours.	During daylight saving clocks are one hour ahead of real time.
deca	• Prefix meaning ten.	Decathlon is an athletics contest with ten events.

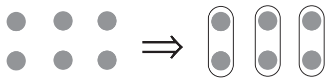
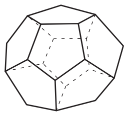
decade	• A *unit* of *time* equal to 10 *years*.	2011 to 2020 is a decade.
decagon	• A shape with 10 *sides*.	
decimal number	• A number based on the ten *place value* system where a *decimal point* separates the *units* and *tenths*.	The decimal number 4.3 represents: 4 - ones 3 - tenths OR 4 and 3 tenths.
decimal place		7 is in the tenths place. 6 is in the hundredths place. 3 is in the thousandths place.
decimal point (.)	• A point that separates the *units* and *tenths* in a *decimal number*.	2.5 is a decimal number where the 2 and the 5 are separated by a decimal point.
decrease	• To make smaller.	8 must decrease by 5 to become 3.
deduct	• To take away.	If you deduct 1 from 3 there are 2 left. 3 – 1 = 2
degree (°)	• A *unit* used to measure the amount of turn in an *angle*.	Angle measures 45°. 
degrees Celsius (°C)	• A *unit* used to measure temperature.	The thermometer shows 14°C. 
denominator	• The number below the fraction bar in a *fraction*. • The number of equal parts in one whole.	Considering fifths, 5 parts would make a whole. 
deposit (money)	• To pay an amount of money into a bank account.	A deposit of $15 into a bank account with a balance of $25 will increase the account balance to $40.

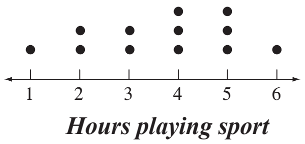
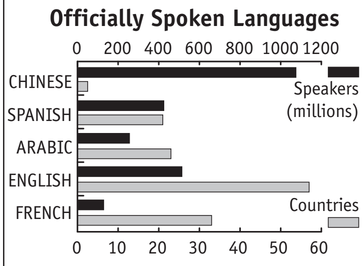
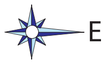
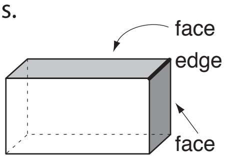
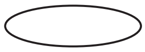
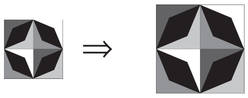
descending order	• Arranged from largest to smallest. Becoming smaller, lesser or lower.	8, 6 and 2 are in descending order.
diagonal	• A straight *line* inside a *polygon* joining any two *vertices* that are not next to each other.	
diameter of a circle	• A *segment* that passes through the *centre* of a *circle* and has both endpoints on the circle. The diameter of a circle is twice the length of its *radius*.	
die	• (pl. **dice**) A numbered *cube* that is used in games. A standard die has 1 to 6 pips (dots) on each *face* with *opposite* faces adding to 7.	
difference	• The result when a number is *subtracted* from another number. • The amount by which one number is bigger or smaller than another number.	The difference between 5 and 3 is 2. $5 - 3 = 2$
digit	• Any of the first ten *whole numbers* from 0 to 9.	There are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9.
digit sum	• The *sum* of the *digits* in a number.	124 has a digit sum of 7. $1 + 2 + 4 = 7$
digital clock	• A clock that uses only numbers to show the *time*. (No hands!)	
dimension	• A measure of size. A *two-dimensional* shape (2D shape) has *length* and *width*. A *three-dimensional* shape (3D shape) has *length*, *width* and *height*.	
direction	• The way something is placed, pointing or moving.	North, east, south, west, up, down, sideways, backwards and forwards. 

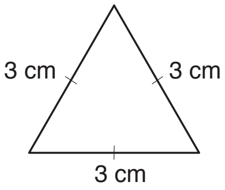
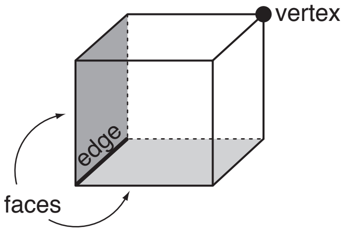
disc	• The union of a *circle* and its interior.	The cross section of a sphere is always a disc. 
discount (money)	• An amount *subtracted* from the regular price of an item to get the sale price.	When $80 track shoes are on sale at 25% off ⇒ discount = 25% of $80 = $20.
distance	• The *length* between two *points*.	The distance between the fish is 3 metres. 
distribution	• To *multiply* out parts of an *expression*. The *opposite* to factorisation. • How objects are spaced.	See *distributive property*.
distributive property (of multiplication over addition and subtraction)	• Rule: You can *multiply* a *sum* (or a *difference*) by multiplying the number outside the brackets by each term of the sum (or the difference).	$a(b + c) = ab + ac$ “+” $2 \times (4 + 3) = 2 \times 4 + 2 \times 3$ $14 = 14$ $a(b - c) = ab - ac$ “–” $2 \times (4 - 3) = 2 \times 4 - 2 \times 3$ $2 = 2$
divide (÷)	• To share into groups.	These 6 cows are divided into 2 groups.  6 ÷ 2 = 3 in each group
dividend	• The first number written in a *division*. It is the number being divided. In a *fraction* the dividend is the *numerator*.	In the division: 144 ÷ 9 = 16 the number 144 is called the dividend.
divisible	• Can be divided without a *remainder*.	20 ÷ 2 = 10 with 0 remainder. So 20 is divisible by 2 and 10.

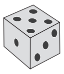
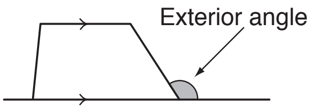
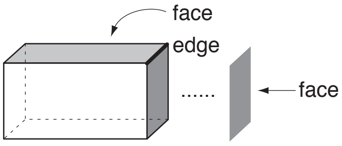
divisibility tests	• Checks performed to help find the *factors* of a number.	

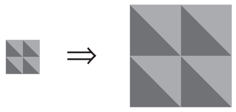
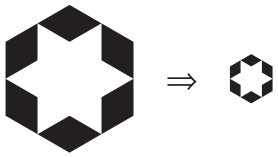
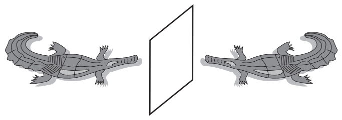
Divisibility tests (factor tricks)	Examples
2 is a factor of all even numbers.	Numbers that end with 0, 2, 4, 6 and 8, e.g. 754, 120
3 is a factor of all numbers with a digit sum that is divisible by 3.	252 has 3 as a factor because 2 + 5 + 2 = 9 and 9 is divisible by 3.
4 is a factor of all numbers where the last two digits are divisible by 4.	1532 has 4 as a factor because 32 is divisible by 4.
5 is a factor of all numbers whose last digit is a 5 or a 0.	120 and 4935 both have 5 as a factor.
6 is a factor of all numbers that have both 2 and 3 as factors.	102 has 6 as a factor because 2 and 3 are both factors.
9 is a factor of all numbers with a digit sum that is divisible by 9.	1764 has 9 as a factor because 1 + 7 + 6 + 4 = 18 and 18 is divisible by 9.
For **11 to be a factor** of a number, the difference between the sum of the even placed digits and the sum of the odd placed digits must be divisible by 11.	81 917 has 11 as a factor because 1 + 1 = 2 8 + 9 + 7 = 24 and 24 − 2 = 22 which is divisible by 11.
For **10, 100, 1000 .... to be a factor** of a number, that number must end in one 0 or two 0's or three 0's, etc.	270 has 10 as a factor, 1400 has 100 as a factor etc.

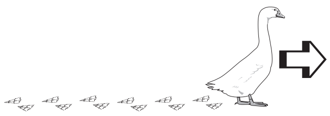
division	• The *operation* of sharing or grouping a number into *equal* parts.	The division 6 ÷ 2 = 3 means: How many groups of 2 can 6 be divided into? OR How many groups of 2 can be taken from 6 before none remain? ⇒3 groups of 2. 
divisor	• The *second* number written in a *division*. • It is the number that will divide the *dividend*. In a *fraction* the divisor is the *denominator*.	$8 \div \boxed{4} = 2$ OR $\frac{8}{\boxed{4}} = 2$ divisor
dodecahedron	• A regular *solid* in which all twelve *faces* are *regular pentagons*.	 

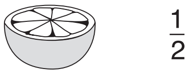
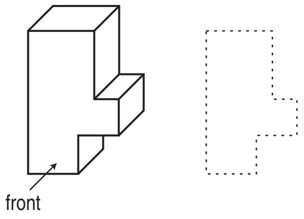
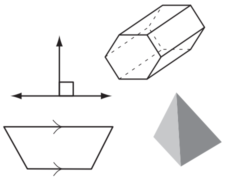
dot plot	• A *graph* showing the frequency of data, using a *number line*. • The number line has all the numbers in the *sample*. Each observation is marked with a point above the *line*.	A dot plot showing how many hours are dedicated to sport by 12 people. 
double	• *Twice* as much. • *Multiplied* by two.	Double 4 is: $4+4=8$ OR $4 \times 2=8$.
double bar graph	• A *bar graph* that shows two sets of *data* on the same graph.	
east	• A *compass direction*.	The sun rises in the east. 
edges of a solid	• The *segment* (line part) where two *faces of a solid* meet.	A rectangular prism has 12 edges. 
eighth	• The position after *seventh*.	1st, 2nd, 3rd, 4th, 5th, 6th, 7th, **8th**......
elapsed time	• The amount of time between the start time and the finish time.	The amount of elapsed time from 2:15 pm to 3:00 pm is 45 minutes.
element	• A number, letter, point, line, or any other object contained in a set.	The elements of the set $\{a, b, c\}$ are the letters a, b and c.
ellipse	• A curved shape that looks like a squashed *circle*.	The approximate orbit of the Earth around the Sun is an ellipse. 
enlargement	• To reproduce and make bigger. See *enlargement factor*.	The second object is an enlargement of the first. 

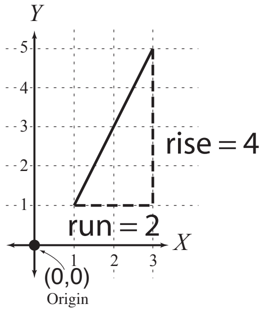
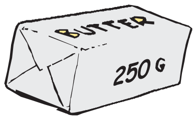
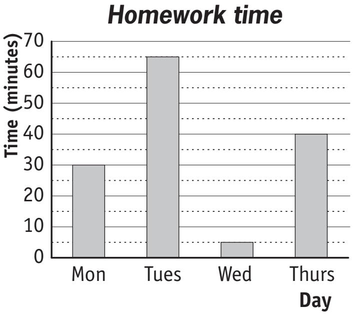
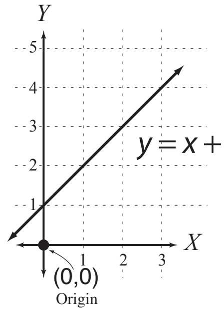
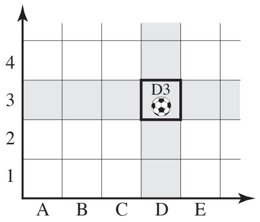
equal (=)	• Exactly the same in value or size.	$7 + 2 = 9$ 100 centimetres is equal to 1 metre: $100\text{ cm} = 1\text{ m}$
equal sets	• The *elements* of the *sets* are identical. Order does not matter.	$A = \{1, 2, 3, 4\}$ $B = \{4, 3, 2, 1\}$ $A = B$
equation	• A mathematical sentence formed by placing an *equals* sign (=) between two *expressions*.	$6 \times 2 = 9 + 3$ $4x - 5 = 0$ $2y^2 - 2 = 0$ are examples of equations.
equilateral triangle	• A *triangle* with all three *sides* of equal *length*.	
equivalent fractions	• *Fractions* that represent the same number.	$\frac{2}{16}$ and $\frac{8}{64}$ are equivalent fractions. They both equal $\frac{1}{8}$.
error	• The variation of a measurement. It may be contributed to by the *precision* of the instrument or the *accuracy* of the measured value. See *relative error.*	"My measuring may be off by 1%!"
estimate	• To make a close guess based on *rounding*.	$48 + 21 = ?$ By rounding to $50 + 20$, the estimation of the sum is 70.
Euler's formula (pronounced - 'oiler')	• In any *polyhedra*: The number of *faces* + the number of *vertices* – the number of *edges* = 2 OR $\mathbf{F + V - E = 2}$ $\mathbf{E = F + V - 2}$	faces + vertices – edges = 2 Cube: $6 + 8 - 12 = 2$ $2 = 2$ (true) 
evaluate	• To work out the value.	Evaluate: $7 \times 3 - 10 = 21 - 10$ $= 11$
even numbers	• A *whole number* that can be *divided* by two. • Even numbers end with 0, 2, 4, 6 or 8.	134 is an even number. **134 ✔** 431 is not an even number. **431 ✘**

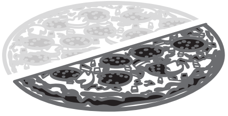
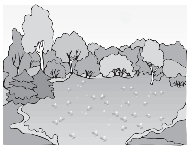
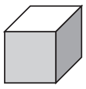
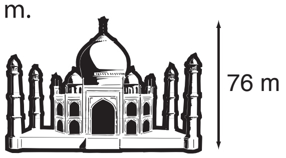
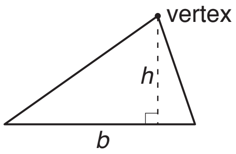
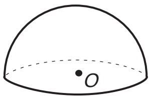
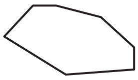
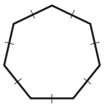
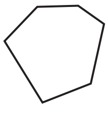
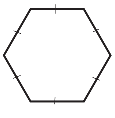
event	• A *set* of possible *outcomes* resulting from a particular *experiment*. Any *subset* of a *sample space*.	A die is rolled - Event = {5, 6} i.e. either a 5 or a 6 may result 
expand	• To *multiply* out parts of an *expression* thereby removing the *brackets*.	Using $a = 1, b = 4, c = 3$, $a(b + c) = ab + ac$ $1 \times (4 + 3) = 1 \times 4 + 1 \times 3$ $= 7$
expense (money)	• The cost involved.	You buy 3 CDs at $15 each. Your expense is $45.
experiment	• A controlled, repeatable process carried out in the study of *probability*.	Tossing a coin is an experiment. 
exponent	• A number placed to the upper right of a base number, showing how many times the base number is multiplied by itself. See *index*.	$7^4 = 7 \times 7 \times 7 \times 7 = 2401$ The exponent is 4. It is read as 'seven to the power of four'.
exponential notation	• Quantities in the form of a *base* number and an *exponent*. Exponential notation indicates what *power* is to be used and makes it easier to use multiple *factors*. See *index notation*.	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ can be more easily written using exponential notation as 3^7.
expression	• A *sequence* of numbers and/or *pronumerals* (letters) connected by *operation* signs.	$42 \div 3 - 10$ $x + 2y$ $2x^2 - 2$ are examples of expressions
exterior angle	• An *angle* that is the *supplement* of an *interior angle* of any *polygon*.	
faces of a solid	• *Polygons* that join on their *edges* to form a *solid*.	A rectangular prism has 6 rectangular faces. 
factor	• When *whole numbers*, other than zero, are multiplied together, each number is a factor of the *product*. OR A whole number that divides exactly into another number. See *divisibility tests*.	Because $1 \times 12 = 12$ $2 \times 6 = 12$ and $3 \times 4 = 12$ 1, 2, 3, 4, 6 and 12 are all factors of 12.

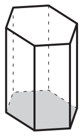
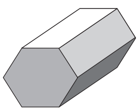
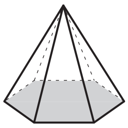
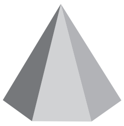
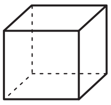
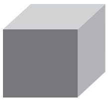
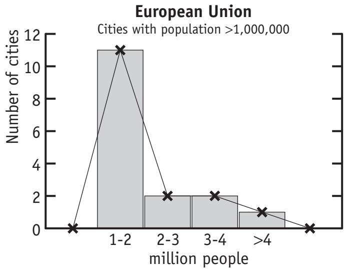
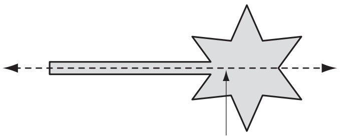
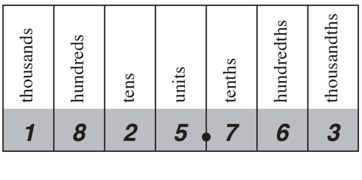
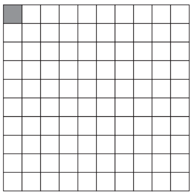
factor of enlargement	• The amount by which an object is made bigger.	The enlargement factor is 3. 
factor of reduction	• The amount by which an object is made smaller.	The reduction factor is 3. 
factor tree	• A diagram that shows all possible *factors* on the different branches of a 'tree'. It is used to find the *prime factors* of a number.	The prime factors of 12 are 2 and 3. 12 = 2 × 6 = 2 × 2 × 3
factorial (!)	• The *product* of a *whole number multiplied* by every number between itself and 1.	$n! = n(n-1)(n-2)$ ..... 1 $5! = 5(5-1)(5-2)(5-3)(5-4)$ $= 5 \times 4 \times 3 \times 2 \times 1$ $= 120$
factorise	• To write a number as a *product* of its *factors*. • To write an *expression* as a product of two or more expressions.	Factorise: $3x + 15 = 3(x + 5)$ because $3 = 3 \times 1$ and $15 = 3 \times 5$
favourable outcome	• The result that you are hoping or testing for.	A die is rolled. Event = {numbers >2} Fav. outcomes = {3,4,5,6}
fifth	• The position after *fourth*.	1st, 2nd, 3rd, 4th, **5th**......
finite	• With limits. Able to be counted.	There are a finite number (12) of months in the year.
first	• Placed before anything else.	The first athlete to cross the finish line won the gold medal.
flip	• To turn across a line so the result is a mirror image. See *reflection*.	
formula	• (pl. **formulae**) A mathematical rule, usually an *equation*, describing a relationship between two or more quantities. For example, the formula describing the *area* of a *circle* is $A = \pi r^2$ where A is the symbol for the area and r is the *symbol* for the *radius*.	Find the area of a circle of radius 10 cm, using $\pi \approx 3.14$ Use the formula $A = \pi r^2$ and substitute $r = 10$ $A = 3.14 \times 10^2$ $= 3.14 \times 100$ $= 314\ \text{cm}^2$

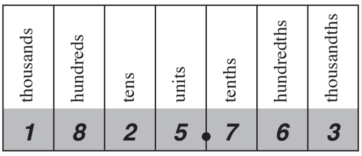
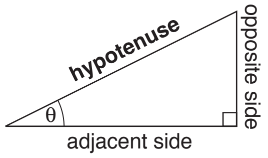
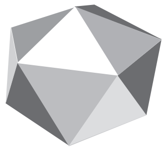
<table>
<tr><td>fortnight</td><td>• A unit of time equal to 2 whole weeks or 14 days.</td><td> </td></tr>
<tr><td>forwards</td><td>• In the direction of your front.
• The usual way.</td><td></td></tr>
<tr><td>fourth</td><td>• The position after third.</td><td>1st, 2nd, 3rd, 4th......</td></tr>
<tr><td>fraction</td><td>• Part of a group.
• Part of a whole.
• A number in the form $\frac{a}{b}$ ($b \neq 0$) where a is the numerator and b is the denominator.
• Fractions can be proper fractions or improper fractions.</td><td>5 out of 8 dots are circled.

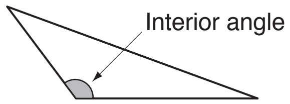
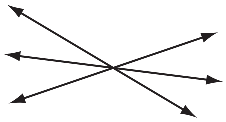
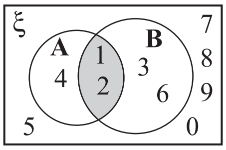
1 half of a whole orange.
 </td></tr>
<tr><td>frequency (f)</td><td>• How often a particular item occurs in a set of data.</td><td>The frequency of a's in the set {m, a, t, h, s, m, a, t, e} is 2.
TRAFFIC SURVEY<table><tr><th>Vehicle</th><th>Tally</th><th>Frequency</th></tr><tr><td>car</td><td>𝍸 III</td><td>8</td></tr><tr><td>truck</td><td>I</td><td>1</td></tr><tr><td>bus</td><td>II</td><td>2</td></tr></table>"The most frequent vehicle to pass by was a car."</td></tr>
<tr><td>frequency table</td><td>• Lists items to show the number of times each event or category occurs for a set of data.</td><td><table><tr><td>Score</td><td>9</td><td>6</td><td>3</td><td>1</td><td>0</td></tr><tr><td>Frequency</td><td>3</td><td>5</td><td>4</td><td>5</td><td>6</td></tr></table></td></tr>
<tr><td>front view</td><td>• What you see of an object looking from a frontal perspective.
• Three-dimensional objects have 3 views: front, top and side.</td><td> </td></tr>
<tr><td>function (f)</td><td>• A relationship or correspondence in which values of one variable determine the values of another. $f(x)$ = rule or y = rule.</td><td>$f(x) = x^2 - 4$ or
$y = x^2 - 4$
See rule, linear function and quadratic function.</td></tr>
<tr><td>geometry</td><td>• A branch of Mathematics studying the properties and relations of lines, surfaces and solids.</td><td></td></tr>
</table>

gradient of a line	• An indication of slope determined by the vertical *rise* of a line (the change in the *y-axis* value) and the horizontal *run* of the line (the change in the *x-axis* value).	If we have a straight line $y = mx + c$ then the gradient of the line is m (the coefficient of x).  $\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$ $= \frac{4}{2} = 2$
gram (g)	• A *unit* of *measurement* for *mass equal* to 1000 *milligrams*.	250 g of butter. 
graph	• A diagram that shows a collection of *data*.	
graph of a function	• The picture obtained by plotting all the *points* of the *rule*.	
greater than (>)	• An *inequality symbol* showing which is bigger.	$10 > 2$ means 10 **is greater than** 2.
grid reference	• A pair of letters and/or numbers that describe *location* within a grid. See also *coordinates*.	The grid reference for the ball is D3. 

GST (money)	• An abbreviation for the Goods and Services Tax which is applied to certain purchases at a designated *rate*.	The standard GST in Australia is 10%. If the price of an item is $150 excluding GST then its GST inclusive price would be $165.
half	• (pl. **halves**) One of two *equal* parts expressed as a fraction.	One half is one of 2 parts of one whole pizza: 
hectare (ha)	• A *unit* of *area equal* to 10 000 square metres (100 m × 100 m).	The field measures 2 hectares. 
hedron	• (pl. **hedra**) Face.	Polyhedron - A solid object that has multiple (poly) faces. 
height	• The *vertical* distance from top to bottom.	The height of the Taj Mahal is 76 m. 
height of a triangle	• The height (*h*) is the *perpendicular* distance between the *base* (*b*) and the *vertex* opposite that side.	
hemisphere	• One half of a *sphere*.	
hepta	• Prefix meaning seven.	See *heptagon*
heptagon	• A *polygon* with 7 sides.	 Heptagon  Regular heptagon
hexa	• Prefix meaning six.	See *hexagon*
hexagon	• A *polygon* with 6 sides.	 Hexagon  Regular hexagon

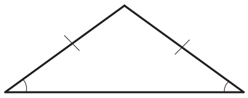
hexagonal prism	• A *three-dimensional* shape. Two identical *bases* are *hexagons*. Six *faces* are *rectangles*.	 
hexagonal pyramid	• A *three-dimensional* shape. The *base* is a *hexagon*. Six faces are *triangles*.	 
hexahedron	• A *regular solid*. Six faces are *square (cube)*.	 
highest common factor (HCF)	• The largest number that is a *factor* of all the given numbers.	Factors of 12: 1, 2, 3, 4, 6, 12 Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30 12 and 30 have a HCF of 6.
histogram	• A *vertical bar graph* used to represent the *frequency* of individual scores.	
horizontal line	• *Parallel* to the horizon.	
horizontal symmetry	• A shape has horizontal symmetry if an *axis of symmetry* is horizontal.	
hour (h)	• A *unit* of *time equal* to 60 *minutes*.	One hour is the amount of time between 1 o'clock and 2 o'clock.
hundreds	• The *place value* between *tens* and *thousands*.	1825.763 has 8 hundreds. 
hundredth	• One part out of 100 parts of one whole.	

hundredths	• The *place value* between *tenths* and *thousandths*.	1825.763 has 6 hundredths.  thousands hundreds tens units tenths hundredths thousandths 1 8 2 5 • 7 6 3
hypotenuse	• The side *opposite* the *right angle* of a *right-angled triangle*. • The longest side of a right-angled triangle.	
icosahedron	• A *regular solid* in which all twenty *faces* are *equilateral triangles*.	
identity element (for addition)	Rule: The *sum* of any number and zero equals that number. • Zero is the identity element for *addition*.	$a + 0 = a$ *OR* $0 + a = a$ $3 + 0 = 3$ $0 + 3 = 3$
identity element (for multiplication)	• Rule: The *product* of any number and one equals that number. • One is the identity element for addition.	$a \times 1 = a$ *OR* $1 \times a = a$ $3 \times 1 = 3$ $1 \times 3 = 3$
improper fraction	• Any *fraction* in which the *numerator* is greater than or equal to the *denominator*.	$\frac{9}{8}$ the numerator is 9 the denominator is 8. $9 \geq 8$ so $\frac{9}{8}$ is an improper fraction.
increase	• To make larger or grow in size.	8 must increase by 5 to get to 13.
independent event	• An *event* that is totally unaffected by whether or not another event does or does not occur.	The toss of the first coin has no effect on the probability of the resulting head or tail on the second toss.
index	• (pl. **indices**) A number placed to the upper right of a base number, showing how many times the base number is multiplied by itself.	$7^4 = 7 \times 7 \times 7 \times 7 = 2401$ The index is 4. It is read as 'seven to the power of four'.
index notation	• Quantities in the form of a *base* number and an *index*. Index notation indicates what *power* is to be used and makes it easier to use multiple *factors*.	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ can be more easily written using index notation as 3^7.
inequality	• See *inequation*.	

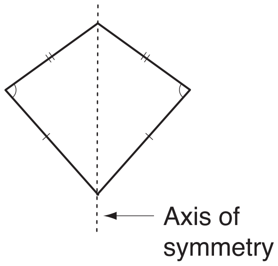
inequality symbols	• Symbols that tell us how the two objects or *expressions* in a mathematical sentence are not *equal*.	$<, >, \leq$ and $\geq$ are inequality symbols.
inequation	• A mathematical sentence that shows the relative size of two objects or *expressions*. • Uses the *inequality symbols*: $<, >, \leq$ or $\geq$	$12 > x$ is an inequation. twelve **is greater than** *x*.
infinite (∞)	• Has no limits. Unable to be counted. • The symbol for infinity is (∞).	There are an infinite number of integers: ..... −3, −2, −1, 0, 1, 2, 3 .....
integer (ℤ)	• Any *negative number*, zero or *positive number*.	−3, −2, −1, 0, 1, 2, 3 are integers. 3.5 and $5\frac{2}{3}$ are not integers.
interquartile range (IQR)	• The *difference* between the *upper quartile* (UQ) and the *lower quartile* (LQ) of a set of *data*. • Used to describe the spread of a set of data.	Data: 2, 2, 3, 3, 4, 5, 7, 8, 9, 9 The lower quartile (LQ) is 3. The upper quartile (UQ) is 8. IQR = UQ − LQ = 8 − 3 = 5 See *box-and-whisker plot*.
interest (money)	• The amount of money paid for the use of money.	See *simple interest*.
interior angle	• An *angle* inside a *polygon*.	
intersecting lines	• *Lines* that meet at a *point*.	
intersection of sets (∩)	• *Elements* common to all sets. • The symbol for intersection is ∩.	 A intersection B $A \cap B = \{1, 2\}$
invalid	• Not a logical result.	If $a = b$ and $a = 4$ saying $b = 5$ would be invalid.
inverse of a number	• That number which *cancels* out a given number. • Can be called *opposite* for *addition* and *reciprocal* for *multiplication*.	The reciprocal of 3 is $\frac{1}{3}$. The opposite of 3 is −3.

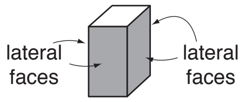
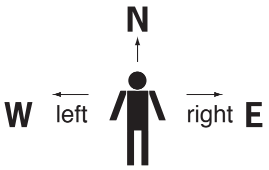
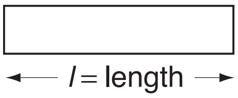
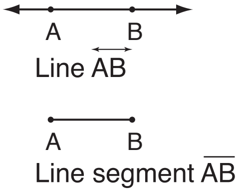
inverse of an operation	• The opposite operation. Operations that undo each other.	+ is opposite − × is opposite ÷

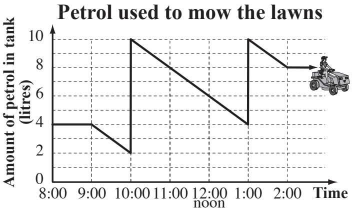
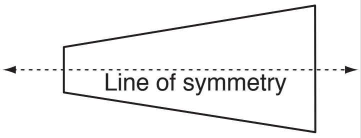
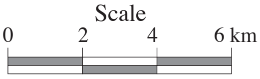
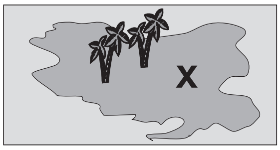
Operation $+$	Inverse Operation $-$	Operation $-$	Inverse Operation $+$	Operation $\times$	Inverse Operation $\div$	Operation $\div$	Inverse Operation $\times$
$x+3=6$ $x+3-3=6-3$ $x=3$		$x-3=6$ $x-3+3=6+3$ $x=9$		$3x=6$ $\frac{3x}{3}=\frac{6}{3}$ $x=2$		$\frac{x}{3}=6$ $\frac{x}{3}\times 3=6\times 3$ $x=18$	

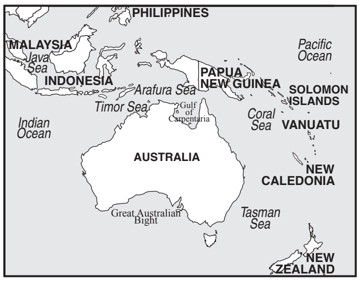
invest (money)	• To put some form of money at risk to make a *profit*.	It is common to invest in shares.
investment (money)	• The act of laying out some form of money in an enterprise to make a *profit*.	
irrational number	• A *real number* that can be written as a non-repeating or non-terminating decimal but not as a *fraction*. • Not a *rational number*.	$\pi, \phi, e, \sqrt{2}, \sqrt{3}, \sqrt{5}$, 2.6293045632.... cos 30° tan 60°
isosceles triangle	• A *triangle* with two sides of equal length.	
Karnaugh map	• A truth table that shows the combinations of *events* possible and their values.	

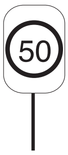
	B	B ′	Total
A	A ∩ B	A ∩ B ′	A
A′	A′ ∩ B	A′ ∩ B′	A′
Total	B	B ′	ξ

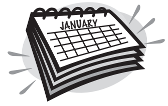
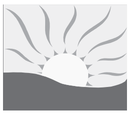
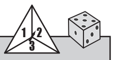
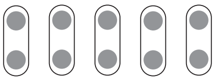
kilogram (kg)	• A *unit* of *weight equal* to 1000 *grams*.	My father weighs 75 kg. 
kilometre (km)	• A *unit* of *distance equal* to 1000 *metres*.	The distance from Melbourne to Sydney is 900 km. 
kite	• A *quadrilateral* where one *diagonal* is an *axis of symmetry*.	

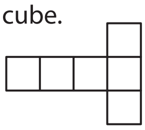
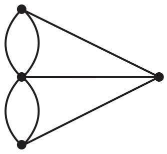
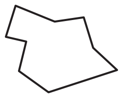
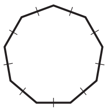
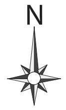
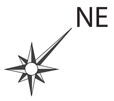
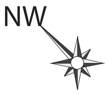
largest to smallest	• Ranking in order from the biggest to the littlest.	
lateral faces	• The *vertical* surfaces on a solid.	A rectangular prism has 4 lateral faces. 
lay-by (money)	• To pay a *deposit* on an object which is held for the buyer until the full price is paid, usually in installments.	Jacinta put the last air hockey table on lay-by. The full price was $240. She paid 25% of the price or $60 so that it would be held for her.
leap year	• A *year* with 366 *days* that falls every *fourth* year and includes the 29th of February as the extra day.	A leap year is divisible by 4. 2012 is a leap year.
left	• The *direction* to the *west* of your body if you are facing *north*.	
length	• The *distance* from one end to the other. • How long a shape is.	
less than (<)	• An *inequality symbol* showing which is smaller.	$2 < 10$ means that 2 **is less than** 10.
like terms	• *Terms* that contain the same *pronumerals* raised to the same *power*. Only the number parts of like terms can be different. Like terms may be added, subtracted, multiplied or divided. *Unlike terms* may not be added or subtracted. However, they may be multiplied and divided.	• $7, \frac{6}{9}$ and -18 are like terms. • $6a, a$ and $-3a$ are like terms. • $xy^2, 5xy^2$ and $-3xy^2$ are like terms. • $7, 6a$ and $-4y^3$ are not like terms. • $5w, \frac{6}{w}$ and $-18w^2$ are not like terms.
line ($\overleftrightarrow{AB}$)	• A set of *points* which extend without end in *opposite* directions. What is commonly called a line is actually a *segment* or part of a line.	

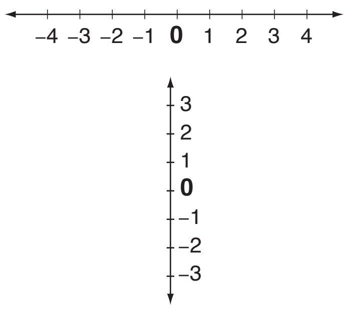
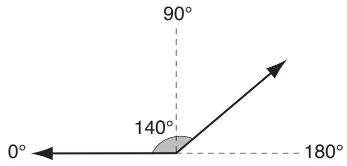
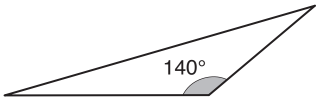
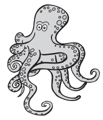
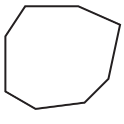
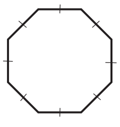
line graph	• A *graph* in which *points* representing *data* pairs are connected by line *segments*. It shows how quantities change over *time*.	
line of symmetry	• A *line* that divides a shape so that one *side* is a mirror image of the other. Both sides match exactly when folded.	
linear equation	• An algebraic *expression* in which the *variable* is in the first *power*. It can be solved for x and the value of x for which the *equation* is true is called the *solution*. The *graph* of a linear equation is always a straight *line*. See *linear function*.	$4x - 2 = x$ is a linear equation.
linear function	• A *function* in which the *variable* is only in the first *power* and has no *products*. It can be represented by an *equation* in the form of $y = ax + b$ where a and b are *real numbers*. The *graph* of this function is a straight line.	Used to describe things like the movement of a car travelling at a constant speed. $y = x + 4$ $y = -4$ $3x - 4y = 0.5$ are linear functions.
linear rule	• See *linear function*.	$y = ax + b$
linear scale	• A *scale* shown on a line. Compares the dimensions on a map to real life.	Every cm on the map represents 2 km in real life. 
litre (L)	• A *unit* of *capacity* equal to 1000 *millilitres*.	1 litre of milk. 
location	• The exact place, where something is situated.	
logic statement	• A proposition that can be classified as true or false without ambiguity. Words like ‘some’, ‘all’ or ‘no’ tend to bind the *variables* in a logical proposition.	‘It is a cloudy day’ is a logic statement. SO ‘Some days are cloudy’ is a logical proposition.

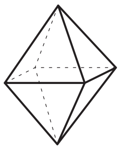
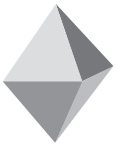
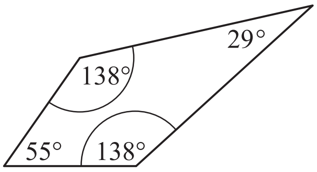
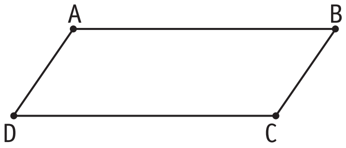
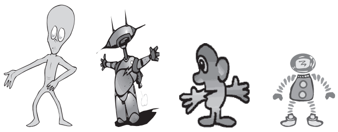
longest	• Having the biggest *length*.	The record length for the reticulated python of S-E Asia is 10 m. The specimen was found in Celebes, Indonesia in 1912. 
loss (money)	• A reduction in the value of an investment. • Expenses > Revenue	Revenue from a business activity is $20. If the expenses are $25 then the loss would be $5.
lower quartile	• Is the *median* of the lower half of scores in a set of *data*. • 25% of the data lies beneath this number.	Data: 2, 2, 3, 3, 4, 5, 7, 8, 9, 9 The lower quartile (LQ) is 3. See *box-and-whisker plot*.
lowest common denominator	• The *lowest common multiple* of the *denominators* of two or more *fractions*.	The lowest common denominator of $\frac{2}{3}$ and $\frac{4}{5}$ is the lowest common multiple of 3 and 5, which is 15.
lowest common multiple (LCM)	• The smallest of the common *multiples* of two or more non-zero *whole numbers*.	The lowest common multiple of 6 and 9 is the smallest of their common multiples 18, 36, 54 ..., so the LCM of 6 and 9 is 18.
magic square	• A square grid filled with numbers. • The *sum* of the numbers in every *row*, *column* and *diagonal* is the same.	4 9 2 3 5 7 8 1 6 Rows: $4+9+2=15$ $3+5+7=15$ $8+1+6=15$ Columns: $4+3+8=15$ $9+5+1=15$ $2+7+6=15$ Diagonals: $4+5+6=15$ $2+5+8=15$
map	• A diagram of a region showing its position in the world.	South Pacific 
mass	• The amount of matter that an object contains. It is measured in *units* like grams (g) and kilograms (kg). Often called weight, but not the same.	The mass of 3 oranges is about 1 kg. The weight of an object changes according to the gravity. A packet of butter would be weightless in space, even though it still has the same mass as on earth.

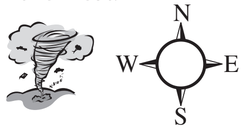
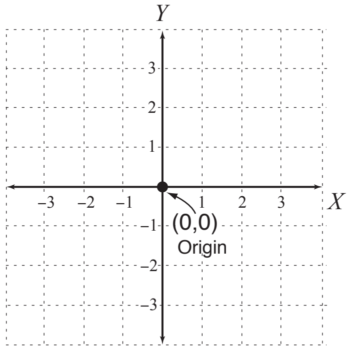
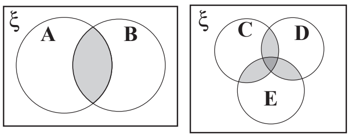
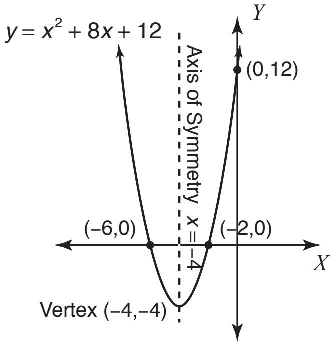
maximum	• The highest value.	The maximum speed in a residential area is 50 kilometres per hour. 
mean	• Or *average*, is the total of all scores divided by how many scores there are. • To calculate the mean: 1) *Add* up the values. 2) *Divide* the total by the number of values.	$\begin{array}{r} 4 \\ 6 \\ 5 \\ +\ 9 \\ \hline 24 \\ \hline \end{array}$ $24 \div 4 = 6$ The average or mean of 4, 6, 5 and 9 is 6.
median	• The middle value of an ordered *set* of values. • If there is an *even number* of values then the median is the *average* of the two middle numbers.	Data: 2, 5, 6, 8, 9 Median is 6 Data: 2, 3, 5, 6, 8, 8 Average the two middle values: $5 + 6 = 11$ $11 \div 2 = 5.5$ Median is 5.5
megalitre (ML)	• A *unit* of *capacity equal* to 1 000 000 *litres*.	Water tanks can hold 1 ML.
metre (m)	• A *unit* of *length equal* to 100 *centimetres*.	Track distances are measured in metres.
millilitre (mL)	• A *unit* of *capacity*. • 1000 millilitres is *equal* to 1 *litre*.	Medicines are measured in mL.
millimetre (mm)	• A *unit* of *length*. • 1000 millimetres is *equal* to 1 *metre*.	Timber length is measured in millimetres.
million	• A thousand thousands.	1 000 000
minimum	• The lowest value.	The minimum temperature reached yesterday was 25°C.
minus (–)	• Another word for *subtract*. To take away.	\$20 minus \$5 is \$15. $20 - 5 = 15$
minute (min)	• A *unit* of *time equal* to 60 *seconds*.	One minute has 60 seconds.
mixed number	• The *sum* of a *whole number* and a *fraction* less than one.	$3\frac{5}{7}$ is a mixed number.

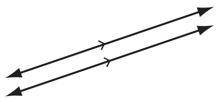
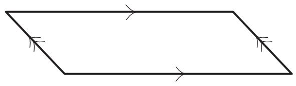
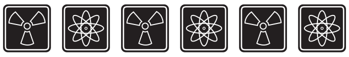
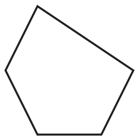
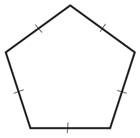
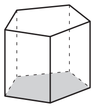
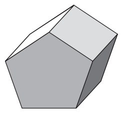
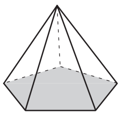
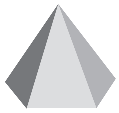
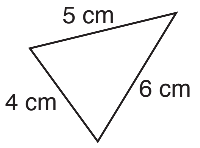
<table>
<tr><td>mode</td><td>• The most frequent score in a set of data.</td><td>Data: 2, 3, 5, 7, 7, 7, 8, 8, 9
Mode is 7 as 7 occurs 3 times.</td></tr>
<tr><td>monomial</td><td>• A <i>polynomial</i> with one <i>term</i>.</td><td>$2, -3gh, 5x^2$
are all monomials.</td></tr>
<tr><td>month</td><td>• A <i>unit</i> of <i>time equal</i> to 28, 29, 30 or 31 <i>days</i>.</td><td>There are 12 months in a year starting with January.

 </td></tr>
<tr><td>morning</td><td>• The early part of the <i>day</i> ending at 12 noon.</td><td></td></tr>
<tr><td>multiple</td><td>• A multiple of a <i>whole number</i> is the <i>product</i> of that number with any non-zero whole number.</td><td>The multiples of 2 are
2, 4, 6, 8, 10, ...
$2 \times 1 = 2$
$2 \times 2 = 4$
$2 \times 3 = 6$ etc.</td></tr>
<tr><td>multiple events</td><td>• See <i>independent events</i>.</td><td>
<table>
<tr><td rowspan="2" colspan="2"><i>Possible outcomes</i></td><td colspan="6">Die</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td rowspan="3">Spinner</td><td>1</td><td>(1,1)</td><td>(1,2)</td><td>(1,3)</td><td>(1,4)</td><td>(1,5)</td><td>(1,6)</td></tr>
<tr><td>2</td><td>(2,1)</td><td>(2,2)</td><td>(2,3)</td><td>(2,4)</td><td>(2,5)</td><td>(2,6)</td></tr>
<tr><td>3</td><td>(3,1)</td><td>(3,2)</td><td>(3,3)</td><td>(3,4)</td><td>(3,5)</td><td>(3,6)</td></tr>
</table></td></tr>
<tr><td>multiplication</td><td>• An <i>operation</i> where a number is added to itself a number of times.</td><td>$2 + 2 + 2 + 2 + 2 = 10$ or
$5 \times 2 = 10$
</td></tr>
<tr><td>multiplication rule of probability</td><td>• A method for finding the <i>likelihood</i> that both of two <i>events</i> occur.</td><td>Multiplication rule for independent events:
P(A and B) = P(A) × P(B)</td></tr>
<tr><td>multiply (×)</td><td>• To find total in a number of groups.</td><td>Three lots of 2 cows is 6.
$3 \times 2 = 6$ or $2 + 2 + 2 = 6$
</td></tr>
<tr><td>mutually exclusive events</td><td>• Two <i>events</i> that have no outcomes in common.</td><td>A 6 sided die is rolled once.
Event A: Roll a 2
Event B: Roll a 3
Events A and B are mutually exclusive since they both can’t happen at the same time.</td></tr>
<tr><td>natural number (ℕ)</td><td>• A counting number from 1 to <i>infinity</i>.</td><td>1, 2, 3, 4, 5..................∞</td></tr>
</table>

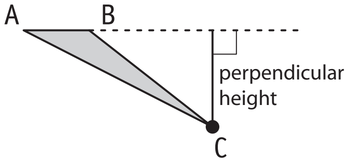
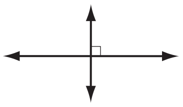
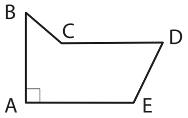
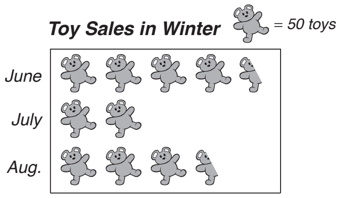
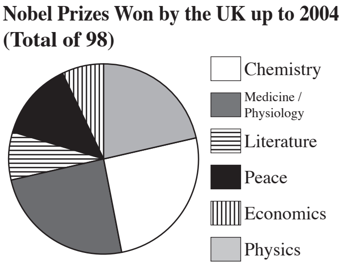
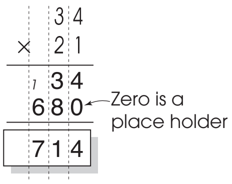
negative number	• A number that is *less than* zero.	−1, −2, −3, −4, −5, ... are negative numbers.
net	• The pattern cut out to form a *3D* shape.	Possible net of a cube. 
network	• A figure made up of *vertices* connected by non-intersecting paths or *arcs*. • Networks are not to *scale*.	Euler's bridges of Konigsberg is a famous network where the land is shown as vertices and the bridges are the paths. 
ninth	• The *position* after *eighth*.	1st, 2nd, 3rd, 4th, 5th, 6th, 7th, 8th, **9th**......
nona	• Prefix meaning nine.	*See nonagon*
nonagon	• A *polygon* with 9 sides.	 Nonagon  Regular nonagon
non-exclusive events	• *Events* that have outcomes in common.	A card is dealt from a pack. Event A: deal a club Event B: deal an even number A and B are non-exclusive events since they can occur at the same time: P(A or B) = P(A) + P(B) − P(A & B)
non-linear function	• Any relationship that is not linear. A *polynomial function* where the *variable* is in the second *power* or higher.	$y = x^3 - 7$ is a non-linear function.
non-recurring decimal	• A *finite* decimal number.	23.375 is a non-recurring decimal.
north	• A *compass direction*.	
north-east	• A *compass direction*.	
north-west	• A *compass direction*.	

null set (∅ or {})	• A *set* with no *elements* also called an empty set.	P = {people 200 years old} is a null set. O = {odd numbers} E = {even numbers} $O \cap E = \varnothing$
number line	• An evenly marked *line* that shows position of numbers. • *Points* are marked with numbers in *ascending order* from left to right (horizontal number line) or from bottom to top (vertical number line). • Zero represents the *origin* of a number line.	
number plane	• See *coordinate plane*.	
number sentence	• A sentence using numbers and *operations* instead of words.	"Mary had four cats and two dogs. How many pets did she have?" Number sentence: $4 + 2 = 6$
numeral	• A symbol used to represent a number.	Arabic numerals: 1, 2, 3, 4, 5 Roman numerals: I, II, III, IV, V
numerator	• The number above the fraction bar in a *fraction*. • The number of parts that are counted.	numerator $\frac{3}{5}$ 3 parts out of 5 count
oblique line	• A line at an *angle* to the horizon.	
obtuse angle	• An *angle* measuring greater than 90° and less than 180°.	
obtuse-angled triangle	• A triangle with one *angle* measuring greater than 90° and less than 180°.	
octa	• Prefix meaning eight.	An octopus has 8 legs. 
octagon	• A *polygon* with 8 sides.	 Octagon  Regular octagon

octahedron	• A *solid* with eight *faces*. • A regular octahedron has faces that are all *equilateral triangles*.	 
odd numbers	• A *whole number* that is not *divisible* by 2.	Odd numbers end with 1, 3, 5, 7 or 9.
of	• Seen in context like 'a *fraction* **of** a number', it means to *multiply*.	A quarter of 100 means $\frac{1}{4}$ of 100, or $\frac{1}{4} \times 100 = 25$
once	• On one occasion.	Just this time!
operation	• A mathematical process performed according to certain *rules*.	There are four basic operations in arithmetic: addition $3 + 12$ subtraction $3 - 1$ multiplication 1×5 division $6 \div 3$ There are many complex operations like: sine 30°, $\sqrt{9}$ and $\log_{10} 100$, 5^4.
opposite angles	• Angles across from each other in a shape.	One pair of opposite angles are equal in a kite. 
opposite sides	• Sides across from each other in a shape.	Side $\overline{AB}$ is opposite to side $\overline{CD}$ Side $\overline{AD}$ is opposite to side $\overline{BC}$ 
opposites	• Two numbers with the same distance to the origin but with different signs.	The opposite of +4 is −4.
order	• Placing a group in a special arrangement.	The aliens are arranged in order of height. 

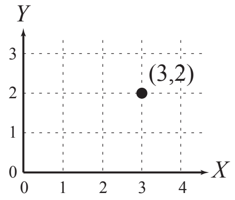
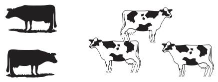
order of operations	• The order of doing *operations* is: 1) *Simplify* inside all *brackets*. 2) *Evaluate powers* and *square roots*. 3) Calculate × and ÷ from left to right. 4) Calculate + and – from left to right.	Calculate $4+3^2\times(6-2)$ by 1) $4+3^2\times(6-2)$ 2) $=4+3^2\times4$ 3) $=4+9\times4$ 4) $=4+36$ $=40$
ordinal numbers	• A *whole number* that shows position.	1st, 2nd, 3rd, 4th, 5th...... are ordinal numbers.
orientation	• Position relative to *direction*.	The tornado is coming from the west. 
origin	• The point of *coordinates* (0,0) on a *coordinate plane*.	
outcome	• Result of an event.	The outcome (result) of tossing a coin was to turn up a head.
overlapping sets	• *Sets* that share one or more common *elements*.	
pair	• Two together.	
palindrome	• A number with 2 or more digits that reads the same *forwards* and *backwards*.	44 or 6116 are palindromic numbers.
parabola	• The shape of the *graph* represented by a *quadratic* function.	A parabola can describe the flight of a ball. 

parallel lines	• *Lines* in the same *plane* that never cross over. They are marked with matching arrows.	
parallelogram	• A special *quadrilateral*. *Opposite* sides are *parallel lines*. *Opposite* sides are equal in length.	
pattern	• Numbers or objects that are arranged following a *rule*.	
penta	• Prefix meaning five.	See *pentagon*
pentagon	• A *polygon* with 5 sides.	 Pentagon  Regular pentagon
pentagonal prism	• A *three-dimensional* shape. Two identical, *parallel bases* are *pentagons*. Five *faces* are *rectangles*.	 
pentagonal pyramid	• A *three-dimensional* shape. *Base* is a *pentagon*. Five *faces* are *triangles*.	 
per	• For each. • Can be written as a forward slash (/).	5 kilometres per hour or 5 km/h means 5 km travelled for each hour.
percentage	• Out of 100. • 'Per' means for each, 'cent' means 100.	$59\% = \frac{59}{100} = 0.59$
percentage change	• The amount of *increase* or *decrease* calculated as a *percentage*.	$\text{\% change} = \frac{\text{amount of change}}{\text{original amount}} \times \frac{100}{1}\%$
perfect square	• Any number that is the result of multiplying two *rational numbers* together.	$0, 1, 4, 9, 16, 25, \frac{1}{25}, \frac{4}{9}$ etc. are all perfect squares.
perimeter	• The *distance* around the outside of a *shape*.	Add the length of all sides. Perimeter = 4 + 5 + 6 = 15 cm 

perpendicular height	• The straight *line* distance between a *vertex* and the *opposite side* in a *2D shape*.	
perpendicular lines	• Lines on a *plane* that *intersect* to form a *right angle*.	
perpendicular sides	• Sides on a *shape* that are at *right angles* to each other.	$\overline{AB}$ is perpendicular to $\overline{AE}$. 
perspective	• The appearance of objects affected by size and *position*.	
pi (π)	• The *ratio* of the *circumference* of a *circle* to its *diameter*. The diameter of a circle wraps around the circle approximately 3.14 times.	3.14 or $\frac{22}{7}$ is the approximate value of π. Pi is an infinite number. $\pi = 3.14159\ 26535\ 89793...$
pictograph	• A *graph* that uses pictures or symbols to represent *data*.	
pie graph	• A *graph* that represents *data* as a *fraction* or *percentage* of a *circle*.	
place holder	• Minds a spot in a number.	Zeros are used as place holders in long multiplication algorithms. 

place value	• Value according to position in a number.	954 5 is in the tens place 5 has a value of 50.

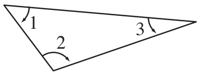
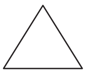
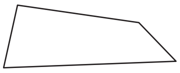
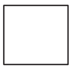
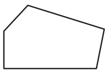
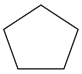
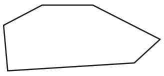
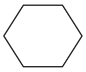
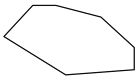
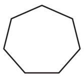
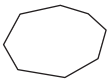
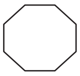
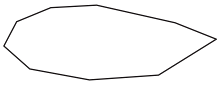
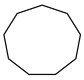
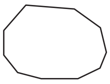
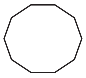
millions	hundreds of thousands	tens of thousands	thousands	hundreds	tens	units	• decimal point	tenths	hundredths	thousandths
1 000 000	100 000	10 000	1000	100	10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

plane	• A flat surface.	
plot	• To mark a *point* on a *coordinate plane*.	The point of coordinate (3,2) 
plus (+)	• Another word for *addition*. To add.	2 cows plus 3 cows gives you 5 cows. 2 + 3 = 5 
pm (**p**ost **m**eridiem)	• The *time* from midday to midnight.	Every night Jimmy starts reading at 9 pm. 
point	• A position in space represented by a dot.	

polygon

- A closed *two-dimensional* shape for which all sides are line segments.
3 or more *sides* and *angles*.

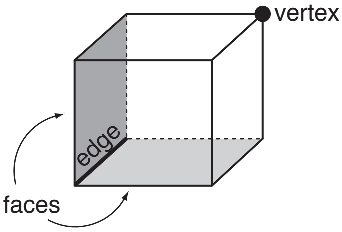
'Poly' means many and 'gon' means angle.
Example: A triangle has 3 angles.

polygon (many angles)		**regular polygon** (all sides and all angles are equal)		Number of Sides	Number of Interior angles
Triangle 3 angles		Equilateral triangle		3	3
Quadrilateral 4 angles		Square		4	4
Pentagon 5 angles		Regular pentagon		5	5
Hexagon 6 angles		Regular hexagon		6	6
Heptagon 7 angles		Regular heptagon		7	7
Octagon 8 angles		Regular octagon		8	8
Nonagon 9 angles		Regular nonagon		9	9
Decagon 10 angles		Regular decagon		10	10

polyhedron

- A *three-dimensional* shape.
Four or more *faces*.
Described by their *faces*, *edges* and *vertices*.

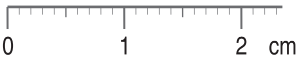
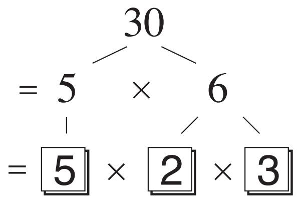
'Poly' means many and 'hedron' means faces.
Example: A hexahedron has 6 faces.

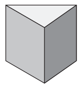


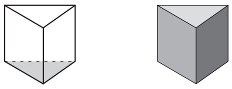
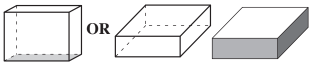
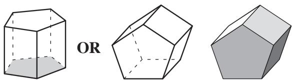
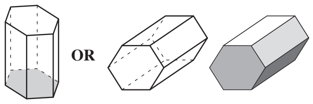
polynomial

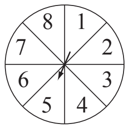
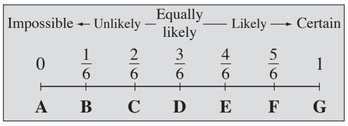
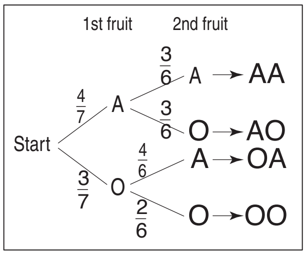
- The *sum* or *difference* of *terms* which have *variables* raised to *positive integer powers* and which have *real coefficients*.

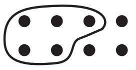
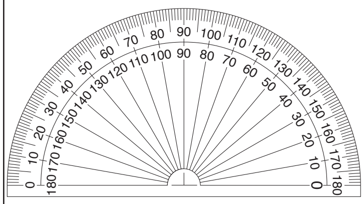
Type of polynomial	# of unlike terms	Examples
Monomial	1	-7 ab $6x^2$
Binomial	2	$7+y$ $gh-4$ $2s^2+s$
Trinomial	3	$x+y+4$ r^2-6s^3+4t $mn+5-2m^2n$

polynomial function	• A *function* where the *variable* is in the second *power* or higher.	$y = x^3 - 4x^2 + 2x - 8$ is a polynomial function.
population	• The entire *set* under consideration in a statistical analysis.	The population of a country is every person who lives in that country.
position	• Where something is in relation to things around it.	In, on, under, behind, next to.
positive numbers	• A number that is greater than zero.	+1, +2, +3, +4, +5, ... are positive numbers.
possible outcomes	• The total number of result options.	When you toss a coin there are 2 possible results: heads or tails.
power	• An *expression*, such as 4^3, in which the base (4) is *multiplied* by itself a number of times equal to the *exponent* (3).	4^3 or 4 to the power of 3 is $4 \times 4 \times 4 = 64$
precision of an instrument	• Considered to be the size of the smallest *unit* on the *scale* of the instrument.	The ruler has a precision of 0.1 cm. 
previous	• The one before.	If the current year is 2006, the previous year is 2005.
prime factor	• A *factor* that is also a *prime number*. *Factor trees* can help to determine a number's prime factors.	The prime factors of 30 are 2, 3 and 5. 
prime factorisation	• Writing a *whole number* as the *product* of its *prime factors*.	Prime factorisation of 30: $30 = 2 \times 3 \times 5$
prime number	• A *whole number* that has exactly two *factors*, 1 and itself. • 1 is not a prime number.	59 is a prime number as its only factors are 1 and 59. The prime numbers between 0 and 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

principal (money)	• Capital sum, distinct from *interest* or income. • The amount of debt on which interest is calculated.	Interest charged = % of principal loan.
prism	• A *three-dimensional* shape. Two *parallel bases* are the same.	

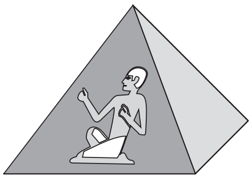
prism	Properties	Number of			Examples
		Faces	Edges	Vertices	
Triangular Prism	Bases are triangles Lateral faces are rectangles	5	9	6	
Square Prism	Bases are squares Lateral faces are rectangles	6	12	8	
Rectangular Prism	Bases are rectangles Lateral faces are rectangles	6	12	8	
Pentagonal Prism	Bases are pentagons Lateral faces are rectangles	7	15	10	
Hexagonal Prism	Bases are hexagons Lateral faces are rectangles	8	18	12	

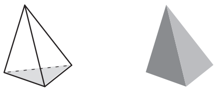
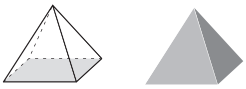
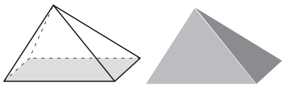
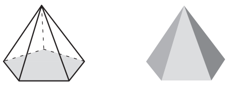
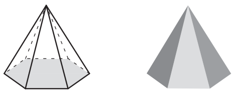
probability	• The likelihood that an event will happen, measured as a *fraction* of the total of possible outcomes. See *chance*.	The probability of spinning the number 5 is $\frac{1}{8}$. 
probability scale	• A measure, from 0 (no chance) to 1 (will happen), of the likelihood of an event occurring.	
probability tree diagram	• A diagram that displays all the possible *outcomes* of an *event*.	When choosing 2 pieces of fruit from a bowl with 4 apples and 3 oranges, there are 4 possible outcomes (branches): AA, AO, OA, OO 

product	• The result when two or more numbers are *multiplied*.	The product of 4 and 5 is 20: $4 \times 5 = 5 \times 4 = 20$
profit (money)	• What is gained, less any *expenses*. Profit = Revenue – Expense.	Revenue from a business activity is $20. If the expenses are $15 then the profit would be $5.
pronumeral	• A letter which stands in for (pro) a number (numeral). A pronumeral takes the place of: an unknown value or a value which may change (vary) in different situations. Any pronumeral can also be called a *variable*.	"s" is a pronumeral in the expression $2s + 1$
proper fraction	• Any *fraction* in which the *numerator* is *less than* the *denominator*.	$\frac{5}{8}$ the numerator is 5 the denominator is 8. $5 < 8$ so $\frac{5}{8}$ is a proper fraction.  $\frac{5}{8}$
proportion	• A comparative *ratio*, showing that two ratios are equivalent.	$\frac{2}{3} = \frac{6}{9}$ is a proportion. 2 : 3 is the same ratio as 6 : 9 2 : 3 is in proportion with 6 : 9
protractor	• A *semi-circular* tool used to measure *degrees*. There are 180° on a protractor.	

pyramid

- A *three-dimensional* shape.
One *base* is a *polygon*.
All other *faces* are *triangles* that meet at one point called *vertex*.
A pyramid is named for the shape of its base.

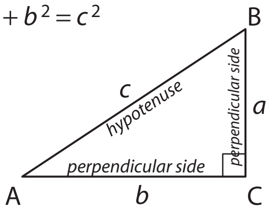


pyramid	Properties	Number of			Examples
		Faces	Edges	Vertices	
Triangular Pyramid	Base is a triangle Lateral faces are triangles	4	6	4	
Square Pyramid	Base is a square Lateral faces are triangles	5	8	5	
Rectangular Pyramid	Base is a rectangle Lateral faces are triangles	5	8	5	
Pentagonal Pyramid	Base is a pentagon Lateral faces are triangles	6	10	6	
Hexagonal Pyramid	Base is a hexagon Lateral faces are triangles	7	12	7	

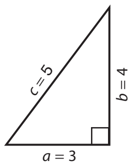
Pythagoras' theorem

- Rule: $a^2 + b^2 = c^2$
For any *right-angled triangle*, the square of the length of the *hypotenuse* (c) equals the sum of the squares of the lengths of the two *perpendicular sides* (a and b).

$a^2 + b^2 = c^2$



$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$ (true)



Pythagorean triad

- A set of 3 *positive integers* that make *Pythagoras' theorem* true.

$a^2 + b^2 = c^2$
$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
so triads include

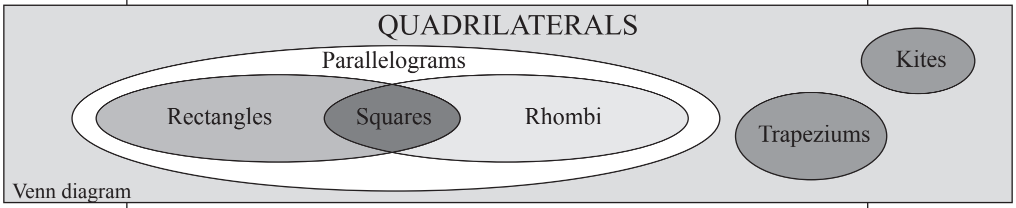
3,4,5	6,8,10
5,12,13	7,24,25
8,15,17	9,40,41,
20,21,29	etc.

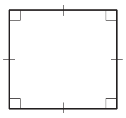
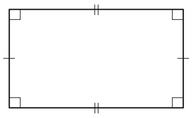
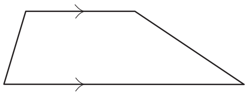
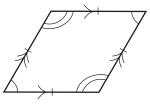
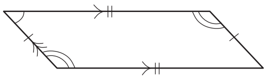
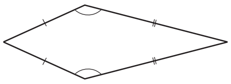
quadratic function

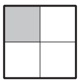
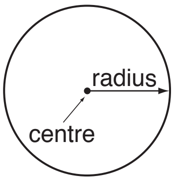
- A *function* that can be represented by an *equation* of the form $y = ax^2 + bx + c$, where a, b and c are *real numbers* and a can't be 0.
The *graph* of this function is a *parabola*.

Used to describe the flight of a ball:
$y = x^2 + 3x - 2$
is a quadratic function.

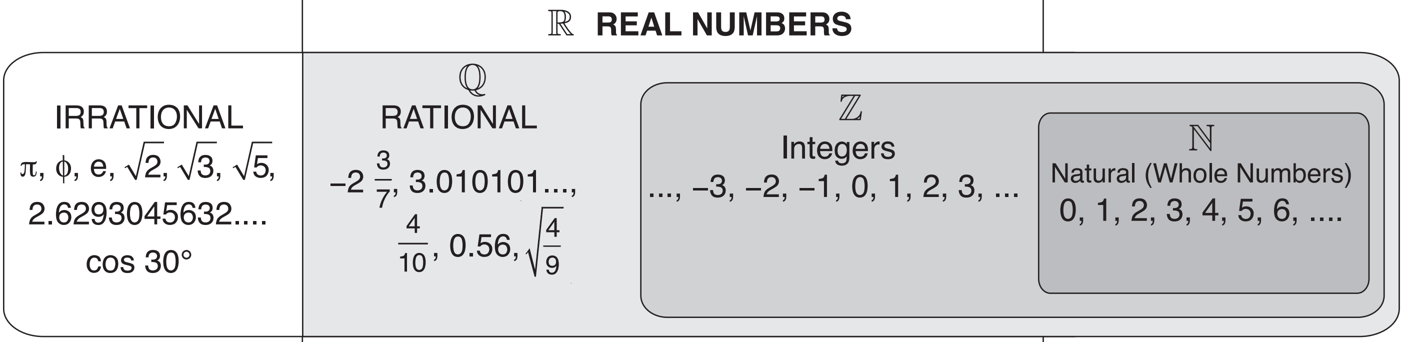
quadratic rule	• $y = ax^2 + bx + c$, where $a \neq 0$.	See *quadratic function.*
quadratic trinomial	• An *expression* with three *terms* with *powers* no higher than two.	$g^2 + 3gh - 2g$ is a quadratic trinomial.
quadrilateral	• A *polygon* with 4 *sides*.	'Quad' means 4 and 'lateral' means side.

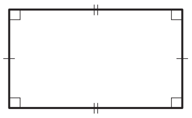
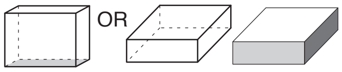
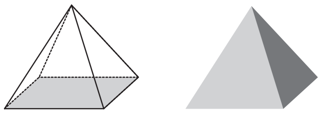
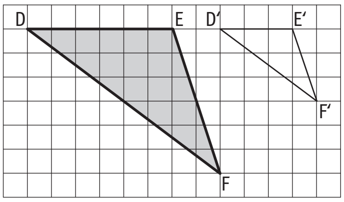
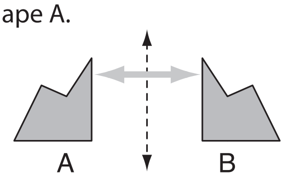
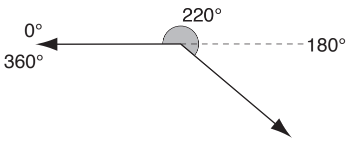
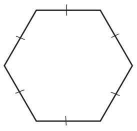


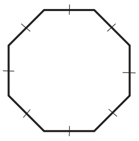
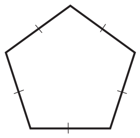
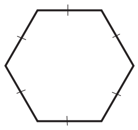
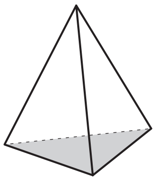
quadrilateral	Sides	Interior angles	Diagonals	Axes of symmetry	Diagram
Square	4 sides of equal length	4 right angles	2 diagonals equal in length and bisecting at right angles	4	
Rectangle	Opposite sides of equal length	4 right angles	2 diagonals equal in length and bisecting each other	2	
Trapezium	2 opposite sides parallel		2 diagonals	0	
Rhombus	4 sides of equal length and opposite sides parallel	Opposite angles equal	2 diagonals bisecting at right angles	2	
Parallelogram	Opposite sides of equal length and parallel	Opposite angles equal	2 diagonals bisecting each other	0	
Kite	4 sides two each of equal length	One pair of opposite angles equal	2 diagonals bisecting each other	1	

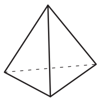
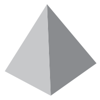
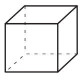
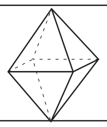
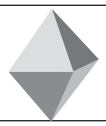
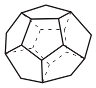
quarter	• One of four *equal* parts of a group or object. • Written as the *fraction* $\frac{1}{4}$.	
quartiles	• The collective term for the *lower quartile* (25th percentile) and the *upper quartile* (75th percentile) of a set of *data*.	See *box-and-whisker plot*
radius of a circle	• (pl. **radii**) The distance from the *centre* to any *point* on the *circle*.	

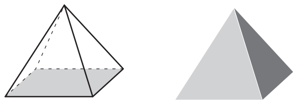
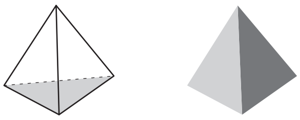
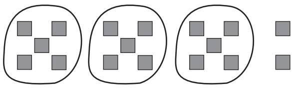
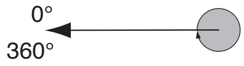
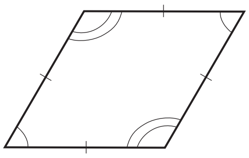
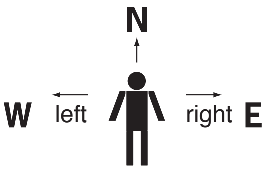
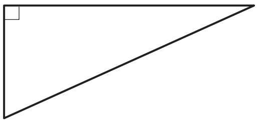
random sample	• A selection taken from a group without method or conscious choice.	Drawing out of a hat is a random selection. 
range	• The *difference* between the greatest and the smallest value.	For the data: 21, 24, 25, 27, 27 and 28 the range is 28 – 21 = 7
rate	• The *ratio* of two measures that have different *units*.	When running, calories burn at a rate of 14 cal/min.
ratio	• The ratio of a number (a) to a non-zero number (b) is the result when a is divided by b. The ratio of a to b can be written as: $\frac{a}{b}$, $a:b$ or 'a to b'. A ratio is made by comparing quantities using the same *unit* e.g. parts, buckets or litres.	If the ratio of cordial to water is 3 : 1 then that would mean 3 parts cordial to 1 part water! Agh, the order of the ratio matters. Map scales are an example of a ratio. See also *ratio scale* and *scale*.
ratio scale	• A *scale* written as a *ratio*. Compares the dimensions on a map or model (first number) to real life (second number).	If the scale on a map is 1 : 10 000 1 cm represents 10 000 cm. 1 cm represents 100 m. Every cm on the drawing represents 100 m in real life.
rational number ($\mathbb{Q}$)	• All *positive* and *negative fractions*, including *integers* and *improper fractions*. • Not an *irrational number*.	$-2\frac{3}{7}$, 3.010101..., $\frac{4}{10}$, 0.56, $\sqrt{\frac{4}{9}}$
real number ($\mathbb{R}$)	• Any number on the *number line*. • Includes all *rational* and *irrational numbers*.	

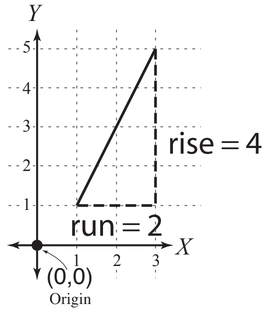
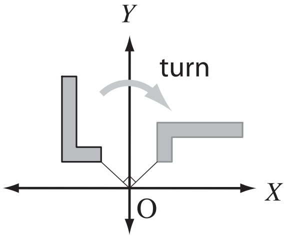
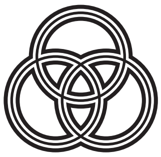
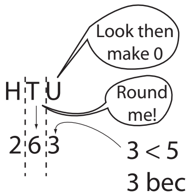
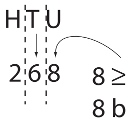


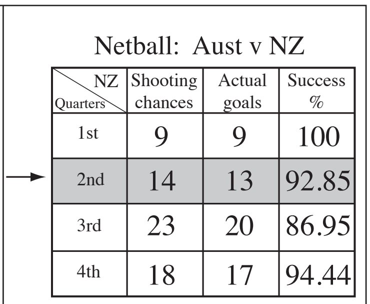
reciprocal	• A *fraction* flipped upside down. • Also called the multiplicative *inverse*. • One of two numbers whose *product* is 1.	The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. $\frac{3}{1} \times \frac{1}{3} = 1$
rectangle	• A special *parallelogram*. Four *right angles*.	
rectangular prism	• A *three-dimensional* shape. Six rectangular faces.	
rectangular pyramid	• A *three-dimensional* shape. One *rectangular base*. All the other *faces* are *triangles*.	
recurring decimal	• A *decimal* that has a repeating *digit* or a repeating pattern of digits. • A repeating digit/s is marked with a dot (•) or a bar (—).	$\frac{2}{9} = 0.22222222 = 0.\dot{2}$ $\frac{1}{6} = 0.1666666 = 0.1\dot{6}$ are repeating decimals, where 2 and 6 are the repeating digits respectively. $\frac{1}{11} = 0.09090909 = 0.\dot{0}\dot{9}$ is a repeating decimal, where 09 is the repeating pattern of digits.
reduction	• Make smaller or *decrease*.	ΔDEF was reduced to ΔD′E′F′ by a scale factor of 2. 
reflection	• A movement that *flips* a figure across a *line* so that the figure is in the mirror image *position*.	Shape B is a reflection of shape A. 
reflex angle	• An *angle* measuring greater than 180° and less than 360°.	
regular hexagon	• A *polygon* with six sides of equal length and six equal angles.	 Regular hexagon

regular octagon	• A *polygon* with eight sides of equal length and eight equal angles.	 Regular octagon
regular pentagon	• A *polygon* with five sides of equal length and five equal angles.	 Regular pentagon
regular polygon	• A shape with all *side*s and all *angles equal*.	A regular hexagon has 6 equal sides and 6 equal angles.  Regular hexagon
regular prism	• A *three-dimensional* shape with *bases* that are *regular polygons* and all the other faces that are rectangles.	A regular hexagonal prism has regular hexagons as its bases.
regular pyramid	• A *three-dimensional* shape with only one *base* which is a regular *polygon* and all the other *faces* that are *triangles*. The base gives the pyramid its name, e.g. regular 'triangular' pyramid.	This regular triangular pyramid has an equilateral triangle as its base. 
regular solid	• A *three-dimensional* shape that encloses a part of space, with all faces being *regular polygons*.	

regular solid	Properties All faces are regular polygons	*In any polyhedron:* $E = F + V - 2$ Number of			Examples
		Faces	Edges	Vertices	
Tetrahedron	All faces are equilateral triangles	4	6	4	 
Hexahedron	All faces are squares	6	12	8	 
Octahedron	All faces are equilateral triangles	8	12	6	 
Dodecahedron	All faces are regular pentagons	12	30	20	 
Icosahedron	All faces are equilateral triangles	20	38	20	

regular square pyramid	• A *pyramid* whose *base* is a *square* and whose *height* intersects the base at its centre. • All 4 *slant heights* and 4 *vertical edges* are *congruent*.	
regular tetrahedron	• A *triangular pyramid* whose four *faces* are equal *equilateral triangles*.	
relative error	• The degree to which a measurement is different to the actual value.	"My measuring may be off by 1%!"
remainder	• The amount left over when one number cannot be *divided* exactly by another.	17 ÷ 5 = 3 with 2 remainder. 
reversible	• Able to be turned in the *opposite* way.	The process of freezing the water is reversible: water ⟶ ice ⟶ water
revolution	• A complete turn. • An *angle* measuring 360°.	
rhombus	• (pl. **rhombi**) A special *parallelogram*. Four *equal sides*. *Opposite angles equal*.	
right	• The *direction* to the *east* of your body if you are facing *north*.	
right angle	• An *angle* measuring exactly 90°. It is marked with a corner.	
right-angled triangle	• A *triangle* with one *right angle*.	

rise	• The vertical change in the *y* value on a straight line. It helps determine the *gradient of a line*. See *gradient of a line*.	The value of the *y*-axis changes from 1 to 5 so the rise is 4.  $\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$ $= \frac{4}{2} = 2$
Roman numerals	• Number system invented by the ancient Romans.	I = 1 V = 5 X = 10 L = 50 C = 100 D = 500 M = 1000
rotation	• A movement that turns a shape about a fixed *point* (the centre of rotation) by a given *angle* (the angle of rotation).	The centre of rotation is the origin O and the angle of rotation is 90°. 
rotational symmetry	• A shape has rotational symmetry if a rotation of 180° or less produces an image that fits exactly on the original shape.	This shape has rotational symmetry, because after a rotation of 120° it looks identical to the original. 
round	• To *approximate* a number to a given *place value*. Look at the next *digit* after the given place value you are rounding to. If this digit is less than 5, keep the digit in the given place value the same. If this digit is greater than or equal to 5, add 1 to the digit in the given place value. Then make the *digit* you were looking at zero.	Round 263 to the nearest 10:  3 < 5 so 6 stays 3 becomes 0 263 ≈ 260 Round 268 to the nearest 10:  8 ≥ 5 so add 1 to 6 8 becomes 0 268 ≈ 270

<table>
<tr><td>**row of a table**</td><td>• A *horizontal* line of *data* in a table.</td><td>
Netball: Aust v NZ
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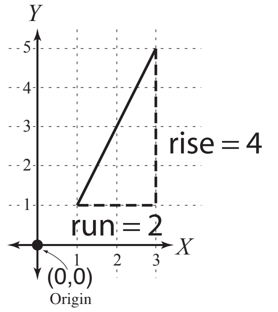
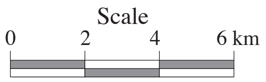
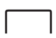
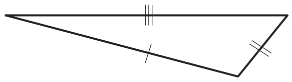
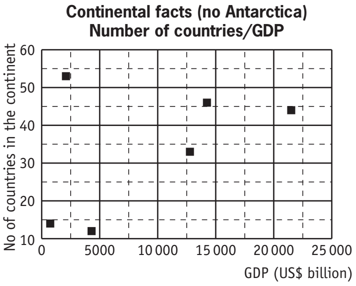
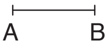
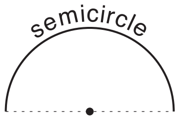
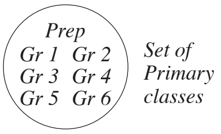
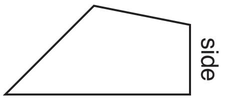
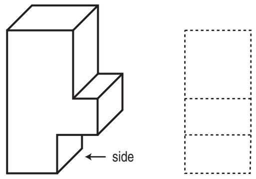
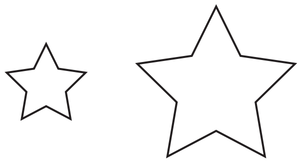
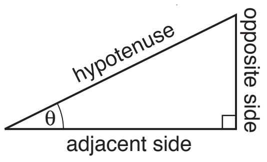
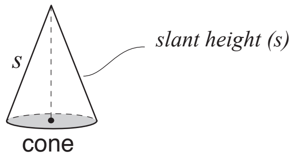
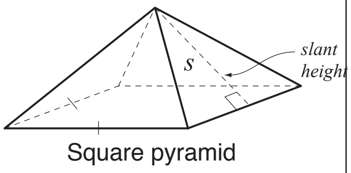
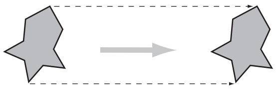
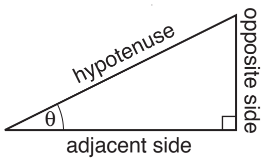
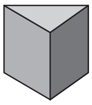
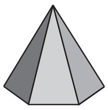
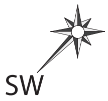
</table></td></tr>
<tr><td>**rule**</td><td>• See *function*.</td><td></td></tr>
<tr><td>**run**</td><td>• The horizontal change in the *x* value on a straight *line*. It helps determine the *gradient of a line*.
See *gradient of a line*.</td><td>The value of the *x*-axis changes from 1 to 3 so the run is 2.


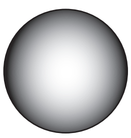
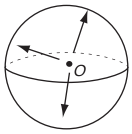
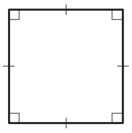
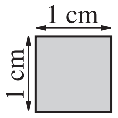
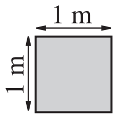
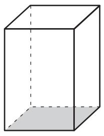
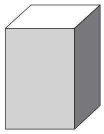
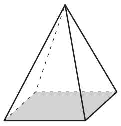
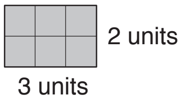
$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$
$= \frac{4}{2} = 2$</td></tr>
<tr><td>**sample**</td><td>• A selection taken from a group or *population*.</td><td>See *random sample*.</td></tr>
<tr><td>**sample space**</td><td>• The *set* of all possible *outcomes* of an *experiment*.</td><td>A coin is flipped -
Sample space = {heads, tails}</td></tr>
<tr><td>**scale**</td><td>• A key on a *scale drawing*/map that tells how the drawing’s *dimensions* and life size dimensions are related.
Can be written as:
1) A *ratio scale* with the first number referring to the map distance and the second number referring to the real distance.
OR
2) A *linear scale* with a set of marks on a line.</td><td>On a map with a ratio scale of 1 : 10 000
1 cm represents 10 000 cm or 100 m
Every centimetre on the drawing represents 100 m in real life.


On a map with this linear scale, every highlighted segment represents 2 km in real life.</td></tr>
<tr><td>**scale drawing**</td><td>• Changing the size of an object but not the shape.</td><td>A life size staple. 
The staple scaled by 50%.
</td></tr>
</table>

scale factor	• The amount used to *enlarge*, *reduce* or find the original size of an object.	To make an object 2 times bigger or 200% of the original size, enlarge the object by a scale factor 2 : 1 To do this multiply each dimension by the fraction $\frac{2}{1}$. To make an object 2 times smaller or 50% of the original size, reduce the object by a scale factor 1 : 2 To do this multiply each dimension by the fraction $\frac{1}{2}$.
scalene triangle	• A *triangle* in which all three sides are a different length.	
scatter plot	• A *graph* in which two sets of data are plotted as ordered pairs in a *coordinate plane*.	 Continental facts (no Antarctica) Number of countries/GDP
second (s)	• A very short unit of *time*.	There are 60 seconds in 1 minute.
second	• The *position* after *first*.	1st, **2nd**......
segment	• Two *points* and all points on the *line* between the two points. Part of a line.	 Segment $\overline{AB}$
semicircle	• Half of a circle.	
sequence of numbers	• A list of numbers that follows a certain *rule*. Each number is called a *term*.	35, 30, 25, 20, … In this sequence of numbers, the next three are 15, 10 and 5.
set {}	• A collection of items. Members of a set are called *elements*.	There are 7 elements in the set. 
seventh	• The *position* after *sixth*.	1st, 2nd, 3rd, 4th, 5th, 6th, **7th**...
side	• One of the lines that form a *polygon*.	

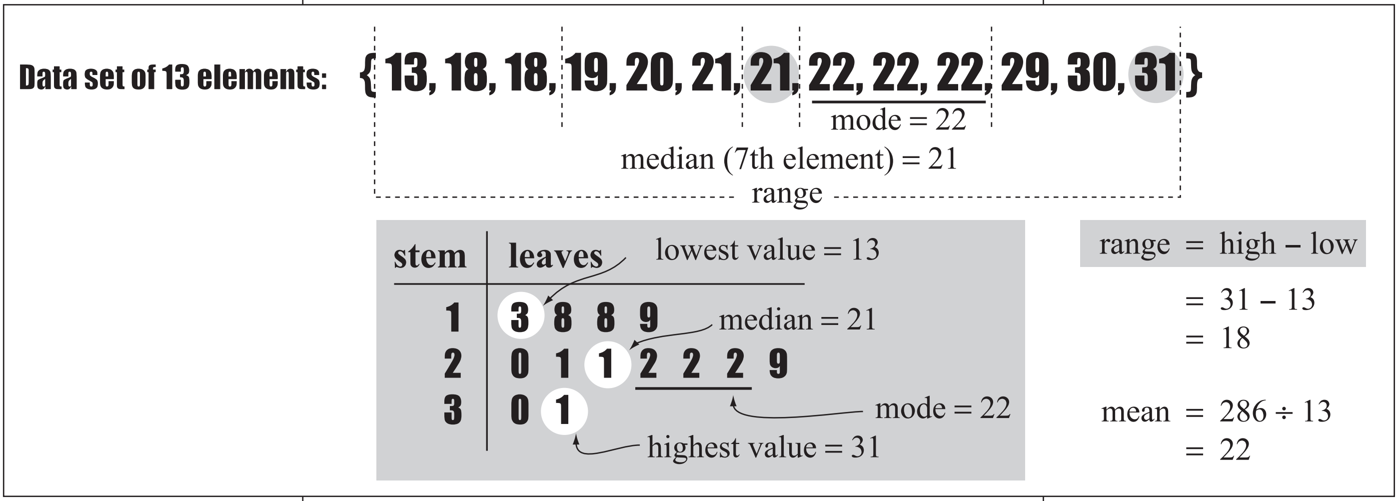
side view	• What you see of an object looking from a side *perspective*. • *Three-dimensional* objects have 3 views: front, top and side.	
sign	• The *positive* or *negative* indicator attached to any *real number* that is *greater than* or *less than* zero respectively.	+ positive sign − negative sign
similar shapes	• Shapes that are identical but not necessarily in size.	These stars are similar. 
simple interest	• *Interest* paid only on the *principal* not on the accruing interest as well. • Interest = principal × rate × time OR $SI = prt$	If you deposit \$100 in a bank which pays 6%, you will earn $100 \times 0.06 = 6$ or \$6 simple interest in a year.
simplest form of a fraction	• A *fraction* is in its simplest form when the only number that divides into both the *numerator* and the *denominator* is 1.	The simplest form of $\frac{6}{9}$ is $\frac{2}{3}$ (Divide 6 and 9 by 3. 2 and 3 can only be divided by 1 so they cannot be reduced.)
simplify	• To reduce to the *simplest form*.	To simplify the ratio 14 : 6 divide both sides by 2. 14 : 6 simplified is 7 : 3.
simultaneous equations	• Two or more equations containing a common variable or variables.	$x + y = 1$ $x^2 + y^2 = 2$ are simultaneous equations.
sine	• A *trigonometric* function. • In a *right-angled triangle*, the sine of an *acute angle* is the *ratio* of the length of the side *opposite* the angle to the length of the *hypotenuse*.	$\sin\theta = \frac{\textbf{O}\text{pposite}}{\textbf{H}\text{ypotenuse}}$ 
sixth	• The *position* after *fifth*.	1st, 2nd, 3rd, 4th, 5th, **6th**......
size	• How big an object is.	The size of the wave is 2 m. 

slant height (cone)	• The distance from the *vertex* to any *point* on the circular *edge* of the *base*.	
slant height (regular pyramid)	• The *length* of an *altitude* of a *lateral face*.	
slide	• Move without changing *direction*. See *translation*.	
smallest to largest	• Ranking in order from the littlest to the biggest.	
SOH - CAH - TOA	• Memory jogger for calculating the *trigonometric ratios* of *sine*, *cosine* and *tangent*.	$\text{Sin}\,\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ SOH $\text{Cos}\,\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ CAH $\text{Tan}\,\theta = \frac{\text{Opposite}}{\text{Adjacent}}$ TOA 
solid	• A *three-dimensional* shape that encloses a part of space.	 OR 
south	• A *compass direction*.	
south-east	• A *compass direction*.	
south-west	• A *compass direction*.	
speed	• The *rate* at which an object moves. Speed is worked out by dividing the distance travelled by the time taken. We call this average speed $v = \frac{d}{t}$	The average speed for a car which travels 270 km in 3 h is: $v = \frac{\text{distance}}{\text{time}} = \frac{270}{3} = 90$ km/h

sphere	• A *set* of *points* in space of equal distance from the central point.	 
square	• A *rectangle* with all *sides* of equal length.	
square centimetre (cm^2)	• A *unit* of *area* *equal* to 1 *centimetre* by 1 centimetre.	
square metre (m^2)	• A *unit* of *area* *equal* to 1 *metre* by 1 metre.	
square number	• A number that results from *multiplying* another number by itself.	9, 6.25 and $\frac{4}{9}$ are all square numbers. $9 = 3 \times 3$ $6.25 = 2.5 \times 2.5$ $\frac{4}{9} = \frac{2}{3} \times \frac{2}{3}$
square prism	• A *three-dimensional* shape. Two identical square *bases*. All the other faces are *rectangles*.	 
square pyramid	• A *three-dimensional* shape. One square *base*. All the other faces are *triangles*.	 
square root of a number (√)	• A *number* which, when *multiplied* by itself, gives the original number. Finding the square root of a number is the *inverse operation* of squaring that number.	$\sqrt{900} = 30$ Square root of 900 is 30, because $30 \times 30 = 900$ or $30^2 = 900$
square units	• A *unit* of *area* equal to the area of a square with side lengths of 1 unit.	$A = lw$ $= 3 \times 2$ $= 6$  Area = 6 square units
squared	• *Multiplied* by itself. A number raised to the second *power*.	4 squared is written as 4^2 $4^2 = 4 \times 4 = 16$

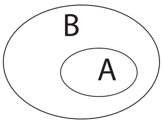
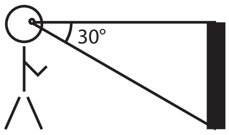
statistics	• Numerical facts systematically collected, organised and analysed.	Data is collected from a sample of the population, organised into a graph and interpreted to summarise some characteristic.
stem-and-leaf plot	• A diagram displaying *data* by *place value*. The data is in order from lowest to highest.	

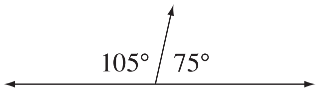
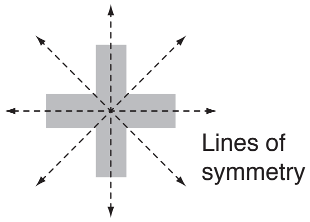
Data set of 13 elements: {13, 18, 18, 19, 20, 21, 21, 22, 22, 22, 29, 30, 31}



range = high − low
= 31 − 13
= 18

mean = 286 ÷ 13
= 22

straight angle	• An *angle* measuring 180°.	
subset (⊂)	• A *set* of *elements* completely contained in a larger set.	$\{a, b\} \subset \{a, b, c, d, e\}$ $A \subset B$ 
substitute	• To replace a number or *function* with another. Often used in *algebra* when a *pronumeral* is replaced by a number.	If $x = 4$, the value of $x + x$ is found by replacing the letter x with 4: $4 + 4 = 8$
subtended angle	• The *angle* formed by an object at a given external point.	The post subtends at an angle of 30° to the observer's eye. 
subtract	• To take away or *minus*.	If you subtract 10 from 15 you are left with 5: $15 - 10 = 5$
sum	• The result when two or more numbers are added.	The sum of 20 and 6 is 26: $20 + 6 = 6 + 20 = 26$
super annuation	• An investment strategy designed to provide for retirement.	In Australia, an employer pays 9% of an employee's base wage into superannuation.

<table>
<tr><td>supplement of an angle</td><td>• An <i>angle</i> that, when added to an <i>adjacent</i> angle, makes a <i>straight angle</i> (or 180° in total).</td><td>75° is the supplement of 105°, because 75° + 105° = 180°
 </td></tr>
<tr><td>surd</td><td>• An <i>irrational number</i>. It can be expressed as an <i>infinite non-recurring decimal</i> but not as a <i>fraction</i>.</td><td>The square roots of prime numbers are all surds.
$\sqrt{7} = 2.64575131$.........
$\sqrt{7}$ is a surd.</td></tr>
<tr><td>survey</td><td>• A method of collecting a <i>sample</i> of <i>data</i> by getting people's responses.</td><td>TV ratings are detemined by surveying viewers.</td></tr>
<tr><td>symmetry</td><td>• A shape has a <i>line of symmetry</i> when a line can be drawn through the shape so that one side of the shape is the mirror image of the other.</td><td>There are 3 kinds of symmetry:
horizontal symmetry
vertical symmetry
rotational symmetry
 </td></tr>
<tr><td>table</td><td>• <i>Data</i> organised in <i>columns</i> and <i>rows</i>.</td><td>Netball: Aust v NZ
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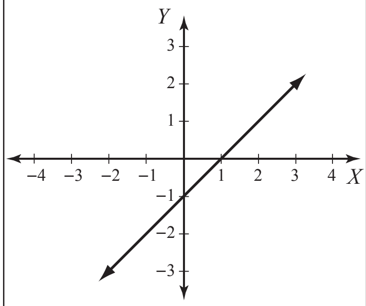
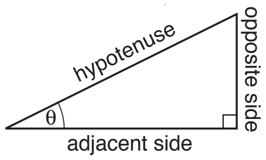
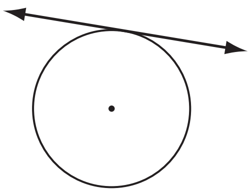
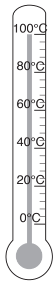
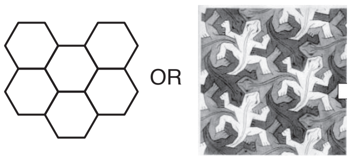
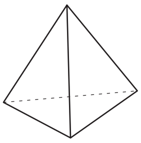
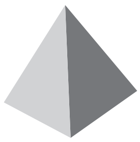
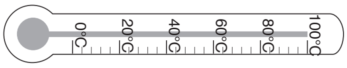
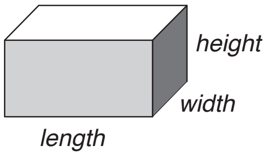
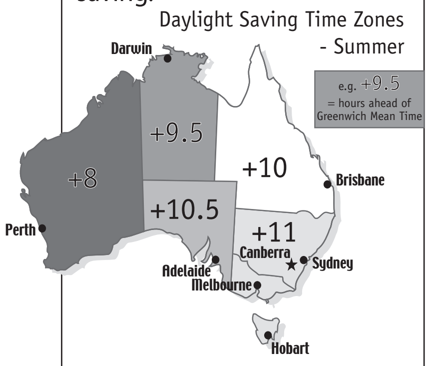
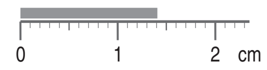
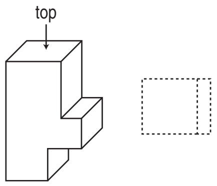
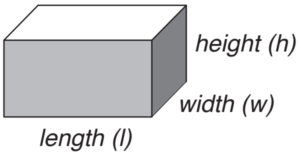
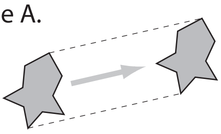
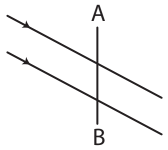
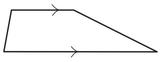
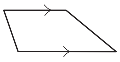
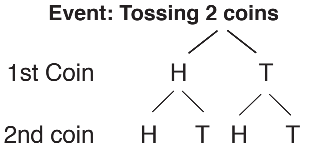
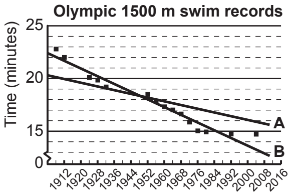
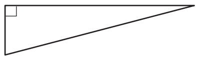
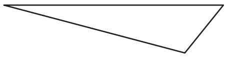
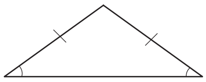
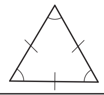
<tr><td>4th</td><td>18</td><td>17</td><td>94.44</td></tr>
</table></td></tr>
<tr><td>table of values</td><td>• Mathematical <i>data</i> organised in <i>rows</i> and <i>columns</i> representing possible solutions for <i>x</i> and <i>y</i>. The solutions can be graphed.</td><td><table>
<tr><td>x</td><td>−2</td><td>−1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>y</td><td>−3</td><td>−2</td><td>−1</td><td>0</td><td>1</td><td>2</td></tr>
</table>
 </td></tr>
<tr><td>tangent</td><td>• A <i>trigonometric</i> function.
• In a <i>right-angled triangle</i>, the tangent of an <i>acute angle</i> is the <i>ratio</i> of the length of the side <i>opposite</i> the angle to the length of the side <i>adjacent</i> to it.</td><td>$\tan\theta = \frac{\mathbf{O}\text{pposite}}{\mathbf{A}\text{djacent}}$
 </td></tr>
</table>

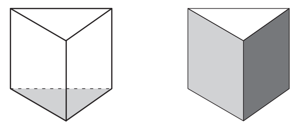
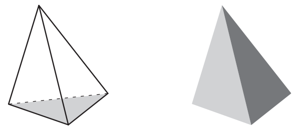
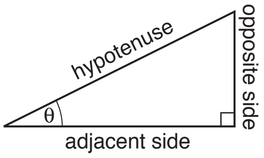
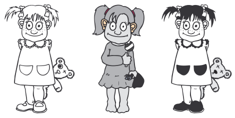
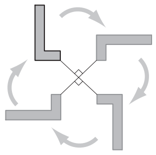
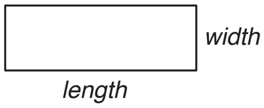
tangent to a circle	• A *line* that touches the *circle* at a *point* without crossing over.	
tax	• A financial charge imposed by the state often calculated as a *percentage*.	
temperature	• How hot or cold a thing is. • Temperature is measured in *degrees* Celsius (°C) with a *thermometer*.	100°C is the temperature at which water boils. 
tens	• The *place value* between the *units* and *hundreds*.	1825.763 has 2 tens. thousands | hundreds | tens | units | tenths | hundredths | thousandths 1 | 8 | 2 | 5 . | 7 | 6 | 3
tenth	• One part out of 10 parts of one whole.	
tenths	• The *place value* after the decimal point between the *units* and *hundredths*.	1825.763 has 7 tenths. thousands | hundreds | tens | units | tenths | hundredths | thousandths 1 | 8 | 2 | 5 . | 7 | 6 | 3
term	• Any part of an expression separated by "+" or "–" signs. • A term can be a: a) *numeral* b) single *pronumeral* (letter) c) *product* of a number and a pronumeral d) product of a number and two or more pronumerals $a+a+a+a+a$ = Five lots of a $= 5 \times a$ $= 5a$ We simplify the writing by removing the "×" sign. We read this as "five a". a = One lot of a $= 1 \times a$ $= 1a$ $= a$ We simplify the writing by removing the "1" and the "×" sign. We read this as "a".	a) 7, $\frac{1}{3}$ or -18 b) a, b or $-c$ c) $7a, \frac{1}{b}, -18g$ or $3x^2$ d) $7ab, 5mn^3$ or $-3jk^2$ A term that has both numerals and pronumerals is always written with the number before the pronumeral. If there is more than one pronumeral in the term then they are usually written in alphabetical order.

<table>
<tr><td>terminating decimal</td><td>• A <i>decimal</i> whose <i>digits</i> end. Every terminating decimal can be written as a <i>fraction</i> with a <i>denominator</i> of 10, 100 or 1000 etc.</td><td>$0.765 = \frac{765}{1000}$</td></tr>
<tr><td>tessellation</td><td>• A repeated shape covering a large <i>plane</i> area with no gaps and no overlaps. Brick walls and tiled floors are examples of tessellations. <i>Maurits Escher</i>, a Dutch mathematician, developed tessellating patterns and artwork by distorting or adding and taking space from the <i>opposite</i> sides of <i>polygons</i>.</td><td>Tessellating patterns

Tessellating shapes
 </td></tr>
<tr><td>tetrahedron</td><td>• A <i>triangular pyramid</i>.
See also regular tetrahedron.</td><td></td></tr>
<tr><td>thermometer</td><td>• An instrument used to measure <i>temperature</i>.</td><td> </td></tr>
<tr><td>third</td><td>• The <i>position</i> after <i>second</i>.</td><td>1st, 2nd, 3rd......</td></tr>
<tr><td>thousands</td><td>• The <i>place value</i> between <i>hundreds</i> and tens of thousands.</td><td>1825.763 has 1 thousand.<table><tr><td>thousands</td><td>hundreds</td><td>tens</td><td>units</td><td>tenths</td><td>hundredths</td><td>thousandths</td></tr><tr><td>1</td><td>8</td><td>2</td><td>5 •</td><td>7</td><td>6</td><td>3</td></tr></table></td></tr>
<tr><td>thousandth</td><td>• One part out of 1000 parts of one whole.</td><td>One gram is a thousandth of a kilogram.</td></tr>
<tr><td>thousandths</td><td>• The <i>place value</i> after <i>hundredths</i>.</td><td>1825.763 has 3 thousandths.<table><tr><td>thousands</td><td>hundreds</td><td>tens</td><td>units</td><td>tenths</td><td>hundredths</td><td>thousandths</td></tr><tr><td>1</td><td>8</td><td>2</td><td>5 •</td><td>7</td><td>6</td><td>3</td></tr></table></td></tr>
<tr><td>three-dimensional (3D)</td><td>• Able to be measured in three directions namely <i>length</i>, <i>width</i> and <i>height</i>.</td><td> </td></tr>
<tr><td>time</td><td>• The continuum from past to present to future.</td><td>The time is 9:25 am.</td></tr>
</table>

time zone	• Regions of different *times* around the world. Based on Greenwich Mean Time (GMT), each 15° of longitude away from Greenwich, England represents 1 hour of time.	NSW time is 3 hours ahead of WA time during daylight saving. 
tip	• Optional payment given in addition to a required payment, usually to express appreciation for excellent service.	The tip added an extra 5% to the cost of the meal.
tolerance	• The greatest *range* of variation that can be allowed. The amount of acceptable *error*.	See *tolerance interval*
tolerance interval	• To calculate the tolerance interval, add and subtract one *half* of the *precision* of the measuring instrument.	The ruler has a precision of 0.1 cm. The tolerance interval in this measurement is: 1.4 ± 0.05 cm or from 1.35 to 1.45 cm 
tonne (t)	• A *unit* of *measurement* for *mass equal* to 1000 *kilograms*.	The humpback whale can weigh 58 tonnes.
top view	• What you see of an object looking from a top *perspective*. • *Three-dimensional* objects have 3 views: front, top and side.	
total	• The whole lot. • The *sum* of two or more quantities.	The total of 2 and 7 and 3 is 12: $2 + 7 + 3 = 12$
total surface area (TSA)	• The complete *area* of the exterior surface of a *solid*.	The *TSA* of a rectangular box is $2lw + 2lh + 2wh$ 
transformation	• A movement of a shape in a *coordinate plane*. Types of transformations are *translations*, *reflections* and *rotations*.	See *translation*, *reflection* and *rotation*

translation	• A movement that *slides* a shape. Each *point* of the shape is moved the same distance, in the same direction, to produce a shape that is *congruent* to the original one.	Shape B is a translation of shape A. 
transversal	• A *line* that crosses a pair of *parallel lines*.	Line $\overleftrightarrow{AB}$ is a transversal. 
trapezium	• A *quadrilateral*. Two *opposite sides* are *parallel*.	 or 
tree diagram	• A tree diagram displays all the possible *outcomes* of an *event*.	 When tossing 2 coins there are 4 possible outcomes (branches): HH, HT, TH, TT
trend line	• A straight or curved *line* which is closest to all the *data* points in a *scatter plot* and gives the best approximation to the trend of the *set* of data.	Line B is a line of best fit, being closest to all the data points. 
tri	• Prefix meaning three.	A tricycle has 3 wheels. 
trial and error	• To try repeatedly and learn from mistakes.	This sum can be solved using trial and error. $\begin{array}{r} \text{TWO} \\ +\,\text{TWO} \\ \hline \text{FOUR} \end{array}$
triangle	• A *polygon* with 3 straight *sides*.	

triangle	Interior angles	Sides	Diagram
Right-angled triangle	1 right angle		
Scalene triangle	0 equal angles	0 sides of equal length	
Isosceles triangle	2 equal angles	2 sides of equal length	
Equilateral triangle	3 equal angles	3 sides of equal length	

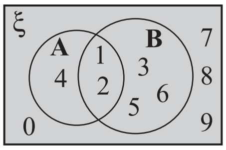
<table>
<tr><td>**triangular prism**</td><td>• A *three-dimensional* shape.
Two identical triangular *bases*.
Three rectangular faces.</td><td></td></tr>
<tr><td>**triangular pyramid**</td><td>• A *three-dimensional* shape.
One triangular *base*.
The other three faces are *triangles*.</td><td></td></tr>
<tr><td>**trigonometric ratios**</td><td>• There are three main trigonometric ratios, *sine*, *cosine* and *tangent*.</td><td>See *SOH - CAH - TOA*</td></tr>
<tr><td>**trigonometry**</td><td>• A branch of Mathematics where the relationship between the *sides* and *angles* of a *right-angled triangle* are studied. It involves the *functions* of *sine*, *cosine* and *tangent*.</td><td>See *SOH - CAH - TOA*
 </td></tr>
<tr><td>**trinomial**</td><td>• A *polynomial* with three *terms*.</td><td>$a + 2b + c$
$g^2 + 3gh - 2g$
$x^3 + 3x^2 + 8$
are all trinomials.</td></tr>
<tr><td>**triple**</td><td>• Multiply by three.</td><td>Children × 3 = triplets!
</td></tr>
<tr><td>**turn**</td><td>• To *rotate* about a point.</td><td></td></tr>
<tr><td>**twenty-four hour time**</td><td>• Time told in 24 hour lots using 4 *digits*.</td><td>Nine thirty am is 0930 or 09:30
Two thirty pm is 1430 or 14:30</td></tr>
<tr><td>**twice**</td><td>• Two times.</td><td>Sam has $5 and Jo has $10.
Jo has twice as much as Sam.</td></tr>
<tr><td>**two-dimensional (2D)**</td><td>• Able to be measured in 2 *directions* (*length* and *width*).</td><td> </td></tr>
<tr><td>**two-way table**</td><td>• A table that shows the combinations of possible outcomes and their values.</td><td>Possible outcomes when spinning a spinner labelled 1, 2, 3, 4 and flipping a coin.
<table>
<tr><td colspan="2">*Possible outcomes (sample space)*</td><td colspan="4">Spinner</td></tr>
<tr><td colspan="2"></td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td rowspan="2">Coin</td><td>H</td><td>H,1</td><td>H,2</td><td>H,3</td><td>H,4</td></tr>
<tr><td>T</td><td>T,1</td><td>T,2</td><td>T,3</td><td>T,4</td></tr>
</table></td></tr>
</table>

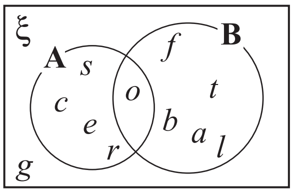
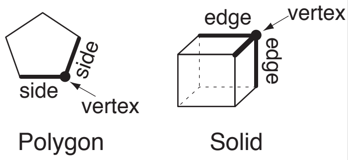
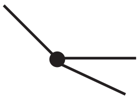
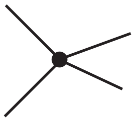
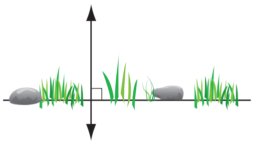
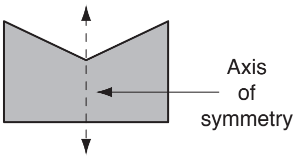
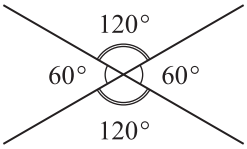
unit	• One.	The unit of measurement for length is metre (m)
units	• The *place value* before the decimal point between the *tens* and *tenths*.	1825.763 has 5 units.

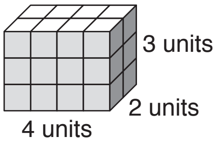
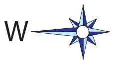
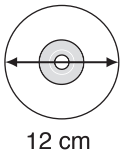
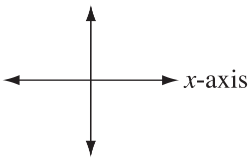
thousands	hundreds	tens	units	tenths	hundredths	thousandths
1	*8*	*2*	*5* •	*7*	*6*	*3*

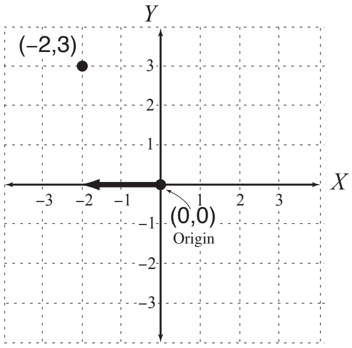
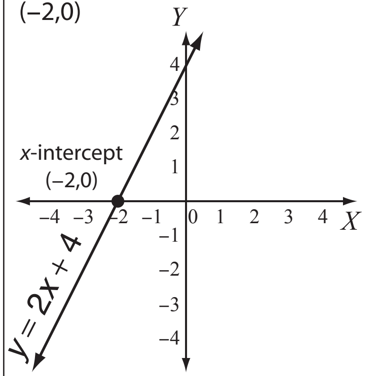
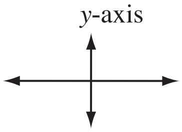
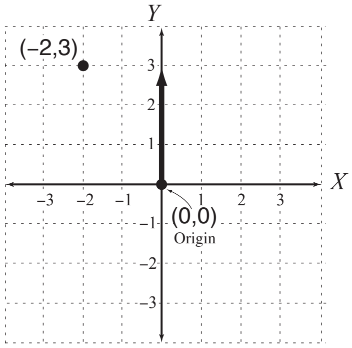
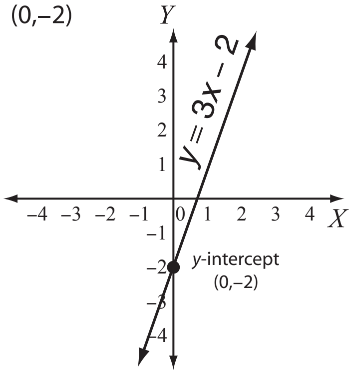
units of measurement	• Standard amount or quantity.	See *cubic unit* and *square unit.*

unit	Abbreviation	Examples	Used for measuring.....
• millimetre	mm	thickness of a plank of wood	LENGTH distance - length, width, height, diameter, perimeter
• centimetre	cm	width of a photo frame	
• metre	m	length of a lap of a stadium	
• kilometre	km	distance between two cities	
• gram	g	weight of an egg	MASS weight - people, animals, objects
• kilogram	kg	weight of a bag of apples	
• tonne	t	weight of an elephant	
• millilitre	mL	liquid in a can	CAPACITY (Liquid Volume) quantity - liquids
• litre	L	liquid in a bucket	
• megalitre	ML	liquid in a water tower	
• square centimetre	cm^2	area of a Maths book cover	AREA surface - objects, territories (countries, continents, oceans)
• square metre	m^2	area of the gym floor	
• square kilometre	km^2	area of Tasmania	
• cubic centimetre	cm^3	volume of water in a fish tank	VOLUME quantity - air, water
• cubic metre	m^3	volume of air in a warehouse	

universal set (ξ)	• A group of items that consists of all the *elements* under consideration. • The symbol for universal set is ξ.	$\xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ 
unlike terms	• Are *terms* that contain different *pronumerals* raised to the different *powers*. Unlike terms cannot be *added* or *subtracted* however they may be *multiplied* and *divided*.	Opposite to *like terms.* $7, 6a$ and $-4y^3$ are not like terms. $5w, \frac{6}{w}$ and $-18w^2$ are not like terms.

upper quartile (UQ)	• Is the *median* of the upper half of scores in a set of *data*. • 25% of the data lies above this number.	Data: 2, 2, 3, 3, 4, 5, 7, 8, 9, 9 The upper quartile (UQ) is 8. See *box-and-whisker plot*.
valid	• Grounded in *logic* or truth.	If A causes B and B causes C then it is valid to propose that A may cause C.
variable	• A *pronumeral* that can take on different values. • Is represented by a letter of the alphabet.	Opposite to a *constant*. In $y = x + 5$ 5 is constant x and y are variables.
Venn diagram	• A diagram using *circles* to show the relationship between *sets* of objects.	
vertex	• (pl. **vertices**) The point at which two *sides* (of a *polygon*) or three *edges* (of a *solid*) meet.	
vertex in a network	• A *point* in a network. A vertex can either be *odd* or *even* depending on the number of *arcs* (paths) leading to it.	Odd vertex - 3 arcs  Even vertex - 4 arcs 
vertical line	• A *line* at a *right angle* to the horizon.	
vertical symmetry	• A shape has vertical symmetry if an *axis of symmetry* is vertical.	
vertically opposite angles	• *Angles* on opposite sides of a *pair* of *intersecting lines*. • Vertically opposite angles are *congruent*.	All vertically opposite angles are equal in a pair of intersecting lines. 

volume	• The amount of space that a *solid* occupies. Volume is measured in *cubic units*. e.g. cubic centimetres (cm^3) or cubic metres (m^3).	Volume of a rectangular prism is calculated by multiplying length by width by height: $V = lwh$ $= 4 \times 2 \times 3$ $= 24$ Volume = 24 cubic units 
week	• A *unit* of *time* equal to 7 days; Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday.	Roger was on holidays for one week (seven days).
weight	• The heaviness of an object. Equals the *mass* of an object times the force of gravity. This means that weight changes with any change in gravity.	A 3 kg brick weighs: 3 kg on Earth, about 0.5 kg on the moon, 0 kg in outer space.
west	• A *compass direction*.	The sun sets in the west. 
whole numbers	• The *counting numbers* from zero to *infinity*.	0, 1, 2, 3, 4, 5, ... are whole numbers.
width	• How wide an object is. The sideways *dimension*.	The width of the CD is 12 cm.  
***x*-axis**	• The *horizontal* axis.	

***x*-coordinate**	• The *first* number in an ordered pair. The position of a *point* along the *x-axis*.	The *x*-coordinate of the ordered pair (–2,3) is –2. 
***x*-intercept**	• The point at which a graph crosses the *x-axis*.	This line crosses the *x*-axis at (–2,0) 
***y*-axis**	• The *vertical* axis.	
***y*-coordinate**	• The *second* number in an ordered pair. The position of a *point* along the *y-axis*.	The *y*-coordinate of the ordered pair (–2,3) is 3. 
***y*-intercept**	• The *point* at which a *graph* crosses the *y-axis*.	This line crosses the *y*-axis at (0,–2) 
year	• A *unit* of *time* equal to 365 days. (366 in a leap year).	1st of January to the 31st of December.

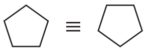
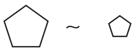
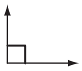
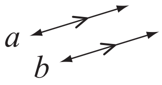
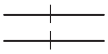
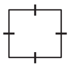
MATHS FACTS

SYMBOLS

Number

$+$	plus or add
$-$	minus or subtract
$\times$	multiplied by, times, lots of
$\div$	divided by, into groups of
$=$	equals, is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$<$	is less than, $4 < 6$
$>$	is greater than, $8 > 5$
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
( )	brackets, a grouping symbol
%	percent, $12\% = \frac{12}{100}$
•	decimal point as in 7.9
-3	negative 3
6^3	6 raised to the 3rd power, $6 \times 6 \times 6$
$\sqrt{9}$	square root of 9
$\frac{4}{7}$	fraction, $4 \div 7$, four sevenths
$a{:}b$ or $\frac{a}{b}$	ratio of a to b
$2.\dot{4}$	recurring decimal
$2.\dot{1}\dot{3}$	recurring decimal

Geometry

π (pi)	≈ 3.14 or $\frac{22}{7}$ ratio of the circumference to the diameter of a circle
$^\circ$	degree (a right angle measures 90°)
$\equiv$	is congruent to, 
$\sim$	is similar to, 
$\parallel$	is parallel to
$\perp$	is perpendicular to
ΔABC	triangle with vertices A, B and C
	right angle
$\overleftrightarrow{AD}$	line AD
$\overline{BC}$	segment BC
	parallel lines (line a is parallel to line b)
	congruent segments
	equal angles
	equal side lengths

Algebra

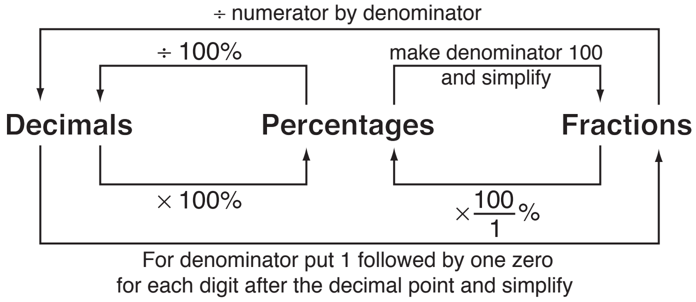
$3x$	3 times x, 3 lots of x, $3 \times x$, $3(x)$
x^2	x raised to the 2nd power, $x \times x$
$-x$	opposite of x
$\frac{1}{x}$	reciprocal of x
(x,y)	coordinates in a Cartesian plane
m	gradient of a linear graph
c	y-intercept of a linear graph

NUMBER FACTS (1)

Place value

millions	*hundreds of thousands*	*tens of thousands*	*thousands*	*hundreds*	*tens*	*units*	*decimal point*	*tenths*	*hundredths*	*thousandths*
1 000 000	*100 000*	*10 000*	*1000*	*100*	*10*	*1*	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Decimals / Fractions / Percentages



Fraction	Decimal	Percentage
$\frac{1}{1}$	1	100%
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	$0.\dot{3}$	33.33%
$\frac{2}{3}$	$0.\dot{6}$	66.66%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{4}{5}$	0.8	80%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{9}$	$0.\dot{1}$	11.11%

0

Subtraction $a - 0 = a$

Multiplication $a \times 0 = 0$ and $0 \times a = 0$

Division $0 \div a = 0$

1

Multiplication $a \times 1 = a$ and $1 \times a = a$

Division $a \div 1 = a$

Prime numbers < 100

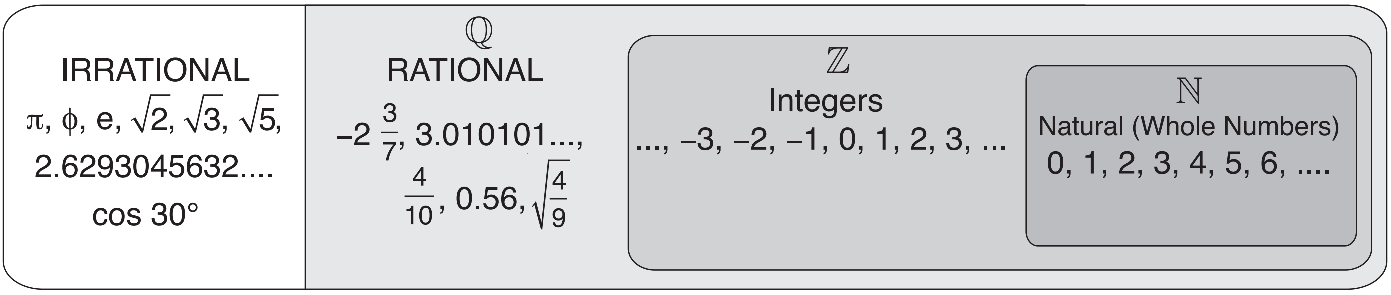
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97

Perfect squares of numbers 0 to 30

0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841 and 900

NUMBER FACTS (2)

Real Numbers $\mathbb{R}$



IRRATIONAL

$\pi, \phi, e, \sqrt{2}, \sqrt{3}, \sqrt{5},$

2.6293045632....

cos 30°

$\mathbb{Q}$

RATIONAL

$-2\frac{3}{7}, 3.010101..., \frac{4}{10}, 0.56, \sqrt{\frac{4}{9}}$

$\mathbb{Z}$

Integers

..., –3, –2, –1, 0, 1, 2, 3, ...

$\mathbb{N}$

Natural (Whole Numbers)

0, 1, 2, 3, 4, 5, 6, ...

Operation terminology

Addition: sum, altogether, in total, more than

Subtraction: difference, less than, change

Multiplication: product, times, lots of

Division: a fraction (half, third, quarter) of, quotient

Order of Operations

The order of doing operations is:

1) Simplify inside all brackets.
2) Evaluate powers and square roots.
3) Calculate × and ÷ from left to right.
4) Calculate + and – from left to right.

Sign Rules

$+ + = +$

$- - = +$

$+ - = -$

$- + = -$

Rates and Proportions

$$a : b = \frac{a}{b}$$

$$a : b = c : d$$

$$\frac{a}{b} \times \frac{c}{d}$$

$$a \times d = b \times c$$

$$ad = bc$$

Applied number - money

$$\text{Percentage} = \text{Fraction} \times \frac{100}{1}\,\%$$

$$\frac{P}{100} = P\%$$

Commission = % × Selling price

Simple Interest = Principal × rate × time

$$SI = PRT$$

$$\text{Percentage change} = \frac{\text{amount of change}}{\text{original amount}} \times \frac{100}{1}\,\%$$

Applied number - distance

Distance (d) = average speed (v) × time taken (t)

$$d = vt$$

$$v = \frac{d}{t}$$

$$t = \frac{d}{v}$$

Applied number - rates

$$\text{Rate } (r) = \frac{\text{amount } (a)}{\text{time } (t)}$$

$$r = \frac{a}{t}$$

$$a = rt$$

$$t = \frac{a}{r}$$

ALGEBRA FACTS

Identity Properties

Addition $a + 0 = a$ and $0 + a = a$

Multiplication $a \times 1 = a$ and $1 \times a = a$

Associative Properties

Addition $(a + b) + c = a + (b + c)$

Multiplication $(a \times b) \times c = a \times (b \times c)$

Commutative Properties

Addition $a + b = b + a$

Multiplication $a \times b = b \times a$

Distributive Properties

$a(b + c) = ab + ac$

$a(b - c) = ab - ac$

Perfect square rules

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Difference of two squares rule

$a^2 - b^2 = (a + b)(a - b)$

Inverse number rules

Addition $a + -a = 0$ and $-a + a = 0$

Multiplication $a \times \frac{1}{a} = 1$ and $\frac{1}{a} \times a = 1$

Index Laws

$a^0 = 1$	Zero exponent
$a^{-n} = \frac{1}{a^n}$	Negative exponent
$a^m \times a^n = a^{m+n}$	Product of powers
$\frac{a^m}{a^n} = a^{m-n}$	Quotient of powers
$(a^m)^n = a^{mn}$	Power to power
$(ab)^n = a^n b^n$	Product to power
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	Quotient to power

Square root rules

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

$\sqrt{a} \times \sqrt{a} = \sqrt{a \times a} = a$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$\frac{\sqrt{a}}{\sqrt{a}} = \sqrt{\frac{a}{a}} = 1$

Properties of Equality

Addition $a = b$

$a + c = b + c$

Subtraction $a = b$

$a - c = b - c$

Multiplication $a = b$

$ac = bc$

Division $a = b$

$\frac{a}{c} = \frac{b}{c},\ c \neq 0$

Inverse operation rules

Operation $+$	Inverse Operation $-$	Operation $-$	Inverse Operation $+$	Operation $\times$	Inverse Operation $\div$	Operation $\div$	Inverse Operation $\times$
$x + 3 = 6$		$x - 3 = 6$		$3x = 6$		$\frac{x}{3} = 6$	
$x + 3 - 3 = 6 - 3$		$x - 3 + 3 = 6 + 3$		$\frac{3x}{3} = \frac{6}{3}$		$\frac{x}{3} \times 3 = 6 \times 3$	
$x = 3$		$x = 9$		$x = 2$		$x = 18$	

MEASUREMENT FACTS (1)

CONVERSIONS

Length

10 millimetres (mm) = 1 centimetre (cm)

$$\left.\begin{array}{r}100\text{ cm} =\\ 1000\text{ mm} =\end{array}\right]\ 1\text{ metre (m)}$$

1000 m = 1 kilometre (km)

Area

100 square mm (mm^2) = 1 square cm (cm^2)
10 000 cm^2 = 1 square metre (m^2)
10 000 m^2 = 1 hectare (ha)
1 000 000 m^2 = 1 square km (km^2)

Time

60 seconds (s) = 1 minute (min)
60 minutes (min) = 1 hour (h)
24 hours (h) = 1 day
7 days = 1 week
2 weeks = 1 fortnight
4 weeks (approx.) = 1 month

$$\left.\begin{array}{r}365 =\\ 52\text{ weeks (approx.)} =\\ 12\text{ months} =\end{array}\right]\ 1\text{ year}$$

366 days = 1 leap year
10 years = 1 decade
100 years = 1 century

Liquid Capacity

1000 millilitres (mL) = 1 litre (L)
1000 000 L = 1 megalitre (ML)
1000 cubic cm (cm^3) = 1 L
1000 L = 1 cubic metre (m^3)

Temperature - degrees Celsius (°C)

0°C = freezing point of water
100°C = boiling point of water
37°C = human body temperature

Volume

1000 cubic mm (mm^3) = 1 cubic cm (cm^3)
1 000 000 cm^3 = 1 cubic metre (m^3)

Mass

1000 milligrams (mg) = 1 gram (g)
1000 g = 1 kilogram (kg)
1000 kg = 1 tonne (t)

METRIC PREFIXES

giga (G) = 1 billion = 1 000 000 000

mega (M) = 1 million = 1 000 000

kilo (k) = 1 thousand = 1000

hecto (h) = 1 hundred = 100

deca (da) = 1 ten = 10

micro (μ) = 1 millionth = $\frac{1}{1\,000\,000}$

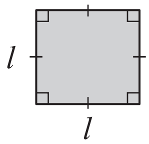
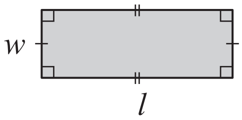
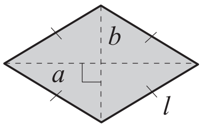
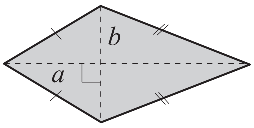
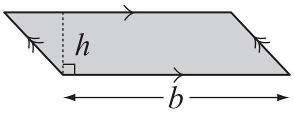
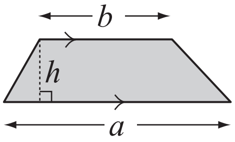
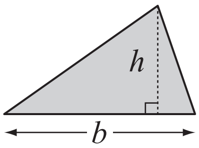
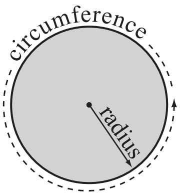
milli (m) = 1 thousandth = $\frac{1}{1000}$

centi (c) = 1 hundredth = $\frac{1}{100}$

deci (d) = 1 tenth = $\frac{1}{10}$

MEASUREMENT FACTS (2)

2D shapes - Formulae

Name	Shape	Perimeter	Area
Square		$P = 4 \times l$ $= 4l$	$A = l \times l$ $= l^2$
Rectangle		$P = 2l + 2w$ $= 2(l + w)$	$A = l \times w$ $= lw$
Rhombus		$P = 4 \times l$ $= 4l$	$A = \frac{a \times b}{2}$ $= \frac{1}{2}ab$
Kite		$P =$ Sum of all sides	$A = \frac{a \times b}{2}$ $= \frac{1}{2}ab$
Parallelogram		$P =$ Sum of all sides	$A = b \times h$ $= bh$
Trapezium		$P =$ Sum of all sides	$A = \frac{1}{2}(a + b)h$
Triangle		$P =$ Sum of all sides	$A = \frac{b \times h}{2}$ $= \frac{1}{2}bh$
Circle		$C = 2\pi r$	$A = \pi r^2$ where $\pi \approx 3.14$ or $\frac{22}{7}$

Prefixes

poly - many
equi - equal
hedra - face
gon - angle
lateral - side

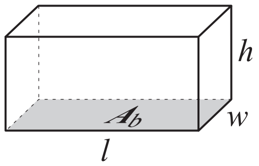
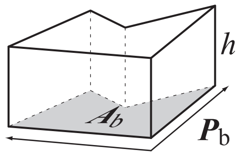
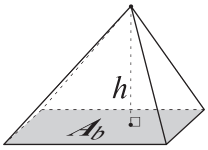
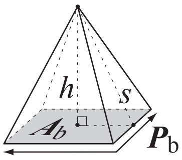
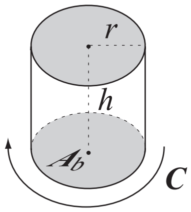
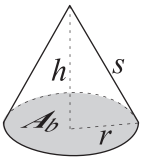
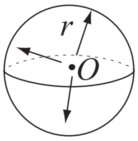
mono - one
bi or **di** - two
tri - three
quad or **tetra** - four
penta - five
hexa - six
hepta - seven
octa - eight
nona - nine
deca - ten

Abbreviations

l	length
w	width
h	height
b	base length
P	perimeter
r	radius
C	circumference
A	area

MEASUREMENT FACTS (3)

3D shapes - Formulae

Name	Shape	Surface Area	Volume
Rectangular Prism		$TSA = 2lw + 2wh + 2lh$ $= 2(lw + wh + lh)$	$V = lwh$ or $= A_bh$
Prism - (All)		$TSA = P_b \times h + 2A_b$ $= P_bh + 2A_b$	$V = A_bh$
Pyramid		$TSA =$ Sum of all areas of faces	$V = \frac{1}{3}A_bh$
Regular Pyramid		$TSA = \frac{P_b \times s}{2} + A_b$ $= \frac{P_b s}{2} + A_b$	$V = \frac{1}{3}A_bh$
Cylinder		$TSA = 2\pi r^2 + 2\pi rh$ $= 2\pi r(r + h)$	$V = A_b \times h$ $= \pi r^2h$
Cone		$TSA = \pi r^2 + \pi rs$ $= \pi r(r + s)$	$V = \frac{1}{3}A_b \times h$ $= \frac{1}{3}\pi r^2h$
Sphere		$TSA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

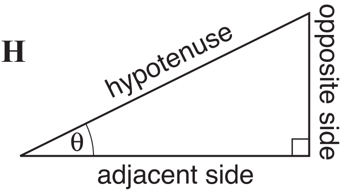
Abbreviations

l	length	TSA	total surface area
w	width	V	volume
h	height	A_b	base area
b	base length	P_b	perimeter of base
P	perimeter	s	slant height
r	radius		
C	circumference		
A	area		

TRIGONOMETRY FACTS

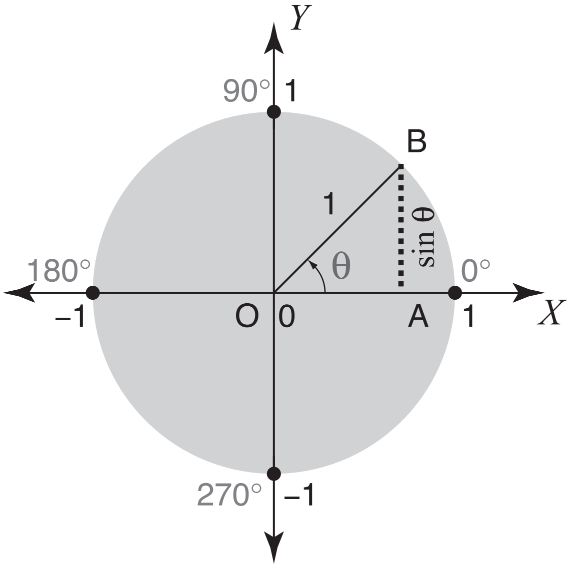
Sine

$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$ **SOH**



$\sin\theta = \dfrac{AB}{OB} = \dfrac{AB}{1} = AB$

Degree	0°	30°	45°	60°	90°	180°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0

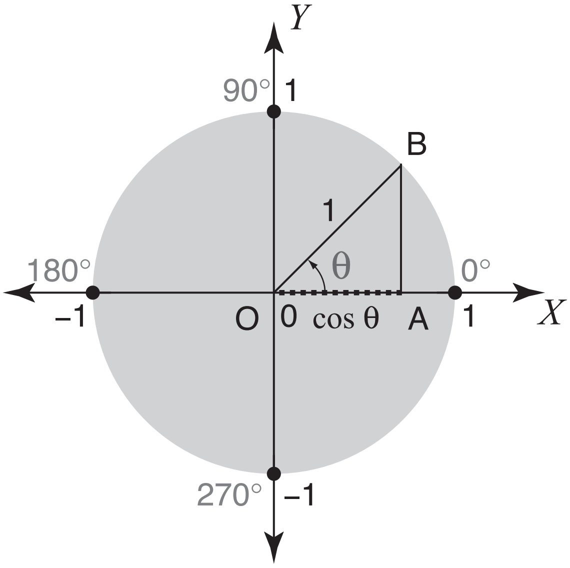


Cosine

$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$ **CAH**

$\cos\theta = \dfrac{OA}{OB} = \dfrac{OA}{1} = OA$

Degree	0°	30°	45°	60°	90°	180°
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1

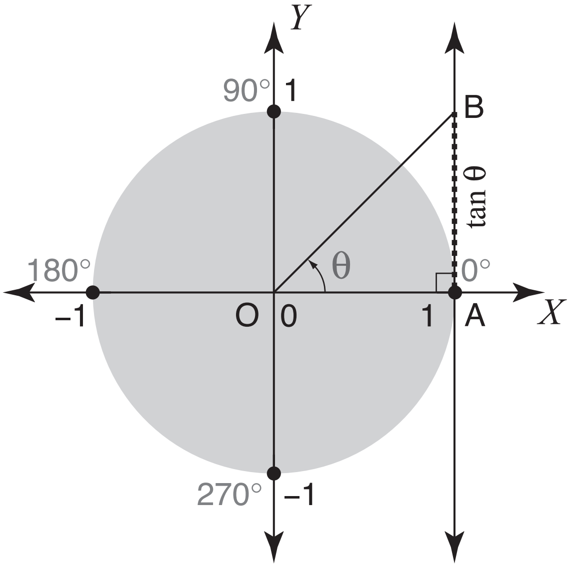


Tangent

$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$ **TOA**

$\tan\theta = \dfrac{AB}{OA} = \dfrac{AB}{1} = AB$

Degree	0°	30°	45°	60°	90°	180°
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	☒	0



Trigonometry hint: SOH - CAH - TOA

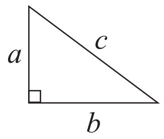
GEOMETRY FACTS (1)

Euler's formula

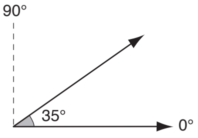
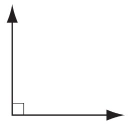
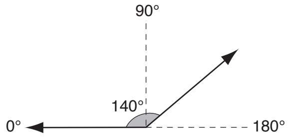
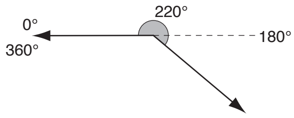
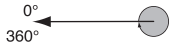
For any polyhedra:
Edges = Vertices + Faces – 2
E = **V** + **F** – 2

Pythagoras' theorem

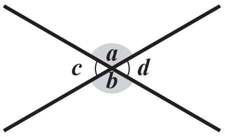
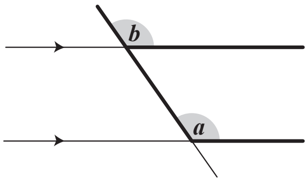
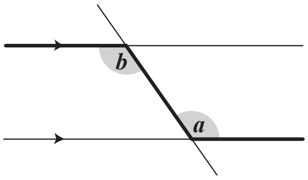
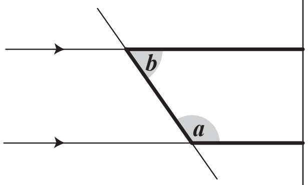
$a^2 + b^2 = c^2$



Angle types

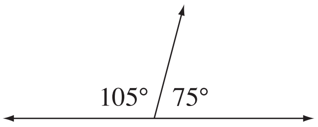
Acute	**Right**	**Obtuse**	**Straight**	**Reflex**	**Revolution**
< 90°	90°	more than 90° less than 180°	180°	more than 180° less than 360°	360°
					

Properties of angles

Vertically opposite	**Corresponding**	**Alternate**	**Co-interior**
			
$\angle a = \angle b$ and $\angle c = \angle d$	$\angle a = \angle b$	$\angle a = \angle b$	$\angle a + \angle b = 180°$

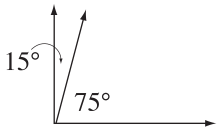
Supplementary Angles

Add to 180°



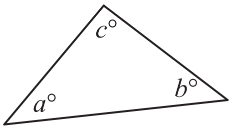
Complementary Angles

Add to 90°

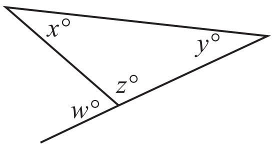


Properties of angles in a triangle

The sum of interior angles of a triangle is 180°.
$a° + b° + c° = 180°$



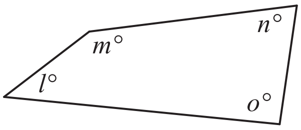
An exterior angle of a triangle is equal to the sum of the two opposite interior angles of the triangle.
$w° = x° + y°$



Sides and angles	Triangle type
no equal sides/angles	**scalene**
two equal sides/angles	**isosceles**
three equal sides/angles	**equilateral**
all acute angles	**acute-angled**
one right angle	**right-angled**
one obtuse angle	**obtuse-angled**

Properties of angles in a quadrilateral

The sum of interior angles of a quadrilateral is 360°.
$l° + m° + n° + o° = 360°$

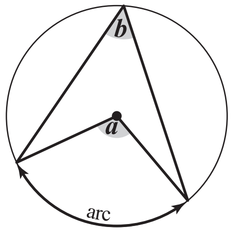


GEOMETRY FACTS (2)

Properties of angles in a circle

Property 1

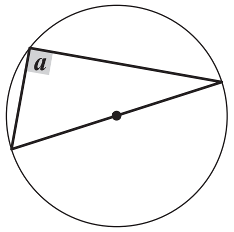
The angle that an arc forms at the centre of a circle is twice the size of the angle formed by the same arc on the circumference.



$\angle a = 2 \times \angle b$

Property 2

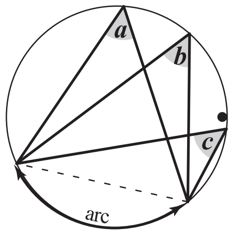
The angle formed on the circumference from a diameter of a circle is always a right angle.



$\angle a = 90°$

Property 3

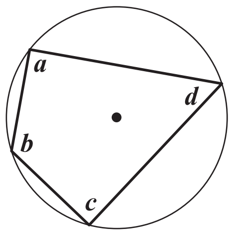
All angles at the circumference standing on the same arc, in the same segment, are equal.



$\angle a = \angle b = \angle c$

Property 4

the opposite angles in a cyclic quadrilateral (all 4 vertices are on the circumference) add up to 180° (are supplementary).

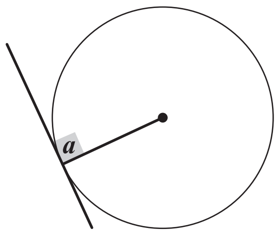


$\angle a + \angle c = 180°$

$\angle b + \angle d = 180°$

Property 5

Any tangent drawn on a circle meets the radius of the circle at right angles.

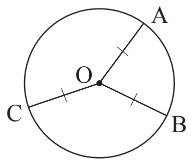


$\angle a = 90°$

Properties of lines related to a circle

Property 1

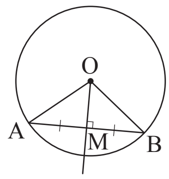
The radii in a circle are the same length.



$\overline{OA} \equiv \overline{OB} \equiv \overline{OC}$

Property 2

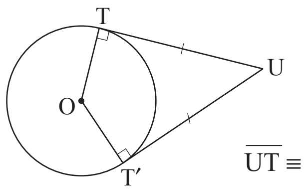
A line through the circle centre perpendicular to a chord bisects the chord.



$\overline{AM} \equiv \overline{MB}$

Property 3

The common tangents from a point to a circle are equal in length.



$\overline{UT} \equiv \overline{UT'}$

ANSWERS

1. [Long ×,÷] page 1

Skill 1.1 a) 2680, b) 5220, c) 4440, d) 6230, e) 24 150, f) 63 200 g) 178 000, h) 258 800, i) 519 200, j) 454 800, k) 620 000 l) 950 000

Skill 1.2 a) 285, b) 351, c) 612, d) 784, e) 4658, f) 8664, g) 8964 h) 6636, i) 23 128, j) 13 583, k) 30 555, l) 29 184, m) 17 028 n) 72 852, o) 38 584, p) 192 864, q) 40 219, r) 238 345 s) 85 527, t) 226 884, u) 522 376

Skill 1.3 a) 46 540, b) 98 490, c) 58 500, d) 89 180, e) 98 440 f) 727 350, g) 1 219 680, h) 394 500, i) 822 600

Skill 1.4 a) 84, b) 17, c) 65, d) 124, e) 264, f) 122, g) 831, h) 497 i) 827, j) 596, k) 945, l) 487, m) 859, n) 1712, o) 1131

Skill 1.5 a) 120, b) 1500, c) 1350, d) 982, e) 32, f) 800, g) 54.3 h) 27.8, i) 546.6, j) 64.5, k) 430.7, l) 55.07, m) 190.34 n) 23.79 o) 42.21

Skill 1.6 a) 82, b) 88, c) 139, d) 137, e) 138, f) 274, g) 978, h) 3799 i) 751, j) 357, k) 626, l) 582, m) 459, n) 563, o) 237

Skill 1.7 a) 28, b) 42, c) 43, d) 66, e) 52, f) 34, g) 29, h) 33, i) 36 j) 24, k) 53, l) 37, m) 537, n) 73, o) 215, p) 158, q) 123 r) 163

Skill 1.8 a) 127.8, b) 52.5, c) 73.2, d) 689.5, e) 571.5, f) 208.75 g) 58.25, h) 73.75, i) 171.5, j) 85.3, k) 345.8, l) 61.75

Skill 1.9 a) $72.\dot{2}$, b) $148.\dot{3}$, c) $61.\dot{6}$, d) $144.\dot{6}$, e) $156.\dot{3}$, f) $106.\dot{8}$ g) $978.\dot{3}$, h) $144.\dot{8}$, i) $89.\dot{5}\dot{4}$, j) $73.\dot{6}$, k) $34.\dot{2}\dot{7}$, l) $305.\dot{7}\dot{3}$

2. [Decimal +,–] page 13

Skill 2.1 a) 0.22, b) 11.3, c) 0.92, d) 8.15, e) 29.514, f) 7.266, g) 5.5 h) 27.429, i) 10.005, j) 10.95, k) 41.46, l) 23.245, m) 11.095 n) 11.263, o) 6.023, p) 20.42, q) 2, r) 16.007, s) 2.354 t) 10.048, u) 52.217, v) 1.012

Skill 2.2 a) 5.11, b) 1.8, c) 4.51, d) 0.971, e) 9.73, f) 5.77, g) 6.072 h) 3.236, i) 1.302, j) 4.65, k) 12.453, l) 0.077, m) 5.965 n) 18.57, o) 0.157, p) 14.589, q) 0.606, r) 6.67, s) 12.73 t) 4.797

Skill 2.3 a) 0.93, b) 0.91, c) 7.3, d) 0.966, e) 9.16, f) 4.363, g) 91.7 h) 99.77, i) 7.38, j) 4.73, k) 8.396, l) 3.29, m) 1.923, n) 9.22 o) 4.433, p) 17.485, q) 17.98, r) 42.693, s) 24.997, t) 12.667

Skill 2.4 a) 4.448 b) 5.25, c) 9.47, d) 2.389, e) 16.68, f) 7.253 g) 1.577, h) 10.44, i) 15.446, j) 6.374, k) 2.917, l) 19.514 m) 5.342, n) 12.96

3. [Decimal ×,÷] page 21

Skill 3.1 a) 2.8, b) 7.2, c) 18.2, d) 24, e) 22.82, f) 6.24, g) 73.38 h) 13.563, i) 149.59, j) 114.4, k) 27.504, l) 144.81

Skill 3.2 a) 637, b) 39.8, c) 0.3, d) 429, e) 300.7, f) 2188, g) 0.5 h) 80, i) 12, j) 0.39, k) 7.3, l) 570, m) 430, n) 0.232, o) 128 p) 0.927, q) 0.8, r) 42.24

Skill 3.3 a) 0.784, b) 0.42, c) 6.85, d) 5.932, e) 4.845, f) 2.237 g) 0.0313, h) 9.0909, i) 1.2345, j) 0.128, k) 0.325, l) 0.139 m) 0.5308, n) 0.0102, o) 0.0054

Skill 3.4 a) 0.42 b) 0.32, c) 0.45, d) 2.16, e) 3.206, f) 0.0136 g) 0.351, h) 0.0894, i) 2.925, j) 18.425, k) 21.045, l) 5.504

Skill 3.5 a) 20.4, b) 19.2, c) 9.3, d) 23.4, e) 48.6, f) 18.7, g) 6.21 h) 39.74, i) 12.07, j) 26.06, k) 17.06, l) 113.54, m) 25.14 n) 65.21, o) 30.125

Skill 3.6 a) 0.067, b) 23.06, c) 1.53, d) 0.335, e) 8.009, f) 0.324 g) 0.036, h) 0.008, i) 0.653, j) 0.492, k) 0.068, l) 0.0074 m) 2.9725, n) 0.0331, o) 0.0005, p) 0.0015, q) 0.006 r) 0.0102, s) 0.00325, t) 0.0426, u) 0.0023

Skill 3.7 a) 56, b) 30.3, c) 24, d) 0.58, e) 427, f) 3.8, g) 76, h) 9 i) 6530, j) 0.5, k) 89, l) 7153, m) 10.48, n) 124, o) 75 p) 120, q) 2320, r) 3580, s) 40, t) 30, u) 8040

Skill 3.8 a) 12.3, b) 3, c) 7, d) 10, e) 11.7, f) 12.3, g) 10, h) 7.1 i) 26.7, j) 40, k) 100, l) 0.8, m) 24.5, n) 31.5, o) 1.25

Skill 3.9 a) 300, b) 350, c) 200, d) 30, e) 450, f) 40, g) 900, h) 200 i) 500, j) 200, k) 120, l) 28

4. [Fraction +,–] page 31

Skill 4.1 a) $\frac{5}{8}$, b) $\frac{5}{6}$, c) $\frac{8}{9}$, d) $\frac{7}{11}$, e) $\frac{7}{9}$, f) $\frac{12}{13}$, g) $1\frac{4}{7}$, h) $1\frac{3}{5}$ i) $1\frac{4}{9}$, j) $1\frac{2}{11}$, k) $1\frac{4}{17}$, l) $1\frac{4}{15}$, m) $\frac{1}{3}$, n) $\frac{1}{2}$, o) $\frac{3}{4}$, p) $\frac{3}{4}$ q) $\frac{2}{3}$, r) $\frac{2}{5}$, s) $1\frac{1}{2}$, t) 1, u) $1\frac{2}{3}$, v) 2, w) $1\frac{1}{3}$, x) $1\frac{1}{3}$ y) 2, z) $1\frac{1}{6}$, zz) $1\frac{1}{5}$

Skill 4.2 a) $\frac{2}{5}$, b) $\frac{3}{8}$, c) $\frac{4}{9}$, d) $1\frac{4}{9}$, e) $1\frac{7}{10}$, f) $2\frac{2}{7}$, g) $\frac{2}{3}$, h) $\frac{1}{6}$ i) $\frac{1}{2}$, j) $\frac{1}{3}$, k) $\frac{1}{5}$, l) $\frac{1}{4}$, m) 3, n) $1\frac{1}{4}$, o) $1\frac{1}{6}$

Skill 4.3 a) $5\frac{6}{7}$, b) $3\frac{8}{9}$, c) $5\frac{10}{11}$, d) 3, e) $5\frac{4}{5}$, f) $3\frac{3}{4}$, g) $4\frac{1}{7}$, h) $5\frac{2}{9}$ i) $9\frac{7}{10}$, j) $4\frac{1}{4}$, k) $6\frac{2}{3}$, l) $6\frac{1}{4}$

Skill 4.4 a) $1\frac{2}{5}$, b) $2\frac{6}{7}$, c) $1\frac{5}{11}$, d) $3\frac{1}{4}$, e) $2\frac{1}{3}$, f) $2\frac{2}{3}$, g) $1\frac{2}{3}$ h) $1\frac{1}{2}$, i) $1\frac{3}{5}$, j) $2\frac{3}{4}$, k) $3\frac{5}{6}$, l) $2\frac{3}{5}$, m) $1\frac{1}{2}$, n) $1\frac{2}{3}$, o) $1\frac{2}{3}$

Skill 4.5 a) $1\frac{5}{7}$, b) $2\frac{5}{9}$, c) $1\frac{4}{7}$, d) $1\frac{7}{13}$, e) $5\frac{3}{8}$, f) $3\frac{7}{12}$, g) $1\frac{5}{6}$ h) $3\frac{4}{7}$, i) $3\frac{3}{8}$, j) $4\frac{1}{3}$, k) $1\frac{1}{4}$, l) $5\frac{3}{7}$, m) $3\frac{1}{2}$, n) $2\frac{1}{8}$ o) $4\frac{3}{10}$, p) $1\frac{2}{5}$, q) $1\frac{5}{8}$, r) $2\frac{1}{6}$

Skill 4.6 a) $\frac{17}{20}$, b) $\frac{8}{21}$, c) $\frac{7}{12}$, d) $\frac{1}{2}$, e) $1\frac{1}{2}$, f) $1\frac{1}{8}$, g) $1\frac{3}{8}$ h) $1\frac{1}{2}$, i) $\frac{1}{4}$, j) $1\frac{2}{3}$, k) $\frac{9}{25}$, l) $\frac{3}{4}$

Skill 4.7 a) $\frac{21}{40}$, b) $\frac{19}{30}$, c) $\frac{25}{33}$, d) $1\frac{1}{12}$, e) $1\frac{1}{15}$, f) $1\frac{7}{20}$, g) $1\frac{3}{10}$ h) $1\frac{1}{14}$, i) $1\frac{5}{24}$

Skill 4.8 a) $\frac{23}{24}$, b) $\frac{11}{20}$, c) $\frac{7}{30}$, d) $\frac{11}{36}$, e) $\frac{9}{50}$, f) $\frac{17}{30}$, g) $1\frac{2}{15}$ h) $1\frac{7}{12}$, i) $1\frac{3}{40}$

Skill 4.9 a) $\frac{5}{8}$, b) $\frac{9}{16}$, c) $\frac{5}{14}$, d) $\frac{2}{15}$, e) $\frac{3}{25}$, f) $\frac{1}{12}$, g) $\frac{3}{5}$, h) $\frac{1}{8}$, i) $\frac{1}{9}$

Skill 4.10 a) $\frac{7}{40}$, b) $\frac{1}{10}$, c) $\frac{13}{30}$, d) $\frac{1}{24}$, e) $\frac{5}{12}$, f) $\frac{5}{18}$, g) $\frac{4}{35}$, h) $\frac{5}{14}$ i) $\frac{3}{20}$, j) $\frac{11}{56}$, k) $\frac{1}{30}$, l) $\frac{5}{33}$

Skill 4.11 a) $\frac{3}{20}$, b) $\frac{11}{40}$, c) $\frac{1}{30}$, d) $\frac{1}{36}$, e) $\frac{7}{24}$, f) $\frac{13}{45}$, g) $\frac{1}{10}$, h) $\frac{1}{10}$ i) $\frac{2}{15}$

Skill 4.12 a) $\frac{5}{8}$, b) $\frac{1}{16}$, c) $\frac{5}{12}$, d) $\frac{7}{30}$, e) $\frac{13}{42}$, f) $\frac{19}{90}$, g) $\frac{4}{11}$, h) $\frac{4}{5}$, i) $\frac{2}{7}$

Skill 4.13 a) $3\frac{7}{18}$, b) $5\frac{1}{20}$, c) $2\frac{19}{40}$, d) $2\frac{5}{6}$, e) $2\frac{9}{10}$, f) $2\frac{19}{21}$, g) $1\frac{3}{14}$ h) $3\frac{5}{12}$, i) $2\frac{1}{6}$

5. [Fraction ×,÷] page 49

Skill 5.1 a) $1\frac{5}{7}$, b) $1\frac{1}{9}$, c) $2\frac{2}{5}$, d) $2\frac{1}{10}$, e) $1\frac{5}{7}$, f) $1\frac{1}{3}$, g) $\frac{1}{2}$
h) $1\frac{2}{3}$, i) $\frac{1}{4}$, j) $1\frac{1}{2}$, k) $1\frac{1}{3}$, l) $6\frac{2}{3}$

Skill 5.2 a) $\frac{3}{20}$, b) $\frac{2}{21}$, c) $\frac{5}{48}$, d) $\frac{3}{16}$, e) $\frac{3}{10}$, f) $\frac{4}{11}$, g) $\frac{1}{4}$, h) $\frac{1}{5}$
i) $\frac{1}{5}$, j) $\frac{1}{4}$, k) $\frac{1}{6}$, l) $\frac{1}{8}$, m) $\frac{1}{8}$, n) $\frac{2}{5}$, o) $\frac{1}{4}$, p) $\frac{1}{9}$, q) $\frac{2}{15}$
r) $\frac{2}{25}$, s) $\frac{2}{15}$, t) $\frac{1}{4}$, u) $\frac{4}{25}$

Skill 5.3 a) $1\frac{1}{4}$, b) 6, c) $4\frac{1}{2}$, d) $1\frac{1}{5}$, e) $2\frac{1}{4}$, f) $\frac{3}{4}$, g) $2\frac{3}{8}$, h) $4\frac{1}{2}$
i) 2

Skill 5.4 a) $\frac{1}{10}$, b) $\frac{2}{15}$, c) $\frac{1}{16}$, d) $\frac{1}{4}$, e) $\frac{3}{16}$, f) $\frac{2}{5}$, g) $\frac{1}{12}$, h) $\frac{4}{15}$
i) $\frac{1}{20}$, j) $\frac{3}{11}$, k) $\frac{1}{8}$, l) $\frac{1}{15}$

Skill 5.5 a) $\frac{5}{24}$, b) $\frac{4}{9}$, c) $\frac{16}{35}$, d) $\frac{2}{5}$, e) $1\frac{1}{4}$, f) 4, g) $1\frac{1}{2}$, h) $2\frac{2}{5}$
i) $\frac{2}{3}$

Skill 5.6 a) 6, b) 7, c) 9, d) 10, e) 24, f) 16, g) $16\frac{1}{2}$, h) $9\frac{1}{3}$
i) $22\frac{1}{2}$

Skill 5.7 a) $\frac{1}{10}$, b) $\frac{1}{15}$, c) $\frac{2}{7}$, d) $\frac{1}{14}$, e) $\frac{1}{40}$, f) $\frac{1}{18}$, g) $\frac{1}{55}$, h) $\frac{1}{60}$, i) $\frac{2}{65}$

Skill 5.8 a) 3, b) 2, c) $1\frac{1}{3}$, d) $\frac{5}{12}$, e) $1\frac{5}{9}$, f) $\frac{8}{9}$, g) 10, h) $1\frac{1}{2}$, i) $1\frac{5}{17}$

6. [Percentages] page 59

SSkill 6.1 a) 40%, b) 50%, c) 75%, d) 38%, e) 90%, f) A, g) 25%
h) 40%, i) 12%, j) 20%

Skill 6.2 a) 19%, b) 20%, c) 90%, d) 7%, e) 76%, f) 65%, g) 33%
h) 18%, i) 21%, j) 36%

Skill 6.3 a) 80, b) 50, c) 60, d) 21, e) 75, f) 32, g) $420, h) $14
i) $15, j) $5.50, k) $15, l) $1.80, m) $9, n) $225, o) $0.40

Skill 6.4 a) 7, b) 18, c) 9, d) 120, e) 2, f) 14, g) 9, h) 5, i) 42, j) 15
k) 12, l) 12.8, m) 7, n) 48, o) 39, p) 21, q) 7.5, r) 15, s) 45
t) 8, u) 20, v) 0.45, w) 2, x) 40, y) 1.3, z) 24, A) 0.3

Skill 6.5 a) 22, b) $7.50, c) $1.487/L, d) 72, e) 400 g
f) 16 320, g) 7.8 billion, h) 4.5 kg, i) 459, j) 3.5 kg

Skill 6.6 a) 36, b) 176, c) 140, d) 84, e) 36, f) 136, g) 675, h) 253
i) 33.6, j) 404, k) 100, l) 399, m) 1250, n) 72, o) 275

Skill 6.7 a) 408, b) 77, c) 465, d) 96, e) 606, f) 70, g) 46, h) 336
i) 78, j) 140, k) 72, l) 306, m) 27, n) 87, o) 144 p) 560
q) 2700, r) 840, s) 975, t) 126, u) 270

Skill 6.8 a) 693, b) 450, c) 3560, d) 2208, e) 125, f) 17, g) 60
h) 114, i) 72, j) 343, k) 30, l) 24, m) 22, n) 728, o) 154
p) 138, q) 108, r) 470, s) 99, t) 234, u) 632

Skill 6.9 a) $65, b) 16 000 L, c) 1600, d) 28, e) 150, f) 50 000 L
e) 76, f) $0.80

Skill 6.10 a) 25%, b) 10%, c) 25%, d) 150%, e) 14%, f) 25%
g) 33.33%, h) 400%

Skill 6.11 a) 450, b) 400, c) 350, d) 250, e) 200, f) 450, g) 95, h) 700
i) 180, j) 500

7. [Dec. / Frac. / Percents] page 71

Skill 7.1 a) 0.06, 0.066, 0.606, 0.66, b) 3.104, 3.041, 3.04, 3.014
c) 0.236, 0.263, 0.326, 0.362, d) 0.205, 0.052, 0.05, 0.025
e) 0.570, 0.507, 0.075, 0.057, f) 1.064, 1.24, 1.246, 1.264
g) 0.076, 0.176, 0.617, 0.706, h) 3.928, 3.893, 3.298, 3.28

Skill 7.2 a) $\frac{2}{3}, \frac{13}{18}, \frac{5}{6}$ b) $\frac{43}{100}, \frac{21}{50}, \frac{2}{5}$ c) $\frac{23}{36}, \frac{3}{4}, \frac{7}{9}$ d) $\frac{71}{100}, \frac{7}{10}$
e) $\frac{1}{4}, \frac{13}{40}, \frac{3}{8}$ f) $\frac{29}{54}, \frac{4}{9}, \frac{7}{18}$

Skill 7.3 a) $\frac{5}{6} = \frac{15}{18}$, b) $\frac{5}{8} = \frac{125}{200}$, c) $\frac{85}{100} = \frac{17}{20}$, d) $\frac{3}{4} = \frac{15}{20} = \frac{75}{100}$
e) $\frac{64}{144} = \frac{16}{36} = \frac{4}{9}$, f) $\frac{20}{70} = \frac{10}{35} = \frac{2}{7}$, g) $\frac{2}{5} = \frac{10}{25} = \frac{30}{75}$
h) $\frac{50}{80} = \frac{25}{40} = \frac{5}{8}$, i) $\frac{4}{9} = \frac{12}{27} = \frac{36}{81}$, j) true, k) false, l) false

Skill 7.4 a) 3%, b) 20%, c) 35%, d) 88%, e) 8%, f) 10%, g) 2%
h) 40%, i) 46.3%, j) 5.5%, k) 1.5%, l) 7.1%, m) 120%
n) 250%, o) 230%, p) 310%, q) 34.3%, r) 21.4%

Skill 7.5 a) 0.09, b) 0.04, c) 0.7, d) 0.86, e) 0.4, f) 0.63, g) 0.025
h) 0.0415, i) 0.115, j) 0.0325, k) 0.15, l) 0.6, m) 0.175
n) 0.0005

Skill 7.6 a) $\frac{3}{5}$, b) $\frac{1}{50}$, c) $\frac{3}{25}$, d) $\frac{1}{20}$, e) $\frac{9}{20}$, f) $\frac{4}{5}$, g) $\frac{1}{5}$, h) $\frac{17}{25}$
i) $\frac{21}{25}$, j) $\frac{1}{25}$

Skill 7.7 a) 0.06, b) 0.45, c) 0.5, d) 0.34, e) 0.56, f) 0.75, g) 0.8
h) 0.44, i) 0.55

Skill 7.8 a) $\frac{9}{25}$, b) $\frac{3}{50}$, c) $\frac{3}{4}$, d) $\frac{3}{10}$, e) $\frac{9}{50}$, f) $\frac{9}{10}$, g) $\frac{1}{4}$, h) $\frac{11}{25}$
i) $\frac{2}{5}$, j) $\frac{14}{25}$, k) $\frac{1}{25}$, l) $\frac{37}{50}$, m) $\frac{13}{50}$, n) $\frac{7}{20}$

Skill 7.9 a) 15%, b) 70%, c) 12%, d) 20%, e) 10%, f) 90%, g) 5%
h) 40%, i) 25%, j) 20%

Skill 7.10 a)

Decimal	Fraction	Percentage
0.045	$\frac{9}{200}$	4.5%

b)

Decimal	Fraction	Percentage
0.75	$\frac{3}{4}$	75%

c)

Decimal	Fraction	Percentage
0.6	$\frac{3}{5}$	60%

d)

Decimal	Fraction	Percentage
0.18	$\frac{9}{50}$	18%

Skill 7.11 a) 48, b) 160, c) 80, d) 2000, e) 6, f) 8, g) 6, h) 12
i) $480, j) $17 500

Skill 7.12 a) 90%, b) 0.8 c) 30%, 0.31, $\frac{1}{3}$ d) 0.66, $\frac{6}{10}$, 6%
e) 0.14, $\frac{1}{4}$, 41% f) 0.83, 81%, $\frac{4}{5}$

Skill 7.13 a) B, b) C, c) B, d) A, e) C, f) A, g) B, h) C, i) A, j) B

Skill 7.14 a) $0.\dot{0}\dot{9}$, b) $0.\dot{3}\dot{6}$, c) $0.\dot{6}$, d) $0.\dot{4}$, e) $0.7\dot{3}$, f) $0.41\dot{6}$

Skill 7.15 a) B, b) C, c) B, d) A, e) C, f) B

8. [Integers +,–] page 87

Skill 8.1 a) –2, b) –12, c) –8, d) –6, e) –2, f) –3, g) 3, h) –3, i) –12
j) –7, k) –23, l) –5

Skill 8.2 a) 1, b) –6, c) –1, d) 14, e) –1, f) 4, g) 13, h) 13, i) –30
j) –25, k) 3, l) –3

Skill 8.3 a) –2, b) 15, c) 5, d) 1, e) 1, f) 6, g) 1, h) –2, i) –3, j) 3
k) 16, l) –12, m) 13, n) –11, o) –9, p) –15, q) –1, r) –3

Skill 8.4 a) 1, b) –3, c) 2, d) 2, e) 7, f) –3, g) 8, h) 3, i) 2, j) –10
k) –2, l) –8, m) 18, n) –10, o) –8

Skill 8.5 a) –8, b) –2, c) –7, d) –15, e) 18, f) 5, g) –4, h) –5, i) –11
j) 11, k) –13, l) –6, m) 10, n) 3, o) –10

9. [Integers ×,÷] page 93

Skill 9.1 a) –24, b) 12, c) –45, d) –100, e) –12, f) 28, g) –21 h) –20, i) 64, j) –34, k) 45, l) 42, m) 70, n) 24, o) –45, p) 32 q) –360, r) –300

Skill 9.2 a) –2, b) –6, c) –3, d) 5, e) –4, f) 1, g) –7, h) –9, i) 4, j) –8 k) 5, l) –6, m) –4, n) 9, o) –8, p) –14, q) 4, r) –5, s) –10 t) 8, u) –7

Skill 9.3 a) –300, b) 80, c) –100, d) 400, e) –800, f) –4200 g) –1000, h) 2000, i) 400, j) –900, k) –800, l) 1200 m) –600, n) 2000, o) 1500, p) –1200, q) 1500, r) 4900

Skill 9.4 a) –4, b) 6, c) –15, d) 4, e) 8, f) –2, g) –9, h) 14, i) 15, j) –2 k) –8, l) 45, m) –6, n) 24, o) 4, p) 160, q) 2, r) 18, s) 12 t) –48, u) 20

Skill 9.5 a) 30, b) –12, c) –12, d) –15, e) 7, f) –16, g) –8, h) 72, i) –9 j) 48, k) –81, l) –21, m) –9, n) –6, o) –20, p) –2, q) –1 r) 4, s) –2, t) –1, u) –3

Skill 9.6 a) –5, b) –6, c) –21, d) 5, e) –9, f) 8, g) 10, h) –66, i) –3 j) –9, k) 11, l) –56, m) –9, n) –15, o) –42

10. [Rates / Ratios] page 99

Skill 10.1 a) 10, b) 84, c) 2, d) 60, e) \$1.25/kg, f) \$77/h, g) \$12.50/h h) \$1.60

Skill 10.2 a) 3:1, b) 4:1, c) 3:4, d) 1:8, e) 5:8, f) 8:1, g) 1:4:8, h) 1:8:5

Skill 10.3 a) 20:4:3, b) 22:1:2, c) 4:1, d) 4:5, e) 6:1, f) 29:24, g) 4:1 h) 7:12

Skill 10.4 a) 38:113, b) 3:10, c) 1:4, d) 2:3, e) 14:25, f) 1:8, g) 3:10 h) 11:20

Skill 10.5 a) 990 cm/h, b) 40 km/h, c) 300 km/h, d) 54 km/h e) 9 km/h, f) 8 km/h

Skill 10.6 a) 900 km, b) 780 km, c) 45 km, d) 350 km, e) 7.2 m f) 800 km

Skill 10.7 a) 1.5 h, b) 5 s, c) 20 s, d) 125 s, e) 84 min, f) 9 min

Skill 10.8 a) false, b) false, c) true, d) true, e) true, f) true, g) true h) true, i) false, j) false

Skill 10.9 a) 24, b) 10, c) 6, d) 12, e) $y = 3$, f) $x = 9$

Skill 10.10 a) 9 g, b) 500, c) 10 000, d) 1260 mL, e) 6, f) \$60

Skill 10.11 a) 18, b) 1260 mL, c) 60, d) 360 g, e) 15, f) 146 cm

Skill 10.12 a) 70 beats/s, b) 150 beats/min, c) 25 min, d) 52 500 L e) 5 016 000, f) 144 000 km^2

Skill 10.13 a) 144 000 mL/h, b) 10 000 m/h, c) 3 kg/min d) 500 cents/kg, e) 3600 mm/min, f) 300 answers/h g) 2 m/s, h) 8 beats/min

Skill 10.14 a) 32, b) \$50 c) 60 min, d) \$1.80, e) 310, f) 40

Skill 10.15 a) 45 m/s, b) 900 km/h, c) 8000 m/s, d) 108 000 km/h e) 110 m/s, f) 306 km/h

Skill 10.16 a) 16 000 km, b) 21 000 km, c) 5000 km, d) 300 km e) 19 cm, f) 50 cm, g) 40 cm, h) 15 cm, i) 1:12 000 000 j) 1:200, k) 1:20 000 000, l) 1:1500

Skill 10.17 a) B, b) B c) B, d) A, e) A, f) A

11. [Indices] page 117

Skill 11.1 a) 81, b) 8, c) 64, d) 25, e) 1, f) 16, g) 49, h) 216, i) 1000 j) 243, k) 343, l) 81, m) 8, n) 1, o) 0, p) 64, q) 1, r) 10 000

Skill 11.2 a) $\frac{27}{1000}$, b) $\frac{1}{125}$, c) $\frac{4}{49}$, d) $\frac{1}{100}$, e) $\frac{8}{27}$, f) $\frac{9}{64}$, g) $\frac{36}{121}$ h) $\frac{16}{81}$, i) $\frac{1}{256}$, j) $\frac{32}{243}$, k) $\frac{64}{125}$, l) $\frac{343}{1000}$, m) $\frac{27}{64}$, n) $\frac{25}{64}$, o) $\frac{25}{144}$ p) $\frac{81}{256}$, q) $\frac{81}{10\,000}$, r) $\frac{81}{169}$

Skill 11.3 a) 16, b) 243, c) 8, d) 625, e) 64, f) 81, g) 243, h) 125 i) 1296, j) x^9, k) z^4, l) y^5, m) d^7, n) a^5b^4, o) f^5g^3, p) c^3d^5 q) b^4c^3, r) u^4v^5, s) l^3m^7, t) a^4b^2, u) g^3h^4

Skill 11.4 a) 16, b) 81, c) 32, d) 216, e) 81, f) 125, g) 512, h) 49 i) 1000, j) t, k) p^6, l) r^7, m) j^5, n) q^5, o) y^2, p) p^3q^2, q) a^4b^2 r) t^3u^4

Skill 11.5 a) $9t^5$, b) $2x^4$, c) $4p^3$, d) $6b^3$, e) $8d^6$, f) $15m^8$, g) $24s^5$ h) $10a^{10}$, i) $7k^9$, j) $18c^{11}$, k) $2y^8$, l) $28w^9$, m) $16r^2$, n) $25g^8$ o) $6y^{13}$, p) $20a^6$, q) $35p^9$, r) $36d^8$

Skill 11.6 a) $4c$, b) $3a^3$, c) $4h^4$, d) $5m^8$, e) z^4, f) $6f^5$, g) $2u^4$, h) $2b^2$ i) $3w^3$, j) $2v^3$, k) $5x^{12}$, l) $8n^5$, m) $7e^4$, n) 2, o) $3w^4$

Skill 11.7 a) $16x^4$, b) $125y^3$, c) $64v^6$, d) t^4u^4,e) f^2g^2, f) d^fe^f g) $216m^3$, h) $49r^2$, i) $81p^4$, j) $4b^2$, k) $125y^3$, l) $64k^3$ m) $48y^3$, n) $8q^2$, o) $32n^3$, p) $160v^5$, q) $48s^2$, r) $72h^2$

Skill 11.8 a) y^8, b) r^{16}, c) x^{10}, d) a^{ef}, e) p^{qr}, f) t^{uv}, g) d^4, h) h^6, i) n^{12} j) $w^0 = 1$, k) a^{20}, l) g^{10}, m) $2b^6$, n) $4q^9$, o) $5z^6$,p) $6c^{12}$ q) $8w^8$, r) $7k^{15}$

Skill 11.9 a) 81, b) 4, c) –1, d) 16, e) 64, f) 16, g) –1, h) 64, i) –27 j) –5, k) 256, l) –343, m) –216, n) 49, o) 625, p) 81 q) –125, r) 10 000, s) –1000, t) 144, u) –1, v) 225 w) –1331, x) –1

Skill 11.10 a) $\frac{1}{1000}$, b) $\frac{1}{27}$, c) $\frac{1}{4}$, d) $\frac{1}{64}$, e) $\frac{1}{7}$, f) $\frac{1}{64}$, g) $\frac{1}{81}$, h) $\frac{1}{16}$ i) $\frac{1}{625}$, j) $\frac{1}{216}$, k) $\frac{1}{25}$, l) $\frac{1}{243}$, m) $\frac{1}{32}$, n) $\frac{1}{10\,000}$, o) $\frac{1}{256}$ p) $\frac{1}{64}$, q) $\frac{1}{27}$, r) $\frac{1}{36}$, s) $\frac{1}{100}$, t) $\frac{1}{125}$, u) 1

12. [Square Roots] page 127

Skill 12.1 a) 5, b) 2, c) 3, d) 7, e) 9, f) 8, g) 11, h) 1, i) 13, j) 4, k) 6 l) 20, m) 10, n) 14, o) 12, p) 15, q) 16, r) 100, s) 60, t) 90 u) 70

Skill 12.2 a) $\frac{1}{6}$, b) $\frac{1}{4}$, c) $\frac{2}{5}$, d) $\frac{5}{6}$, e) $\frac{4}{11}$, f) $\frac{5}{9}$, g) $1\frac{1}{3}$, h) $1\frac{1}{2}$ i) $1\frac{1}{5}$, j) $4\frac{1}{2}$, k) $1\frac{1}{8}$, l) $1\frac{3}{4}$, m) $3\frac{1}{3}$, n) $1\frac{1}{7}$, o) $2\frac{1}{5}$

Skill 12.3 a) 0.1, b) 0.4, c) 0.5, d) 0.6, e) 1.3, f) 0.8, g) 1.4, h) 0.3 i) 1.6, j) 0.7, k) 2.5, l) 0.9, m) 1.2, n) 1.1, o) 1.5, p) 2.1 q) 2.4, r) 2.6

Skill 12.4 a) 16, b) 12, c) 12, d) 14, e) 20, f) 24, g) 60, h) 42, i) 32 j) 26, k) 60, l) 55, m) 60, n) 50, o) 200, p) 60, q) 300, r) 5

Skill 12.5 a) 36, b) 7, c) 25, d) 45, e) 12, f) 56, g) 99, h) 120, i) 120 j) 42, k) 96, l) 60, m) 80, n) 78, o) 132, p) 240, q) 336 r) 720

Skill 12.6 a) 2, b) 2, c) 2, d) 5, e) 6, f) 2, g) 5, h) 4, i) 6, j) 4, k) 4, l) 20 m) 4, n) 20, o) 5, p) 4, q) 10, r) 6

Skill 12.7 a) 5, b) 16, c) 12, d) 7, e) 4, f) 20, g) 7, h) 14, i) 2, j) 15 k) 19, l) 28, m) 11, n) 1, o) 1, p) false, q) false, r) false s) true, t) false, u) false

Skill 12.8 a) 8 & 9, b) 3 & 4, c) 2 & 3, d) 7 & 8, e) 5 & 6, f) 4 & 5 g) 8 & 9, h) 12 & 13, i) 7 & 8, j) 12 & 13, k) 9 & 10 l) 11 & 12

13. [Exploring Number] page 135

Skill 13.1 a) 15, b) 66, c) 11, d) 48, e) 10, f) 6, g) 7, h) 20, i)126, j) 10
k) 39, l) 2, m) 10, n) 34

Skill 13.2 a) 225, b) 196, c) 400, d) 196, e) 25, f) 18, g) 41, h) 33
i) 100, j) 400, k) 84, l) 70, m) 3, n) 7

Skill 13.3 a) 0.1, b) 7.9, c) 12.5, d) 31.58, e) 24.79, f) 4.23, g) 3.9
h) 50.3, i) 4.78, j) 3.42, k) 0.67, l) 1.733, m) 4.289, n) 0.162

Skill 13.4 a) 3.46, b) 4.47, c) 4.90, d) 5.48, e) 3.142, f) 1.618
g) 0.259, h) 1.732, i) 2.72, j) 3.162

Skill 13.5 a) C, b) A, c) 1.5×10^5, d) 6.84×10^9, e) B, f) A
g) 2.4×10^{-7}, h) 1.42×10^{-7}

Skill 13.6 a) B, b) A, c) C, d) B, e) 965 000 m, f) 1 400 000 000, g) C
h) B, i) 0.000000000025, j) 0.0000005

Skill 13.7 a) –180, b) –64, c) –2, d) –1, e) 48, f) 27, g) –55, h) 30
i) –24, j) –10, k) –13, l) 15, m) 21, n) –65, o) –14, p) –15

Skill 13.8 a) 89, 14 b) 567, 12, 0 c) 18, 143 d) 34,2 e) 11, –1, 2
f) $\frac{16}{4}$, –3, $\sqrt{25}$ g) –75, $-\frac{8}{2}$, 10.00 h) $-\sqrt{4}$, 18, 0

Skill 13.9 a) irrational, b) rational, c) rational, d) irrational, e) integer
f) rational, g) rational, h) integer, i) C, j) B, k) D, l) C
m) B & D, n) –2012, 0, $\frac{28}{11}$

Skill 13.10 a)

	Integer	Rational	Irrational	Real
4.327	false	true	false	true

b)

	Integer	Rational	Irrational	Real
–500	true	true	false	true

c)

	Integer	Rational	Irrational	Real
π	false	false	true	true

d)

	Integer	Rational	Irrational	Real
$\frac{3}{14}$	false	true	false	true

e)

	Integer	Rational	Irrational	Real
$\sqrt{26}$	false	false	true	true

f)

	Integer	Rational	Irrational	Real
$\frac{36}{9}$	true	true	false	true

g) D, h) A, i) D, j) C

Skill 13.11 a) 5, b) $\sqrt{72}$, c) 1.41, d) 3, e) π, f) $\sqrt{6}$, g) $\sqrt{18}$, h) $\sqrt{25}$
l) $2.\dot{2}$, $\sqrt{5}$, $\frac{7}{3}$, 2.4, $\frac{5}{2}$ m) $\frac{10}{4}$, $\sqrt{7}$, $\frac{8}{3}$, 2.76, $\sqrt{8}$

14. [Financial Mathematics] page 147

Skill 14.1 a) \$155, b) \$450, c) \$7.00, d) A, e) \$4.95, f) \$3.00
g) \$65.60, h) \$96.80

Skill 14.2 a) \$3.00, b) 1.35 kg (accept 1.2 kg to 1.5 kg)
c) 2500 (accept \$2500 to \$2510)
d) 15 m^2 (accept 12.5 m^2 to 15 m^2)
e) 18 (accept \$15 to \$18)
f) 1 920 000 (accept 1 600 000 to 2 000 000)
g) 32 200 (accept 30 000 to 35 000)
h) 6.5 million (accept 6 million to 7 million)
i) 980 000 (accept 900 000 to 1 000 000)
j) \$40 (accept \$39 to \$41)

Skill 14.3 a) \$192.50, b) \$7.50, c) \$960, d) \$76, e) \$28, f) \$61
g) \$12 000, h) \$759, i) \$119, j) 20%

Skill 14.4 a) \$2700, b) \$700, c) \$616, d) \$1380, e) \$13 500, f) \$1680
g) \$5700, h) \$442 000, i) \$25, j) \$70

Skill 14.5 a) \$19, b) \$12, c) \$94.50, d) \$240, e) \$23 000, f) \$115
g) \$2700, h) \$1170

Skill 14.6 a) \$1917, b) \$95 000
c) Total deductions = \$416.87 Net wage = \$526.28
d) \$6500, e) \$16 572, f) \$3230

Skill 14.7 a) \$240, b) \$14, c) \$110, d) \$100, e) \$150, f) \$432
g) \$242, h) \$990

Skill 14.8 a) <, b) <, c) \$25, d) \$700, e) \$3750, f) \$680, g) \$80, h) \$50

Skill 14.9 a) \$98.88, b) \$11 236, c) \$3640, d) \$1891.50

Skill 14.10 a) \$36 450, b) 1210, c) 26 620, d) \$204.80

15. [Number Patterns] page 157

Skill 15.1 a) 19, b) 21, c) –8, d) –9, e) –3125, f) –64, g) 320, h) –6250

Skill 15.2 a) 25, 31 b) 17, 23 c) 23, 33 d) 21, 31 e) 45, 63 f) 24, 13
g) 26, 34 h) 0, –12 i) 31, 46 j) 30, 41

Skill 15.3 a) –25, –40 b) –7, –10 c) 10, 16 d) 9, 14 e) –11, –15
f) –5, –8 g) 0, 7 h) 1, 5 i) –14, –20 j) –13, –18 k) –14, –26
l) 25, 34

Skill 15.4 a) 243, 729 b) 96, 192 c) 486, 1458 d) 625, 3125
e) 36, 216 f) 25, 125 g) 80, 160 h) 343, 2401 i) –64, 128
j) 256, –1024 k) –243, 729 l) 2500, –12 500

Skill 15.5 a) –9, 4.5 b) 6, 2 c) 5, 1 d) 50, 25 e) –20, 2 f) 31, –3.1
g) –10, 2 h) –4, 2 i) $\frac{7}{2}, \frac{7}{4}$ j) $\frac{2}{3}, \frac{2}{9}$ k) $\frac{3}{5}, \frac{3}{25}$ l) $\frac{3}{7}, \frac{3}{49}$

Skill 15.6 a) 60, b) 90, c) 103, d) 52, e) 58, f) 128

Skill 15.7 a) 146, b) 53, c) –42, d) –15, e) 450, f) –1000, g) –10, h) 8
i) –120, j) 54

Skill 15.8 a) D, b) B, c) $2n + 1$, d) $6 - n$, e) $3n - 1$, f) $4n + 1$, g) $5n$
h) $3n + 4$, i) $3 - n$, j) $-3n$

Skill 15.9 a) 11, 14.5 b) 8.8, 11 c) 16.5, 21 d) 16.5, 20.5 e) 20, 40
f) $\frac{9}{8}, \frac{9}{16}$ g) $\frac{1}{32}, \frac{1}{128}$ h) 7, $7\frac{3}{4}$ i) $4\frac{13}{15}$, $2\frac{2}{15}$

16. [Expressions] page 167

Skill 16.1 a) $d + 20$, b) $7y$, c) $p - 15$, d) $9s$, e) $-8t$, f) $2u + 3v$, g) $\frac{d}{3}$
h) $\frac{t}{4}$, i) $50 - p$, j) $a + b$, k) $2pq$, l) $ab - 6$

Skill 16.2 a) $5j$, b) $7y$, c) mn, d) gh, e) $6yz$, f) $4ru$, g) $\frac{3x}{2}$, h) $\frac{6z}{5}$, i) $4b^2$
j) $-3a^2$, k) w^2z, l) $-c^2d$, m) $2r^2s$, n) $-jk^2$, o) $\frac{5rs}{t}$, p) $\frac{2ab}{c}$
q) $-\frac{10uv}{w}$, r) $-\frac{6gh}{i}$

Skill 16.3 a) $8a$, $5a$, b) $-2m$, $3m$, c) m^2, $3m^2$, d) t^2, $-t^2$, e) $3cd$, dc
f) $-bc$, $5cb$, g) $-2t$, $3t$, h) $-6w$, w, i) $3s$, $2.3s$, j) $-0.2y$, $2y$
k) v^2, $-2v^2$, l) $4k^2$, $-k^2$, m) z^2, $-8z^2$, n) g^2, $-4g^2$
o) $-5w^4$, w^4, p) a^2b, $2ba^2$, q) x^2y, $2yx^2$, r) $-tu^2$, $3u^2t$

Skill 16.4 a) $3m$, b) $6cd$, c) $3j$, d) $3xy$, e) $3a + 3b$, f) $3t^2 + 3t$
g) $ad + 5d$, h) $-m + 4n$, i) $5p^2 - 3p$, j) $2y^2 + yz$, k) $3r^2 - 3s^2$
l) $3x^2 - 2x$, m) $-3d^2e - d$, n) $ab^2 - 3a^2b$

Skill 16.5 a) $12v$, b) $15xy$, c) $14mn$, d) $-40jk$, e) $20de$, f) $18b^2$
g) $-24vw$, h) $-28ab^2$, i) $-30xz^2$, j) $-20g^2h$, k) $-30s^2t$
l) $-24p^2q$, m) $15j^2k^2$, n) $25bc^3$

Skill 16.6 a) $4y$, b) $6pq$, c) 7, d) $7m$, e) $-5z$, f) $-6x$, g) $1\frac{1}{5}$, h) 20
i) $-3t$, j) $-a$, k) $-5v^2$, l) $-5ab$, m) $5xyz$, n) $3ghi$

17. [Substitution] page 173

Skill 17.1 a) 12, b) 45, c) 100, d) 6, e) 8, f) 17, g) –40, h) –200, i) 0
j) 0, k) 0, l) 0, m) 0, n) 0, o) 0, p) 0

Skill 17.2 a) 41, b) 63, c) 100, d) 200, e) 5, f) 12, g) 5, h) 56, i) 144
j) 110, k) 16, l) 14, m) 1, n) 7, o) 22, p) 33

Skill 17.3 a) 0, b) 3, c) 47, d) 9, e) 12, f) 4, g) 2, h) 1, i) 5, j) 2

Skill 17.4 a) 3, b) 18, c) 16, d) 18, e) 9, f) 31, g) 2, h) –6, i) 27, j) 9
k) 77, l) 2, m) 6, n) 12

Skill 17.5 a) 18, b) 1200, c) 96, d) 47.1, e) 45, f) 18, g) 80, h) 6.92
i) 150, j) 2400, k) 1256, l) $a = 12$

Skill 17.6 a) 12, b) 20, c) –12, d) –32, e) 18, f) 0, g) 5, h) 160, i) 25
j) 72, k) 0, l) 100, m) 720°, n) 31.4

Skill 17.7 a) 8, b) –9, c) –32, d) 6, e) –3, f) –2, g) –11, h) –7, i) –10
j) –34, k) –8, l) 22

Skill 17.8 a) 29, b) 25, c) $2\frac{1}{15}$, d) $\frac{34}{35}$, e) 2, f) –9, g) –44, h) 45, i) 14
j) 12

Skill 17.9 a) 24, b) 0, c) 0, d) 0, e) 11, f) –5, g) 0, h) 0, i) 11, j) –1
k) 28, l) –4, m) 12, n) 0, o) 6, p) –6

18. [Expansion] page 183

Skill 18.1 a) $12b - 15$, b) $2z + 8$, c) $15 + 3w$, d) $7n - 14$, e) $36 - 9u$
f) $5e - 40$, g) $8 + 16a$, h) $8g - 24$, i) $4k - 6$, j) $18h + 27$
k) $42 - 12c$, l) $32x - 40y + 24$, m) $12 - 18w + 12x$
n) $10 - 14d + 8e$

Skill 18.2 a) $2a - 2a^2$, b) $e^2 + 4e$, c) $9r + r^2$, d) $5s - s^2$, e) $d^2 + 3d$
f) $e^2 - 7e$, g) $a + 2a^2$, h) $5d^2 + 6d$, i) $4p + 2p^2$, j) $6z - 6z^2$
k) $2c^2 - 3c$, l) $4w - 5w^2$, m) $3x^2 - 2xy + 7x$
n) $tu - 5t + 9t^2$, o) $7st - 4s^2 - 8s$, p) $ef + 4e - 9e^2$,

Skill 18.3 a) $6d^2 + 12d$, b) $3a^2 - 15a$, c) $10s - 20s^2$, d) $12y^2 - 9y$
e) $15k + 6k^2$, f) $10g^2 - 20g$, g) $8d^2 + 12d$, h) $21a + 6a^2$
i) $36c + 18c^2$, j) $30h^2 - 12h$, k) $21e^2 + 24e$, l) $32z - 8z^2$
m) $12q - 4qr$, n) $24ij + 16i$, o) $28p^2 + 7pq$, p) $5mn - 25n^2$

Skill 18.4 a) $-5m^2 - 20m$, b) $-4f - 12$, c) $-b - 9$, d) $-3r - 18$
e) $-8a^2 + 16a$, f) $-6w - 8w^2$, g) $-7q^2 - 21q$, h) $-24b + 30b^2$
i) $-4cd + 6cd^2$, j) $-5t^2u - 2tu^2$, k) $-40jk + 20j^2k$
l) $-7g^2h + 3gh^2$, m) $-24hi^2 - 8hi$, n) $-9y^2z - 18yz$
o) $-16s^2t - 6st$, p) $-18m^2n + 12mn$

Skill 18.5 a) $9c + 8$, b) $10x + 3$, c) $-2x + 2$, d) $-s + 2s^2$, e) $3pq - 14p$
f) $5yz + 7z$, g) $-3hi - 39$, h) $n^2 + n + 21$, i) $3de + 36$
j) $w^2 - 4w + 14$, k) $2b^2 - 18b + 40$, l) $abc - 2a - 15$

Skill 18.6 a) $3a^2 + 8a - 7b$, b) $2x^2 - 21$ c) $4t - 12 + t^2$
d) $-10s^3 - 7s^2 + s$, e) $9t^2u - 9tu$, f) $3ef - 11e^2 + 8ef^2$
g) $-10k^2l + 16kl$, h) $m^2 + 5m - 6$, i) $-p^2q - 3pq$
j) $r^2 + 10r - 12$, k) $8y + 4x^2y$, l) $q^2 + q$

Skill 18.7 a) $h^2 - 3h - 10$, b) $x^2 + 4x + 3$, c) $w^2 + w - 12$
d) $-u^2 + u + 20$, e) $fg + 8f - 2g - 16$, f) $jk - 3j - 5k + 15$
g) $2h^2 + 6h - 20$, h) $3r^2 + 11r - 42$, i) $3v^2 - 23v - 36$
j) $5y^2 - 16y + 12$

Skill 18.8 a) $s^2 + 8s + 16$, b) $y^2 + 2y + 1$, c) $h^2 + 4h + 4$
d) $t^2 + 12t + 36$, e) $p^2 + 14p + 49$, f) $m^2 + 10m + 25$
g) $a^2 + 6a + 9$, h) $c^2 + 20c + 25$, i) $r^2 + 16r + 68$
j) $g^2 + 3g + 9$

Skill 18.9 a) $s^2 - 8s + 16$, b) $k^2 - 2k + 1$, c) $m^2 - 4m + 4$
d) $q^2 - 10q + 25$, e) $j^2 - 14j + 49$, f) $e^2 - 18e + 81$
g) $x^2 - 16x + 64$, h) $x^2 - 5x + 100$, i) $z^2 - 12z + 44$
j) $b^2 - 11b + 16$

19. [Factorisation] page 193

Skill 19.1 a) $4(k - 4)$, b) $4(x + 2)$, c) $6(s + 3)$, d) $3(u - 5)$, e) $3(3m - 8)$
f) $7(2n + 3)$, g) $2(y + 5z)$, h) $4(a - 3b)$, i) $2(3d + 7e)$
j) $8(2uv - 5)$, k) $4(3k - 2l)$, l) $2(2g + 2h - 3)$
m) $3(m - 2n + 3)$, n) $5(2v - w + 3)$, o) $5(h^2 - 2i + 5j)$
p) $3(2r^2 - 9s + 3t)$

Skill 19.2 a) $b(a + 5)$, b) $d(e + 1)$, c) $e(7 + f)$, d) $s(3t + 4)$
e) $4b(2a - 1)$, f) $5g(3 + 4h)$, g) $x(w - y)$, h) $2k(j + l)$
i) $v(u - 3w)$, j) $4b(2a + c)$, k) $4r(3q + 2s)$, l) $3e(5d - 2f)$
m) $2d(5c - 4)$, n) $5m(3 - 2n)$, o) $7q(3r + 2p)$, p) $6u(t + 3v)$
q) $3y(2x + 3z)$, r) $5g(2h - 5i)$

Skill 19.3 a) 99, b) 300, c) 9870, d) 800, e) 2300, f) 1250, g) 999
h) 870, i) –80, j) –900

Skill 19.4 a) $j(2jk + 5)$, b) $e(e + 7)$, c) $h(1 + 4h)$, d) $m(m - 9)$
e) $3c(1 - 4c)$, f) $2f(2f + 3)$, g) $f(g^2 + 1)$, h) $2b(5 - 8ab)$
i) $p(pq - 3)$, j) $4i(3 - 6hi)$
k) $2bc(7 + b)$, l) $r^2(5s - t)$, m) $v(w + 7v - 3wx)$
n) $4j(2j - 6k + 3l)$, o) $fg^2(f^2 + 1)$, p) $pq(p^2q + p + 1)$

Skill 19.5 a) $-7(a + 3)$, b) $-4(k + 3)$, c) $-3(2g + 5)$, d) $-2(3e + 7)$
e) $-2h(h + 3)$, f) $-4z(2z + 7)$, g) $-3i(4i^2 + 3j)$, h) $-t^2(t + 5u)$
i) $-3c^2(2b + 1)$, j) $-xy^2(5x + y)$, k) $-2x(x^2 + 2y)$
l) $-2m(2m^2 + 6n^2 - 9)$, m) $-2k(k^2 - 3k^2l - 4)$
n) $-h(2i^3 - 3hi + 5h)$

Skill 19.6 a) $(d + 2)(d + 8)$, b) $(h - 3)(2 + h)$, c) $(x + 4)(5 + x)$
d) $(b - 7)(b + 6)$, e) $(a + 2)(a - 9)$, f) $(z - 5)(z - 1)$
g) $(j + 4)(j + 1)$, h) $(n - 2)(m + 4)$, i) $(2x - 5)(3x - 4)$
j) $(c + 5)(d - 1)$, k) $(s - 3)(q + t)$
l) $(2w - 1)(6v + 4)$ or $2(2w - 1)(3v + 2)$

Skill 19.7 a) $(c + 8)(c + 3)$, b) $(a + 3)(a + 2)$, c) $(s + 6)(s + 5)$
d) $(h + 5)(h + 4)$, e) $(v + 7)(v + 3)$
f) $(4 + n)(n + 4)$ or $(n + 4)^2$, g) $(6 + t)(t - 7)$, h) $(b + 1)(4 - b)$
i) $(5 + p)(p - 2)$, j) $(q - 4)(q + 5)$

Skill 19.8 a) $(c + 9)(c - 9)$, b) $(y + 2)(y - 2)$, c) $(d + e)(d - e)$
d) $(6 + h)(6 - h)$, e) $(2j + 3)(2j - 3)$, f) $2(c + 5)(c - 5)$
g) $(p + 9q)(p - 9q)$, h) $5(4 + y)(4 - y)$, i) $9(a + 2b)(a - 2b)$
j) $3(5 + z)(5 - z)$, k) $3(d + 3)(d - 3)$, l) $4(5 + k)(5 - k)$

Skill 19.9 a) $(x + 5)(x + 2)$, b) $(d - 2)(d - 2)$, c) $(s + 3)(s + 1)$
d) $(g + 5)(g + 3)$, e) $(m + 6)(m - 4)$, f) $(j + 3)(j + 8)$
g) $(y + 1)(y + 4)$, h) $(z - 4)(z - 2)$, i) $(c - 1)(c - 5)$
j) $(p + 2)(p - 8)$

20. [Equations] page 203

Skill 20.1 a) 3, b) 5, c) 3, d) –2, e) –10, f) –3, g) 8, h) 12, i) 16, j) 4
k) –2, l) –3, m) 11, n) 13, o) 21, p) 17, q) 9, r) 8, s) 9, t) 36
u) 8, v) 16, w) 14, x) 21, y) 5, z) 9, A) 8, B) 40, C) 20, D) 37

Skill 20.2 a) 28, b) 9, c) 6, d) 4, e) 4, f) 7, g) 12, h) 10, i) 48, j) 9
k) 7, l) 9, m) 40, n) 42, o) 12, p) 40, q) 99, r) 20, s) 49
t) 120, u) 30, v) 17, w) 50, x) 10, y) 180, z) 24, A) 8, B) 10
C) 6, D) 6

Skill 20.3 a) 3, b) 5, c) –5, d) 5, e) 3, f) –3, g) 8, h) –10, i) $\frac{4}{5}$, j) 11
k) 2, l) 5, m) –5, n) –1, o) 4, p) $\frac{5}{6}$, q) $-\frac{1}{2}$, r) –7, s) 8, t) –25
u) –9

Skill 20.4 a) 4, b) 5, c) 10, d) 3, e) 3, f) 4, g) 7, h) 5, i) 4, j) 11, k) 0
l) 2, m) 9, n) 4, o) 1, p) 2, q) 1, r) 13, s) 2, t) 2, u) 0

Skill 20.5 a) 1, b) 2, c) 5, d) 4, e) 3, f) 8, g) –2, h) 3, i) –3, j) 3, k) 2
l) 6, m) 0, n) 1, o) 3, p) 10, q) 6, r) –12, s) 7, t) 8, u) 8

Skill 20.6 a) 8, b) 9, c) 20, d) 2, e) 4, f) 14, g) 7, h) 14, i) 2, j) 5, k) $\frac{40}{7}$
l) 3, m) 34, n) 2, o) 12

Skill 20.7 a) $x \geq -2$, b) $x < 5$, c) $x \leq 2$, d) $x \leq 8$, e) $x > \frac{13}{5}$, f) $x \leq -2$
g) $x \geq 12$, h) $x < 33$, i) $x \geq 42$, j) $x < 2$, k) $x \leq -9$, l) $x < -2$
m) $x < 2$, n) $x \geq 2$, o) $x \leq -2$, p) $x > 6$, q) $x \geq 9$, r) $x > -3$
s) $x > \frac{28}{5}$, t) $x \leq -\frac{13}{3}$, u) $x < -63$

Skill 20.8 a) 6, 5 b) –7, 2 c) –9, 2 d) –4, –3 e) –7, 4 f) –9, –1
g) 0, 8 h) –3, 0 i) 0, 4 j) –11, 0 k) –2, 10 l) 4, 6
m) –3, 5 n) –8, –7 o) –4, 1 p) 3, 9 q) –6, 7 r) –5, –2

Skill 20.9 a) (4,3) b) (2,3) c) (3,7) d) (3,–2) e) (2,–1) f) (4,2)
g) (–3,–5) h) (2,–2) i) (5,0) j) (3,1) k) (2,0) l) (3,–1)
m) (3,5) n) (6,2)

Skill 20.10 a) 4, –4 b) –2, 2 c) 0, 2 d) 0, 3 e) –4, 0 f) –5, 0 g) –8, 8
h) –12, 12 i) –5, 5 j) 0, 6 k) 0, 11 l) –9, 9 m) –10, 10
n) –7, 0 o) –6, 6 p) –9, 0 q) 0, 15 r) –11, 11

21. [Coordinate Geometry] page 223

Skill 21.1

a)

x	$y = x - 1$	y	(x, y)
–2	$y = -2 - 1$	–3	(–2 , –3)
–1	$y = -1 - 1$	–2	(–1 , –2)
0	$y = 0 - 1$	–1	(0 , –1)
1	$y = 1 - 1$	0	(1 , 0)
2	$y = 2 - 1$	1	(2 , 1)

b)

x	$y = 6x$	y	(x, y)
–2	$y = 6 \times (-2)$	–12	(–2 , –12)
–1	$y = 6 \times (-1)$	–6	(–1 , –6)
0	$y = 6 \times 0$	0	(0 , 0)
1	$y = 6 \times 1$	6	(1 , 6)
2	$y = 6 \times 2$	12	(2 , 12)

c)

x	$y = x + 7$	y	(x, y)
–7	$y = -7 + 7$	0	(–7 , 0)
–2	$y = -2 + 7$	5	(–2 , 5)
0	$y = 0 + 7$	7	(0 , 7)
2	$y = 2 + 7$	9	(2 , 9)
7	$y = 7 + 7$	14	(7 , 14)

d)

x	$y = x - 4$	y	(x, y)
–4	$y = -4 - 4$	–8	(–4 , –8)
–2	$y = -2 - 4$	–6	(–2 , –6)
0	$y = 0 - 4$	–4	(0 , –4)
2	$y = 2 - 4$	–2	(2 , –2)
4	$y = 4 - 4$	0	(4 , 0)

e)

x	$y = -x - 3$	y	(x, y)
–3	$y = -(-3) - 3$	0	(–3 , 0)
–1	$y = -(-1) - 3$	–2	(–1 , –2)
0	$y = -0 - 3$	–3	(0 , –3)
1	$y = -1 - 3$	–4	(1 , –4)
3	$y = -3 - 3$	–6	(3 , –6)

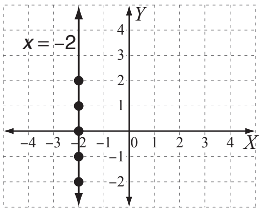
f)

x	$y = -5x + 1$	y	(x, y)
–2	$y = -5 \times (-2) + 1$	11	(–2 , 11)
–1	$y = -5 \times (-1) + 1$	6	(–1 , 6)
0	$y = -5 \times 0 + 1$	1	(0 , 1)
1	$y = -5 \times 1 + 1$	–4	(1 , –4)
2	$y = -5 \times 2 + 1$	–9	(2 , –9)

21. [Coordinate Geometry] (cont.)

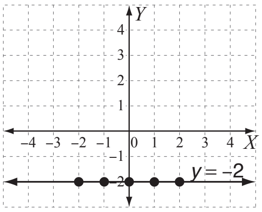
Skill 21.2 a)

x	−2	−2	−2	−2	−2
y	−2	−1	0	1	2
(x,y)	(−2,−2)	(−2,−1)	(−2, 0)	(−2, 1)	(−2, 2)



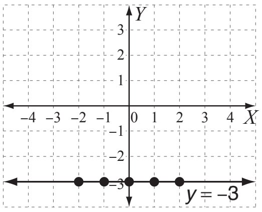
b)

x	−2	−1	0	1	2
y	2	−2	−2	−2	−2
(x,y)	(−2,−2)	(−1,−2)	(0 ,−2)	(1 ,−2)	(2 ,−2)



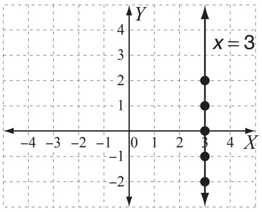
c)

x	−2	−1	0	1	2
y	−3	−3	−3	−3	−3



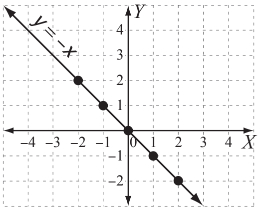
d)

x	3	3	3	3	3
y	−2	−1	0	1	2



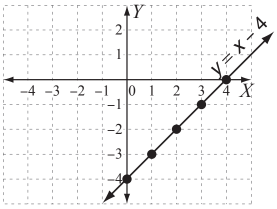
Skill 21.3 a)

x	−2	−1	0	1	2
y	2	1	0	−1	−2
(x,y)	(−2,2)	(−1, 1)	(0, 0)	(1,−1)	(2,−2)



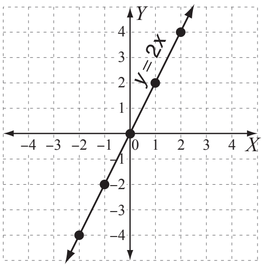
b)

x	0	1	2	3	4
y	−4	−3	−2	−1	0
(x,y)	(0,−4)	(1,−3)	(2,−2)	(3,−1)	(4, 0)



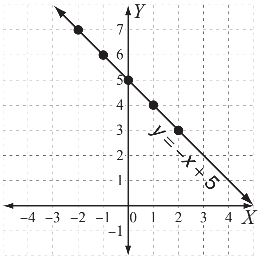
c)

x	−2	−1	0	1	2
y	−4	−2	0	2	4



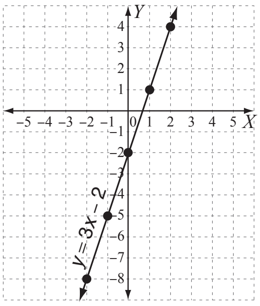
d)

x	−2	−1	0	1	2
y	7	6	5	4	3



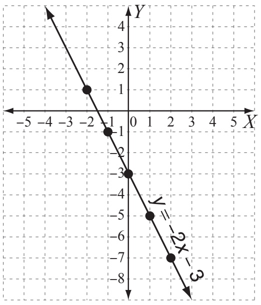
e)

x	−2	−1	0	1	2
y	−8	−5	−2	1	4

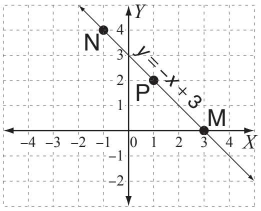


f)

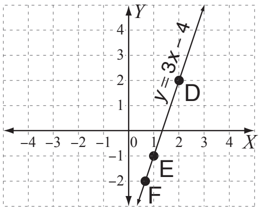
x	−2	−1	0	1	2
y	1	−1	−3	−5	−7



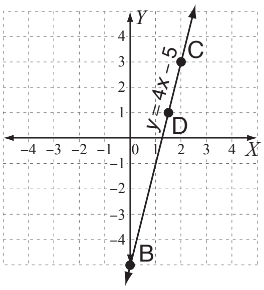
Skill 21.4 a) M(3,0), N(−1,4), P(1,2)



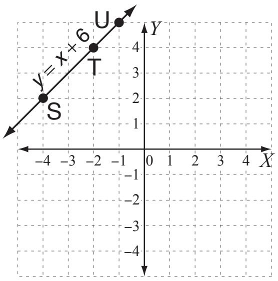
b) D(2,2), E(1,−1), F($\frac{2}{3}$,−2)



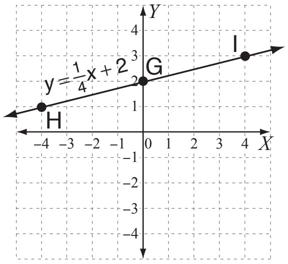
c) B(0,−5), C(2,3), D($\frac{3}{2}$,1)



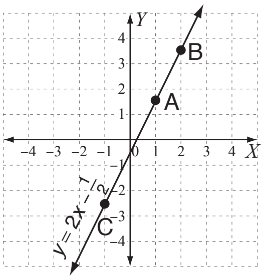
d) S(−4,2), T(−2,4), U(−1,5)



e) G(0,2), H(−4,1), I(4,3)



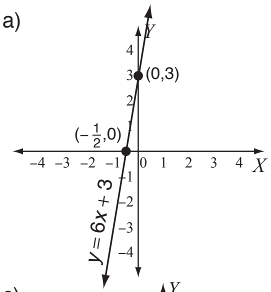
f) A(1,$\frac{3}{2}$), B(2,$\frac{7}{2}$), C(−1,−$\frac{5}{2}$)



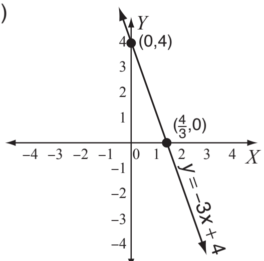
Skill 21.5 a) C, b) B, c) B, d) C

Skill 21.6 a) (2,0), b) (−4,0), c) (0,−3), d) (0,4), e) (−4,0), f) (0,4)
g) (5,0), h) (0,−9), i) (0,−8), j) ($\frac{1}{5}$,0), k) (0,5), l) (0,−2)
m) (2,0), n) (−7,0)

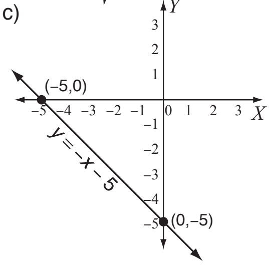
Skill 21.7 a)



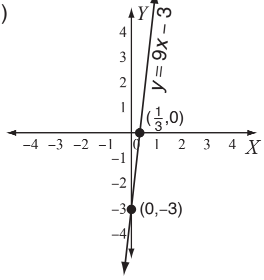
b)



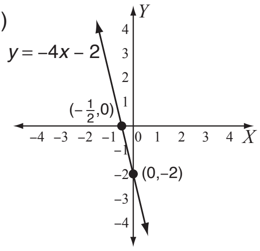
c)



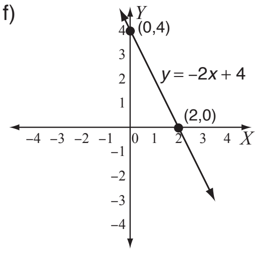
d)



e)



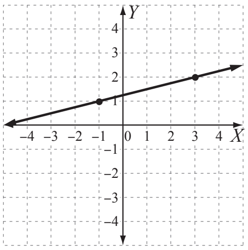
f)



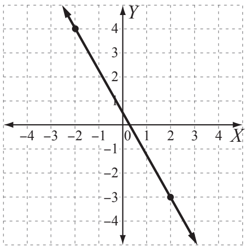
21. [Coordinate Geometry] (cont.)

Skill 21.8 a) –4, b) $\frac{1}{3}$, c) $-\frac{2}{3}$, d) $\frac{1}{2}$

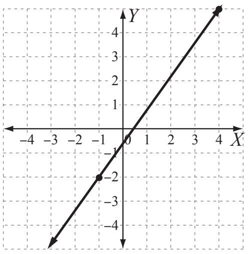
e) $\frac{1}{4}$



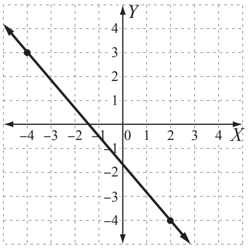
f) $-\frac{7}{4}$



g) $\frac{7}{5}$



h) $-\frac{7}{6}$



Skill 21.9 a) $(-2,-\frac{7}{2})$ b) $(\frac{1}{2},5)$ c) $(6,\frac{9}{2})$ d) $(-\frac{3}{2},\frac{3}{2})$ e) $(-\frac{3}{2},-\frac{5}{2})$
f) (–1,1)

Skill 21.10 a) $y = -4x + 5$, b) $y = \frac{1}{5}x - 1$, c) $y = -\frac{3}{2}x + \frac{1}{2}$
d) $y = 2x - \frac{3}{2}$, e) $y = -2x - 6$, f) $y = -\frac{3}{4}x + 3$

Skill 21.11 a)

equation	gradient (m)	x-intercept	y-intercept (b)
$y = 2x - 6$	2	(3,0)	(0,–6)

b)

equation	gradient (m)	x-intercept	y-intercept (b)
$y = -x + 5$	–1	(5,0)	(0,5)

c)

equation	gradient (m)	x-intercept	y-intercept (b)
$y = \frac{1}{3}x - 2$	$\frac{1}{3}$	(6,0)	(0,–2)

d)

equation	gradient (m)	x-intercept	y-intercept (b)
$y = \frac{2}{5}x + 4$	$\frac{2}{5}$	(–10,0)	(0,4)

e)

equation	gradient (m)	x-intercept	y-intercept (b)
$y = 5x + 3$	5	$(-\frac{3}{5},0)$	(0,3)

f)

equation	gradient (m)	x-intercept	y-intercept (b)
$y = -2x + 1$	–2	$(\frac{1}{2},0)$	(0,1)

Skill 21.12 a) –1, b) 2, c) 2, d) $\frac{1}{4}$, e) $\frac{1}{2}$, f) $-\frac{1}{3}$, g) $-\frac{3}{2}$, h) $\frac{5}{3}$

Skill 21.13 a) $y = -x + 7$, b) $y = 2x + 13$, c) $y = 3x + 2$, d) $y = -\frac{1}{5}x - \frac{14}{5}$
e) $y = -x + 1$, f) $y = -x - 4$, g) $y = -\frac{5}{4}x + \frac{15}{4}$, h) $y = -\frac{3}{2}x - 1$

Skill 21.14 a)

x	–2	–1	0	1	2
y	8	2	0	2	8
(x,y)	(–2,8)	(–1,2)	(0,0)	(1,2)	(2,8)

b)

x	–2	–1	0	1	2
y	6	3	2	3	6
(x,y)	(–2,6)	(–1,3)	(0,2)	(1,3)	(2,6)

c)

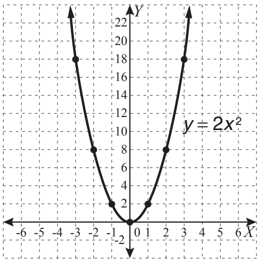
x	–2	–1	0	1	2
y	$\frac{1}{2}$	1	☒	–1	$-\frac{1}{2}$

d)

x	–2	–1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

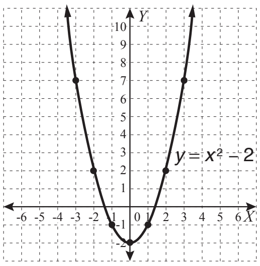
Skill 21.15 a)

x	–3	–2	–1	0	1	2	3
y	18	8	2	0	2	8	18

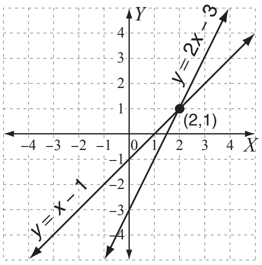


b)

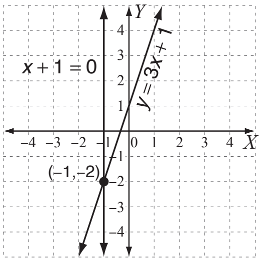
x	–3	–2	–1	0	1	2	3
y	7	2	–1	–2	–1	2	7



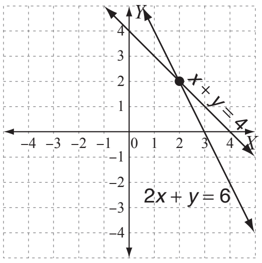
Skill 21.16 a) (2,1)



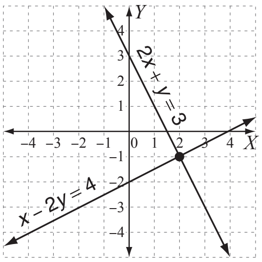
b) (–1,–2)



c) (2,2)



d) (2,–1)



Skill 21.17 a) $\sqrt{82}$, b) $\sqrt{109}$, c) $\sqrt{50}$, d) $\sqrt{2}$, e) $\sqrt{40}$, f) $\sqrt{128}$

22. [Units of Measurement / Time] page 249

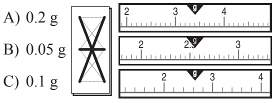
Skill 22.1 a) 4750 RPM, b) 0.5 cm, c) 52 km/h, d) 735 mmHg
e) 29 kg, f) 9 mL

Skill 22.2 a) A, b) C, c) C, d) D, e) B, f) A, g) B, h) A, i) C, j) A

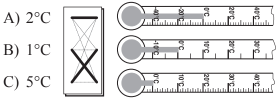
Skill 22.3 a) milli, b) centi, c) kilo, d) deci, e) micro, f) Giga, g) centi
h) milli, i) D, j) B, k) C, l) D

Skill 22.4 a) 2.9 kg, b) 70 cm, c) 22.9 cm, d) 55.5 L, e) 119.5 mm
f) 37.8°C

g)



h)



i) C, j) B

Skill 22.5 a) 340 min, b) 755 min, c) 232 min, d) 314 s
e) Thursday 2020, f) 17 h 15 min, g) 8 h 48 min
h) 5:58 am

Skill 22.6 a) 63 cm, b) 6450 mm, c) 1012 mm, d) 24 m, e) 276 cm
f) >, g) <, h) B, i) A, j) 4, k) 3.1 m, 301 cm, 3001 mm
l) 5900 cm, 590 m, 5.9 km

Skill 22.7 a) 7.5 g, b) 1 g, c) 1.3 kg, d) 80.8 kg, e) =, f) 11 000 000 kg
g) 27 200 000 mg, h) B, i) 3000 g, j) 0.71 kg
k) 2 kg, 2002 g, 0.02 tonne, l) 5.5 kg, 550 g, 55 000 mg

Skill 22.8 a) 0.05 L, b) 15 ML, c) 3075 mL, d) 603 L, e) 3 000 200 L
f) 1032 mL, g) 5.8 L, h) 5, i) >, j) <, k) 182.5 mL
l) 68 L, 680 000 mL, 0.0068 ML

Skill 22.9 a) 4400 mm^2, b) 3000 mm^2, c) >, d) 3 760 000 mm^2
e) >, f) 1325 km^2, g) =, h) 121 000 m^2, i) Indonesia
j) 17.8 m^2, k) 0.02 m^2, 2000 mm^2, 2 cm^2
l) 7 000 000 mm^2, 0.7 m^2, 700 cm^2

Skill 22.10 a) 500 L, b) 2300 cm^3, c) 24 000 L, d) 30 000 mL, e) 6 L
f) 3500 L, g) 350 000 cm^3, h) 26 m^3, i) 5000 mm^3, j) 1 m^3
k) 45 m^3, 4500 L, 45 000 mL, l) 85 cm^3, 850 mL, 8.5 L

23. [Perimeter / Area] page 259

Skill 23.1 a) 92 m, b) 4.7 m, c) 42 mm, d) 720 mm, e) 98 m, f) 50 cm
g) 3150 cm, h) 424 cm, i) 3.45 cm, j) 418 mm, k) 80 m
l) 510 cm, m) 240 m, n) 43 mm, o) $P = 7r$, p) $P = 8s$

Skill 23.2 a) 38.5 mm, b) 26 mm, c) 640 cm, d) 8.6 cm, e) 20 m
f) 34 cm

Skill 23.3 a) 31.4 mm, b) 12.56 m, c) 176 cm, d) 47.1 m, e) 7.85 m
f) 12.56 mm

Skill 23.4 a) 406.128 m, b) 14.4 cm, c) 55.7 mm, d) 106 m
e) 115.12 mm, f) 76 cm, g) $P = \pi r + 2r$ or $r(\pi + 2)$
h) $P = 4\pi k$

Skill 23.5 a) 7.03 m^2, b) 169 cm^2, c) 47.3 mm^2, d) 729 m^2
e) 93 600 m^2, f) 81 m^2, g) 80 cm, h) 108 mm

Skill 23.6 a) 45 m^2, b) 40 mm^2, c) 48 mm^2, d) 12 cm^2, e) 22, f) 15

Skill 23.7 a) 350 cm^2, b) 77 cm^2, c) 510 cm^2, d) 3 mm^2, e) 11 m^2
f) 273 mm^2

Skill 23.8 a) 137.5 cm^2, b) 20 m^2, c) 305 cm^2, d) 56 mm^2, e) 14, f) 12

Skill 23.9 a) 192 m^2, b) 160 mm^2, c) 450 cm^2, d) 72 cm^2, e) 30, f) 24

Skill 23.10 a) 237.5 m^2, b) 616 cm^2, c) 267 mm^2, d) 540 cm^2
e) 575 cm^2, f) 345 m^2, g) $A = 2a^2 + ab$ or $a(2a + b)$
h) $A = 2b^2 + 3ab$ or $b(2b + 3a)$

Skill 23.11 a) 314 mm^2, b) 78.5 m^2, c) 1386 m^2, d) 308 mm^2
e) 38.5 cm^2, f) 942 m^2

Skill 23.12 a) 283 cm^2, b) 420 m^2, c) 846 cm^2, d) 478.5 mm^2
e) 75.36 m^2, f) 367.38 m^2, g) $A = \pi r^2 - r^2$ or $r^2\ (\pi - 1)$
h) $A = r(\pi r - l)$ or $\pi r^2 - lr$, i) $A = 2r^2$, j) $A = d^2 + \frac{\pi d^2}{4}$ or
$d^2(1 + \frac{\pi}{4})$, k) $A = \frac{\pi a^2}{8} + 2ab$, l) $A = \frac{ab}{2} + \frac{\pi a^2}{8} + \frac{\pi b^2}{8}$

24. [Surface Area] page 277

kill 24.1 a) 1500 cm^2, b) 216 cm^2, c) 78 m^2, d) 352 cm^2
e) 210 mm^2, f) 1420 cm^2

Skill 24.2 a) 68 750 cm^2, b) 9.5 m^2, c) 8600 cm^2, d) 208 cm^2

Skill 24.3 a) 238 cm^2, b) 430 mm^2, c) 300 m^2, d) 168 m^2, e) 49.4 m^2
f) 432 mm^2, g) 2800 mm^2, h) 882 cm^2

Skill 24.4 a) 996 cm^2, b) 120 cm^2, c) 432 mm^2, d) 14 cm^2
e) 192 cm^2, f) 480 m^2

Skill 24.5 a) 85 cm^2, b) 33 cm^2, c) 140 000 m^2, d) 756 mm^2
e) 9216 m^2, f) 1496 m^2, g) $36\sqrt{3}$ cm^2, h) $144\sqrt{3}$ m^2

Skill 24.6 a) 43 680 cm^2, b) 263.04 m^2, c) 484 m^2, d) 328 000 cm^2
e) 880 m^2, f) 1620 cm^2, g) 780 cm^2, h) 680 cm^2
i) 344 cm^2, j) 720 m^2, k) 51 m^2, l) 205 m^2

Skill 24.7 a) 75.36 cm^2, b) 5544 m^2, c) 61 600 mm^2, d) 101.3 cm^2
e) 11 304 cm^2, f) 660 cm^2, g) 2024 cm^2, h) 628 cm^2

Skill 24.8 a) 150.72 mm^2, b) 2119.5 cm^2, c) 342.46 cm^2, d) 2728 m^2

Skill 24.9 a) $TSA = 12\pi d^2$, b) $TSA = 3\pi r^2$, c) $TSA = 7a^2$
d) $TSA = 54d^2$, e) $TSA = 78\pi x^2$, f) $TSA = 18\pi p^2$

25. [Volume] page 293

Skill 25.1 25 mm^3, b) 216 mm^3, c) 0.288 m^3, d) 375 cm^3

Skill 25.2 a) 270 mm^3, b) 242 cm^3, c) 600 m^3, d) 375 mm^3, e) 4.5 m^3
f) 2640 mm^3, g) 18 m^3, h) 1260 cm^3, i) 3960 cm^3, j) 99 m^3

Skill 25.3 a) 126 cm^3, b) 63 m^3, c) 297 mm^3, d) 360 m^3

Skill 25.4 a) 113.04 cm^3, b) 385 000 mm^3, c) 75.36 m^3
d) 4186.67 cm^3

Skill 25.5 a) $V = 6\pi d^3$, b) $V = 240x^3$, c) $V = 33k^3$, d) $V = 45\pi a^3$
e) $V = \frac{22\pi x^3}{3}$, f) $V = 12x^3 - 3\pi x^3$ or $3x^3(4 - \pi)$

Skill 25.6 a) 20 cm, b) 1500 mL, c) 113.04 mL, d) 1155 L

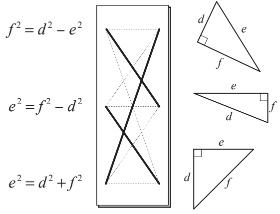
Skill 25.7 a) 27 cm^3, b) 9 mm, c) 64 mm^3, d) 125 cm^3, e) 0.5 m, f) 8
g) 50 cm, h) 27

Skill 25.8 a) 539 cm^3, b) 4504.5 m^3, c) 395.64 mm^3, d) 264 cm^3

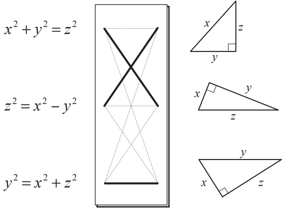
26. [Pythagoras Trigonometry] page 303

Skill 26.1 a) 26, b) 21, c) 15, d) 40, e) 2.5, f) 0.4, g) 25, h) 40
i) 15

Skill 26.2 a) *d*, b) A & C
c)



d)



Skill 26.3 a) 20, b) 8, c) 12, d) 15, e) 7, f) 15, g) 26, h) 30, i) 25

Skill 26.4 a) 150, b) 34, c) 50, d) 35

Skill 26.5 a) 20, b) 10, c) 16, d) 40

Skill 26.6 a) yes, b) no, c) 650 m, d) 120 cm, e) 250 cm, f) 2 m

Skill 26.7 a) 102 cm, b) 56 mm, c) 48 cm, d) 48 m

Skill 26.8 a) 75, b) 26, c) 12, d) 30

Skill 26.9 a) $\sqrt{18}$ m, b) $\sqrt{16\,200}$ cm, c) $\sqrt{2}$ m, d) $\sqrt{7200}$ cm, e) $\sqrt{128}$ m
f) $\sqrt{12\,800}$ cm

Skill 26.10 a) $\sqrt{80}$, b) $\sqrt{68}$, c) $\sqrt{52}$, d) $\sqrt{41}$, e) $\sqrt{72}$, f) $\sqrt{45}$

Skill 26.11 a) 168 m^2, b) 180 cm^2, c) 72 m^2, d) 540 m^2

Skill 26.12 a) B, b) B, c) A, d) C, e) C, f) A

Skill 26.13 a) $\frac{5}{13}$, b) $\frac{4}{3}$ or 1.33, c) $\frac{15}{17}$, d) $\frac{4}{5}$ or 0.8

Skill 26.14 a) 2.67 m, b) 800 cm, c) 3.53 m, d) 100.1 cm, e) 2.05 m
f) 2.35 km

Skill 26.15 a) $\frac{24}{7}$, b) $\frac{3}{5}$, c) $\frac{12}{13}$, d) $\frac{4}{5}$, d) $\frac{4}{5}$, e) $\frac{8}{17}$, f) $\frac{5}{12}$, g) $\frac{40}{9}$, h) $\frac{8}{17}$
i) $\frac{4}{5}$

27. [Angles] page 323

Skill 27.1 a) B, b) B, c) C, d) A, e) B, f) C, g) A, h) C

Skill 27.2 a) 42º, b) 36º, c) 44º, d) 30º, e) $x° = 55°$, $y° = 84°$
f) $x° = 24°$, $y° = 156°$

Skill 27.3 a) 26º, b) 78º, c) $x° = 69°$, $y° = 111°$, d) $x° = 145°$, $y° = 35°$
e) $x° = 60°$, $y° = 30°$, f) $x° = 36°$, $y° = 115°$

Skill 27.4 a) 90º, b) 55º, c) 63º, d) 84º, e) 30º, f) 54º

Skill 27.5 a) 40º, b) 160º, c) 25º, d) 40º, e) 120º, f) 35º

Skill 27.6 a) 55º, b) 110º, c) 130º, d) 75º, e) 115º, f) 60º

Skill 27.7 a) 58º, b) 72º, c) 55º, d) 65º, e) 36º, f) 39º

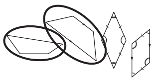
Skill 27.8 a) 40º, b) 40º, c) 45º, d) 30º, e) 50º, f) 32º

Skill 27.9 a) 18º, b) 28º, c) 70º, d) 52º, e) 75º, f) 50º, g) 75º
h) $x° = 70°$, $y° = 76°$, i) 27º, j) 48º

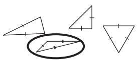
28. [Geometric Reasoning] page 333

Skill 28.1 a)

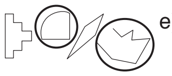
b)



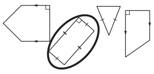
c)



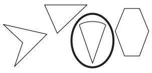
d)



e)



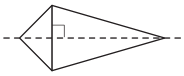
f)



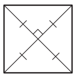
Skill 28.2 a) B&C, b) A&C, c) A&B, d) B&C, e) A&C, f) A&B, g) B&C h) A&C

Skill 28.3 a) B, b) D, c) C, d) C

e) kite

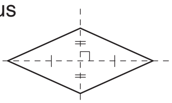


f) square

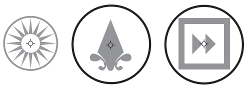


g) parallelogram

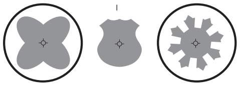
h) rhombus



Skill 28.4 a)



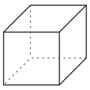
b)



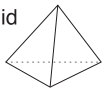
c) 5, d) 3, e) 4, f) 6

Skill 28.5 a) 4, b) 5, c) 8, d) 6, e) 6, f) 8, g) 6, h) 5

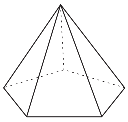
e) cube



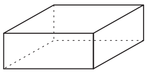
f) triangular pyramid (or tetrahedron)



g) pentagonal pyramid

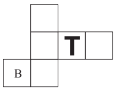


h) rectangular prism

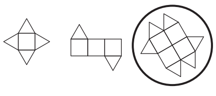


Skill 28.6 a) 12 = 7 + 7 − 2, b) 6 = 4 + 4 − 2, c) 15 = 10 + 7 − 2
d) 12 = 8 + 6 − 2, e) 18 = 12 + 8 − 2, f) 24 = 16 + 10 − 2

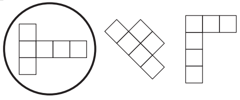
Skill 28.7 a) hexagonal pyramid, b) triangular prism
c) pentagonal prism, d)



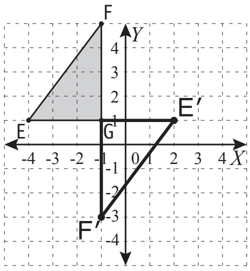
e)



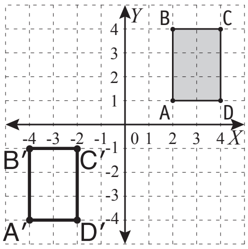
f)



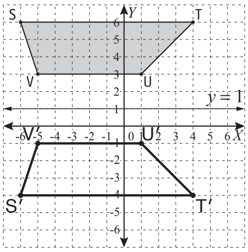
Skill 28.8 a)



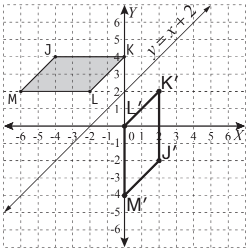
b)



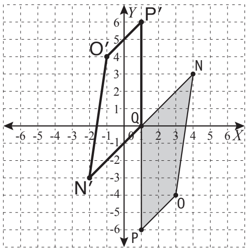
c)



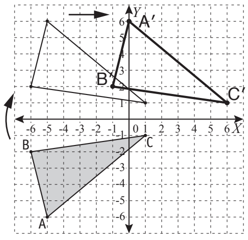
d)



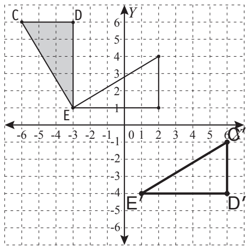
e)



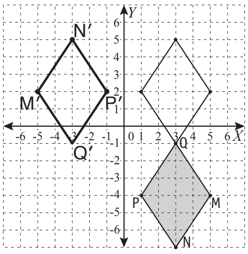
f)



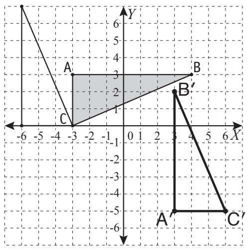
g)



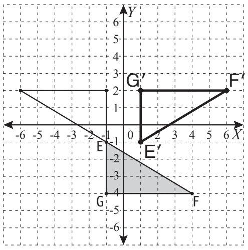
h)



i)

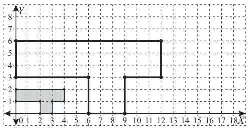


j)

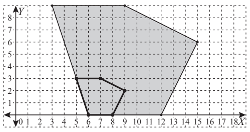


Skill 28.9 a) $\frac{1}{4}$, b) $\frac{1}{2}$

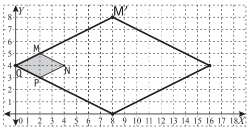
c)



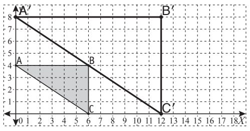
d)



e)



f) similar



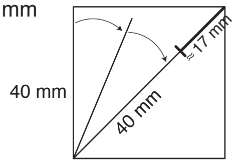
Skill 28.10 a) SAS, b) AAS, c) RHS, d) SAS, e) RHS, f) SSS

Skill 28.11 a) ∠C = ∠G, ∠D = ∠F, ∠E = ∠H, b) $\frac{AB}{GH} = \frac{BC}{GI} = \frac{AC}{HI}$
c) $x = 18$, d) $x = 12.8$, e) $x = 9$, f) $x = 16$

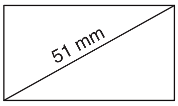
Skill 28.12 a) NQ = QP, NO = OP, QO = QO, SSS
b) AC = AB, AD = AD, ∠ADC = ∠ADB, RHS
c) DF = FG, EF = FH, ∠DFE = ∠GFH, SAS
d) ST = TU, RT = RT, ∠RTS = ∠RTU, SAS

Skill 28.13 a) 3, b) 6, c) $x = 5$ $y = 3$, d) 55°, e) 13, f) 25°
g) $x° = 45°$ $y° = 35°$, h) $x = 6$ $y = 8$

Skill 28.14 a) 57 mm (actual length = 56.57 mm accept 50 mm to 60 mm)



b) 51 mm (actual length = 51.48 mm accept 50 mm to 55 mm)



c) 1 : 60 000, d) 1 : 3000

Skill 28.15 a) 27 cm, b) 19 cm, c) 24 cm, d) 26 cm

29. [Statistics] page 349

Skill 29.1 a) Ryder cup - golf, b) 1998, c) 3, d) Falls Creek, e) 1999
f) London, g) Italy & New Zealand, h) 10%, i) 25%
j) Nigeria

Skill 29.2 a) Rutherglen, b) China, c) 70-79, d) 16

Skill 29.3 a) 1993, b) 1982-1983, c) 1900, d) C, e) 30 km, f) 5 min
g) 1985 - 1990, h) $26, i) drivers, j) employee

Skill 29.4 a) Italy, b) B, c) B, d) Dane Swann, e) Northern Territory
f) USA

Skill 29.5 a) 4, b) 4, c) 3, d) 4.5, e) 12, f) 3, g) 3, h) 10.5

Skill 29.6 a) 21, b) 2, c) 18, d) 98, e) 4, f) 12, g) mode = 4, range = 5
h) mode = 32, range = 6, i) mode = 3.6, range = 2.1
j) mode = 19, range = 15

Skill 29.7 a) 16, b) 6, c) 10, d) 2.5, e) 5, f) 10, g) 8, h) 2, i) 16, j) 9.5

Skill 29.8 a) B, b) A, c) B, d) C

Skill 29.9 a) 11, b) 5, c) 5, d) B

Skill 29.10 a)

stem	leaves
0	3
1	2 6 7
2	0 1
3	2 5 5 7 9
4	3 8

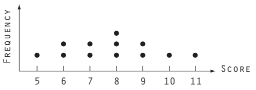
b)

stem	leaves
20	2 4 7
21	0
22	3 3 6 8 9
23	0 1 2 6

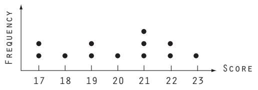
c) median = 46, range = 65
d) median = 103.5, mode = 102
e) male
f) 10, g) 8, h) 9.5

male		female
8 7	2	2 3 6 7 9 9
9 6 3	3	4 6 8 8
9 8 7 7 0	4	1 3 4 4 5 8 9
5 2 0	5	0 6 9
	6	1

Skill 29.11 a) 4, b) 8 h
c) 8



d) 20.5



e) 64.5, f) 1

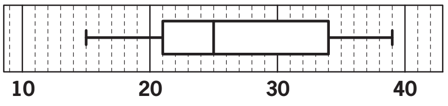
Skill 29.12 a) 5, b) 10, c) median = 17.5, range = 4
d) mean = 3.5, mode = 2, e) median = 2, range = 4
f) mean = 1.65, mode = 1

Skill 29.13 a) median = 30, LQ = 25, b) LQ = 19, UQ = 26
c) median = 55, UQ = 58, d) median = 93, IQR = 28

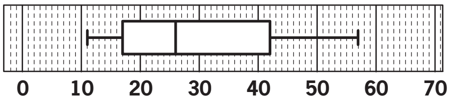
Skill 29.14 a) B, b) 3, c) A, d) C

Skill 29.15 a) range = 6, median = 5 b) range = 9, median = 2
c) range = 4, mean = 2.75 d) median = 70, mean = 74

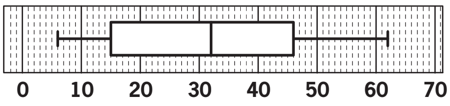
Skill 29.16 a)



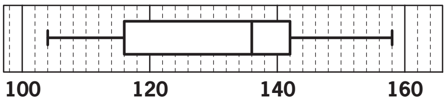
b)



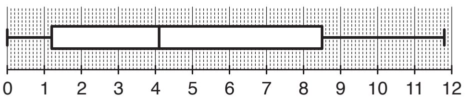
c)



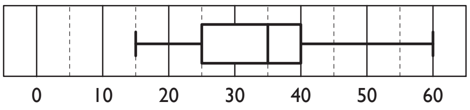
d)



e)



f)



Skill 29.17 a) median = 21, LQ = 18.5, b) median = 2, LQ = 1
c) median = 69, UQ = 71, d) median = 10.5, LQ = 9
e) IQR = 18, f) IQR = 8

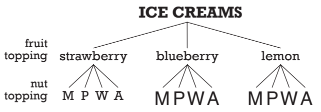
30. [Probability] page 369

Skill 30.1 a) E, b) C, c) A, d) F, e) C, f) E, g) F, h) B

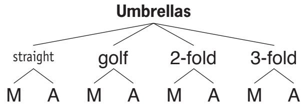
Skill 30.2 a) $\frac{1}{2}$ or 0.5, b) $\frac{1}{13}$, c) $\frac{5}{8}$ or 0.625, d) $\frac{3}{8}$ or 0.375, e) $\frac{1}{16}$, f) $\frac{1}{8}$
g) $\frac{2}{3}$, h) $\frac{1}{2}$ or 0.5, i) $\frac{1}{5}$ or 0.2, j) $\frac{1}{7}$, k) $\frac{3}{8}$ or 0.375, l) $\frac{4}{15}$

Skill 30.3 a) $\frac{13}{20}$, b) $\frac{2}{5}$, c) 0.63, d) $\frac{3}{4}$, e) 0.52, f) $\frac{5}{9}$

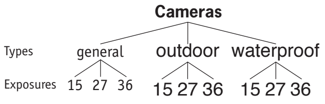
Skill 30.4 a) $\frac{1}{3}$



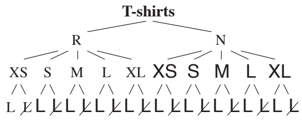
b) $\frac{1}{2}$



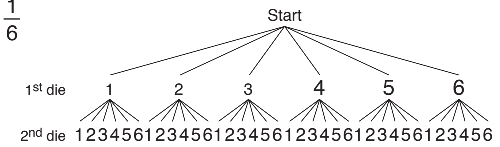
c) $\frac{1}{3}$



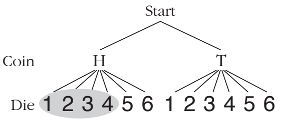
d) $\frac{1}{4}$



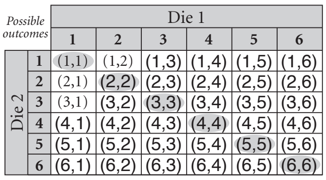
Skill 30.5 a) $\frac{1}{6}$



b) $\frac{1}{3}$

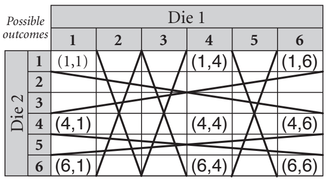


c) $\frac{1}{6}$



Possible outcomes		Die 1					
		1	2	3	4	5	6
Die 2	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

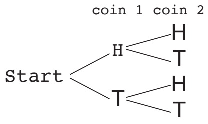
d) $\frac{1}{4}$



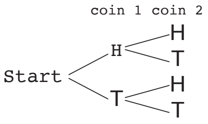
Possible outcomes		Die 1					
		1	2	3	4	5	6
Die 2	1	(1,1)			(1,4)		(1,6)
	2						
	3						
	4	(4,1)			(4,4)		(4,6)
	5						
	6	(6,1)			(6,4)		(6,6)

30. [Probability] (cont.)

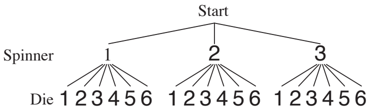
e) $\frac{3}{4}$



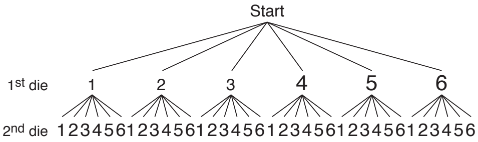
f) $\frac{1}{4}$



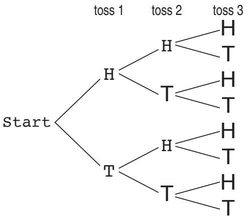
g) $\frac{1}{3}$



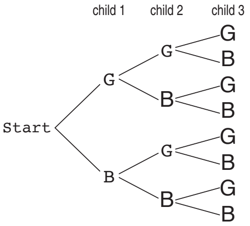
h) $\frac{1}{4}$



i) $\frac{7}{8}$



j) $\frac{1}{4}$



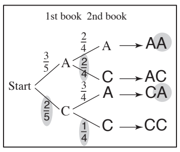
Skill 30.6 a) $\frac{2}{3}$, b) $\frac{5}{12}$, c) $\frac{11}{40}$ or 0.275, d) $\frac{3}{4}$ or 0.75

Skill 30.7 a) $\frac{11}{16}$, b) $\frac{7}{10}$, c) $\frac{4}{13}$, d) $\frac{1}{2}$

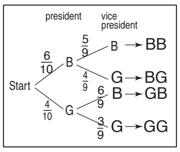
Skill 30.8 a) 4, b) 16, c) 25, d) 5, e) 225, f) 350

Skill 30.9 a) 0.216, b) $\frac{1}{4}$, c) $\frac{3}{10}$, d) $\frac{21}{100}$, e) $\frac{1}{63}$, f) $\frac{9}{625}$

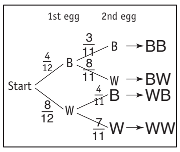
Skill 30.10 a) $\frac{3}{5}$,



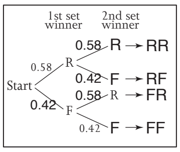
b) $\frac{1}{3}$



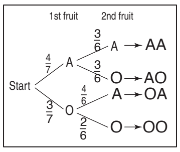
c) $\frac{14}{33}$,



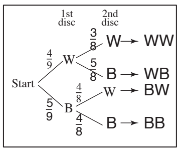
d) 0.34



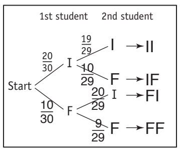
e) $\frac{1}{7}$,



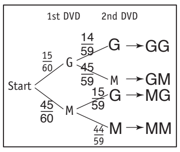
f) $\frac{5}{18}$



g) $\frac{40}{87}$,



h) $\frac{177}{236}$



Skill 30.11 a) $\frac{1}{2}$, b) $\frac{1}{7}$, c) $\frac{7}{22}$, d) $\frac{1}{2}$

Skill 30.12 a) $\frac{49}{176}$

	Women	Men	Total
Yes	245	180	425
No	335	120	455
Total	580	300	880

b) $\frac{17}{135}$

	5-door	3-door	**Total**
Manual	28	17	45
Automatic	37	53	90
Total	65	70	135

c) $\frac{37}{100}$

	Boys	Girls	**Total**
Red hair	16	9	25
Blonde hair	105	210	315
Brown hair	75	85	160
Total	196	304	500

d) $\frac{3}{26}$

	Black	Red	**Total**
Ace	2	2	4
Number	18	18	36
Court card	6	6	12
Total	26	26	52